Modelling and System Identification – Microexam 3

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Fach: Studiengang: Bachelor ☐ Master ☐ Lehramt ☐ Sonstiges ☐

Please fill in your name above and tick exactly one box for the right answer of each question below.

1. Regard the discrete time LTI system $y(k+1) = \theta y(k) + u(k) + \epsilon(k)$ with scalar input u, output y and noise ϵ. Which of the following shorthands describes this model best?

(a) ☐ ARX-IIR	(b) ☐ ARX-FIR	(c) ☐ ARMA-IIR	(d) ☐ ARMA-FIR

2. Given measurement sequences $u(k)$ and $y(k)$ we try to identify a model by solving the following optimization problem: $\min_\theta \sum_{k=3}^{N} |y(k) - \theta_1 u(k-1) - \theta_2 u(k-2)|^4$. What model assumptions do we make?

(a) ☐ FIR model with Gaussian equation errors	(b) ☐ IIR model with Gaussian equation errors
(c) ☐ FIR model with non-Gaussian equation errors	(d) ☐ IIR model with non-Gaussian equation errors

3. Given measurement sequences $u(k)$ and $y(k)$ we try to identify a model by solving the following optimization problem: $\min_\theta \sum_{k=3}^{N} |y(k) - \theta_1 y(k-1) - \theta_2 u(k-2)|^2$. What model assumptions do we make?

(a) ☐ FIR model with Gaussian equation errors	(b) ☐ IIR model with Gaussian equation errors
(c) ☐ FIR model with non-Gaussian equation errors	(d) ☐ IIR model with non-Gaussian equation errors

4. Given measurement sequences $u(k)$ and $y(k)$ we try to identify a model by solving the following optimization problem: $\min_{\theta,\tilde{y}} \frac{1}{\sigma_1^2} \sum_{k=3}^{N} |\tilde{y}(k) - \theta_1 \tilde{y}(k-1) - \theta_2 u(k-2)|^2 + \frac{1}{\sigma_2^2} \sum_{k=1}^{N} (y(k) - \tilde{y}(k))^2$. What model assumptions do we make?

(a) ☐ Gaussian input and output noise	(b) ☐ non-Gaussian input and output noise
(c) ☐ Gaussian output noise and equation errors	(d) ☐ Gaussian input noise and equation errors

5. Given a one-step ahead prediction model $y(k) = \theta_1 y(k-1) + \theta_2 u(k-2)^2 + \epsilon(k)$ with unknown parameter vector $\theta = (\theta_1, \theta_2)^\top$, and assuming i.i.d. Gaussian noise $\epsilon(k)$ with zero mean, and given a sequence of N scalar input and output measurements $u(1), \ldots, u(N)$ and $y(1), \ldots, y(N)$, we want to compute the maximum likelihood estimate $\hat{\theta}$ by minimizing the function $f(\theta) = \|y_N - \Phi_N \theta\|_2^2$. If $y_N = (y(3), \ldots, y(N))^T$, how do we need to choose the matrix $\Phi_N \in \mathbb{R}^{(N-2)\times 2}$?

(a) ☐ $\begin{bmatrix} y(1) & u(2) \\ \vdots & \vdots \\ y(N-2) & u(N-1) \end{bmatrix}$	(b) ☐ $\begin{bmatrix} y(1) & u(2)^2 \\ \vdots & \vdots \\ y(N-2) & u(N-1)^2 \end{bmatrix}$	(c) ☐ $\begin{bmatrix} y(2) & u(1)^2 \\ \vdots & \vdots \\ y(N-1) & u(N-2)^2 \end{bmatrix}$	(d) ☐ $\begin{bmatrix} y(2) & 1 \\ \vdots & \vdots \\ y(N-1) & 1 \end{bmatrix}$

6. What quantity of a continuous time transfer function $G(s)$ shows the Bode amplitude diagram in doubly logarithmic scale?

(a) ☐ $\lvert G(e^{j\omega})\rvert$	(b) ☐ $\arg G(j\omega)$	(c) ☐ $G(j\omega)$	(d) ☐ $\lvert G(j\omega)\rvert$

7. Which slope has the Bode amplitude diagram of $G(s) = \frac{1}{1+s+s^2}$ for high frequencies?

(a) ☐ 20 dB/decade	(b) ☐ 0 dB/decade	(c) ☐ -20 dB/decade	(d) ☐ -40 dB/decade

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8. At which angular frequency ω is the resonance peak of the Bode amplitude diagram of the oscillator $G(s) = \frac{1}{c+s^2}$?

