Exercises for Lecture Course on Numerical Optimization (NUMOPT)
Albert-Ludwigs-Universität Freiburg – Winter Term 2015-2016

Exercise 1: Introduction to YALMIP

(to be completed during exercise session on Oct 28, 2015 or sent by email to dimitris.kouzoupis@imtek.uni-freiburg.de before Oct 30, 2015)

Prof. Dr. Moritz Diehl and Dimitris Kouzoupis

The aim of this exercise is to learn how to use MATLAB and how to formulate and solve an optimization problem using YALMIP, namely the minimization of the potential energy of a chain of masses connected by springs.

Prepare your laptop

1. **MATLAB:** The exercises of this course are exclusively done in MATLAB. Instructions on how to get a free student license from the online software shop can be found here:

   `https://www.rz.uni-freiburg.de/services-en/beschaffung-em/software-en/matlab-license?set_language=en`

   If you are unfamiliar with MATLAB, here are some useful tutorials:

   - `http://www.maths.dundee.ac.uk/ftp/na-reports/MatlabNotes.pdf`
   - `http://www.math.mtu.edu/~msgocken/intro/intro.html`

2. **YALMIP:** For this and future exercises we need to install the YALMIP toolbox (Yet Another Linear Matrix Inequality Parser). YALMIP is a modeling language for advanced modeling and solution of convex and nonconvex optimization problems. Further information can be found at:

   `http://users.isy.liu.se/johanl/yalmip/`

   Note: YALMIP is only the modeling language, to solve the problems, it needs some underlying solvers installed such as quadprog, CPLEX, Sedumi, SDPT3, etc.

   To download and install YALMIP follow the instructions bellow:

   (a) Download and unzip the toolbox from

   `http://www.control.isy.liu.se/~johanl/YALMIP.zip`

   (b) Move the folder called yalmip to the default MATLAB directory or any directory of your choice.

   (c) Start MATLAB and go to the directory that you chose in Step 2.

   (d) Add the path of YALMIP to the MATLAB path, by typing

   ```
   >> addpath(genpath('yalmip'))
   ```

   in the command line of MATLAB.

   (e) Test your YALMIP by invoking

   ```
   >> yalmiptest
   ```

Exercise Tasks

3. **A tutorial example**: Lets first look at the following unconstrained optimization problem

$$\min_x x^2 - 2x$$

   (a) Derive first the optimal value for x on paper. Then, download the code provided for exercise 1 from the course homepage and run `ex1_toy_example.m` in MATLAB to solve the same problem with YALMIP. Is the result the same?

   $x =$ (1 point)

   (b) Have a closer look at the template and adapt it to include the inequality constraint $x \geq 1.5$. What is the new result? Is it what you would intuitively expect?

   $x =$ (1 point)

   (c) Now modify the template to solve the two-dimensional problem:

$$\min_{x,y} x^2 - 2x + y^2 + y \tag{1a}$$

$$\text{s.t. } x \geq 1.5 \tag{1b}$$

$$x + y \geq 0 \tag{1c}$$

   Which are the optimal values for x and y returned by YALMIP?

   $x =$ $y =$ (2 points)

4. **Equilibrium position for a hanging chain:** We want to model a chain attached to two supports and hanging in between. Let us discretize it with N mass points connected by $N - 1$ springs. Each mass i has position $(y_i, z_i),\ i = 1, \ldots, N$. The equilibrium point of the system minimizes the potential energy. The potential energy of each spring is:

$$V_{\text{el}}^i = \frac{1}{2} D \left((y_i - y_{i+1})^2 + (z_i - z_{i+1})^2 \right).$$

The gravitational potential energy of each mass is:

$$V_{\text{g}}^i = m\, g_0\, z_i.$$

The total potential energy is thus given by:

$$V_{\text{chain}}(y, z) = \frac{1}{2} \sum_{i=1}^{N-1} D \left((y_i - y_{i+1})^2 + (z_i - z_{i+1})^2 \right) + g_0 \sum_{i=1}^{N} m\, z_i,$$

where $y = [y_1, \cdots, y_N]^T$ and $z = [z_1, \cdots, z_N]^T$. We are interested in solving the optimization problem:

$$\begin{array}{ll} \underset{y,z}{\text{minimize}} & V_{\text{chain}}(y, z) \\ \text{subject to} & (y_1, z_1) \;=\; (-2, 1) \\ & (y_N, z_N) = \;(2, 1) \end{array}$$

   (a) Formulate the problem using $N = 40,\ m = 4/N$ kg, $D = \frac{70}{40} N$ N/m, $g_0 = 9.81$ m/s^2 with the first and last mass point fixed to $(-2, 1)$ and $(2, 1)$, respectively (you can start from the template code `ex1_hanging_chain.m`). Solve the problem using `quadprog` from YALMIP and interpret the results.

   (4 points)

   (b) Introduce ground constraints: $z_i \geq 0.5$ and $z_i - 0.1\, y_i \geq 0.5$. Solve the resulting Quadratic Program (QP) and plot the result. Compare the result with the previous one.

   (2 points)

This sheet gives in total 10 points.