(a) ☐ $\omega = \frac{2\pi}{c}$	(b) ☐ $\omega = \frac{2\pi}{\sqrt{c}}$	(c) ☐ $\omega = \sqrt{c}$	(d) ☐ $\omega = \frac{c}{2}$

9. Regard a periodic signal with period T that is sampled with sampling time Δt (with T a multiple of Δt). How many discrete time samples N are in one period of the discretized signal ? (Tip: this is easy)

(a) ☐ $\frac{\Delta t}{T}$	(b) ☐ $\frac{T}{\Delta t}$	(c) ☐ $\frac{2\pi T}{\Delta t}$	(d) ☐ $\frac{T}{2\Delta t}$

10. Regard a periodic signal with period T that is sampled with sampling time Δt (with T a multiple of Δt). How many different frequencies are contained in the discretized signal ?

(a) ☐ $\frac{\Delta t}{T}$	(b) ☐ $\frac{T}{\Delta t}$	(c) ☐ $\frac{2\pi T}{\Delta t}$	(d) ☐ $\frac{T}{2\Delta t}$

11. Regard a continuous time signal that contains a significant contribution of a given frequency f_0. You want to sample this signal with a sampling time Δt on a window of length T (with T a multiple of Δt). Which condition helps to avoid **aliasing errors** in the frequency f_0 ?

(a) ☐ $\Delta t > \frac{2\pi}{f_0}$	(b) ☐ $\Delta t < \frac{1}{2f_0}$	(c) ☐ $T^2 f_0 = \Delta t$	(d) ☐ $T f_0$ integer

12. Regard the same situation as in the previous question. Which condition helps to avoid **leakage errors** in the frequency f_0 ?

(a) ☐ $\Delta t > \frac{2\pi}{f_0}$	(b) ☐ $\Delta t < \frac{1}{2f_0}$	(c) ☐ $T^2 f_0 = \Delta t$	(d) ☐ $T f_0$ integer

13. A system is excited with a periodic excitation signal $u(t)$ of period T that is for $t \in [0, T/2]$ given by $u(t) = 10$ and for $t \in [T/2, T]$ by $u(t) = -10$. What is the **crest factor** of this signal?

(a) ☐ 10^{-2}	(b) ☐ 1	(c) ☐ 10	(d) ☐ 10^2

14. When working with periodic multisine excitations, for what reason does one usually like to work with input signals that have a small crest factor?

(a) ☐ leakage errors	(b) ☐ aliasing errors
(c) ☐ frequency limitations	(d) ☐ amplitude limitations

15. You want to identify the transfer function $G(j\omega)$ of an LTI system in the frequency band $\omega \in [\omega_{\min}, \omega_{\max}]$ with periodic multisine excitations. You choose a period length T that is an integer multiple of the sampling time Δt. Which other conditions should Δt and T satisfy?

(a) ☐ $\Delta t < \frac{2\pi}{\omega_{\max}}, \quad T < \frac{\pi}{\omega_{\min}}$	(b) ☐ $\Delta t < \frac{\pi}{\omega_{\max}}, \quad T < \frac{2\pi}{\omega_{\min}}$
(c) ☐ $\Delta t < \frac{\pi}{\omega_{\max}}, \quad T > \frac{2\pi}{\omega_{\min}}$	(d) ☐ $\Delta t < \frac{2\pi}{\omega_{\max}}, \quad T > \frac{\pi}{\omega_{\min}}$

16. Regard for some fixed large integer N the Discrete Fourier Transform (DFT) of a real-valued signal $u(0), \ldots, u(N-1)$ with $u(t) = \sin(\frac{10\pi}{N} t)$. Most values of the DFT $U(0), \ldots, U(N-1)$ are zero, but some are non-zero. Which?

(a) ☐ $U(5), U(10)$	(b) ☐ $U(5), U(N-5)$	(c) ☐ $U(10), U(N-10)$	(d) ☐ $U(10)$

17. You measure a signal where the signal-to-noise-ratio (SNR) at a certain frequency f_0 is given by 20 dB. How accurately can you estimate the amplitude of this frequency component (approximately)?

(a) ☐ 0.1 %	(b) ☐ 1 %	(c) ☐ 10 %	(d) ☐ 50 %

18. You identify an LTI system with periodic multisine excitations, where each window has length T and the total duration of your experiment is MT with a large integer M. Which procedure should you **not** follow to identify the transfer function $\hat{G}(j\omega_k)$ at a given frequency $\omega_k = \frac{2\pi k}{T}$?

(a) ☐ compute the DFTs of each window, then average the DFTs, then estimate the transfer function	(b) ☐ compute the DFTs and estimate the transfer function on each window, then average the M estimates
(c) ☐ average the M windows, then compute the DFT, then estimate the transfer function	(d) ☐ ensure your input signal contains sufficient power in the frequency ω_k

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