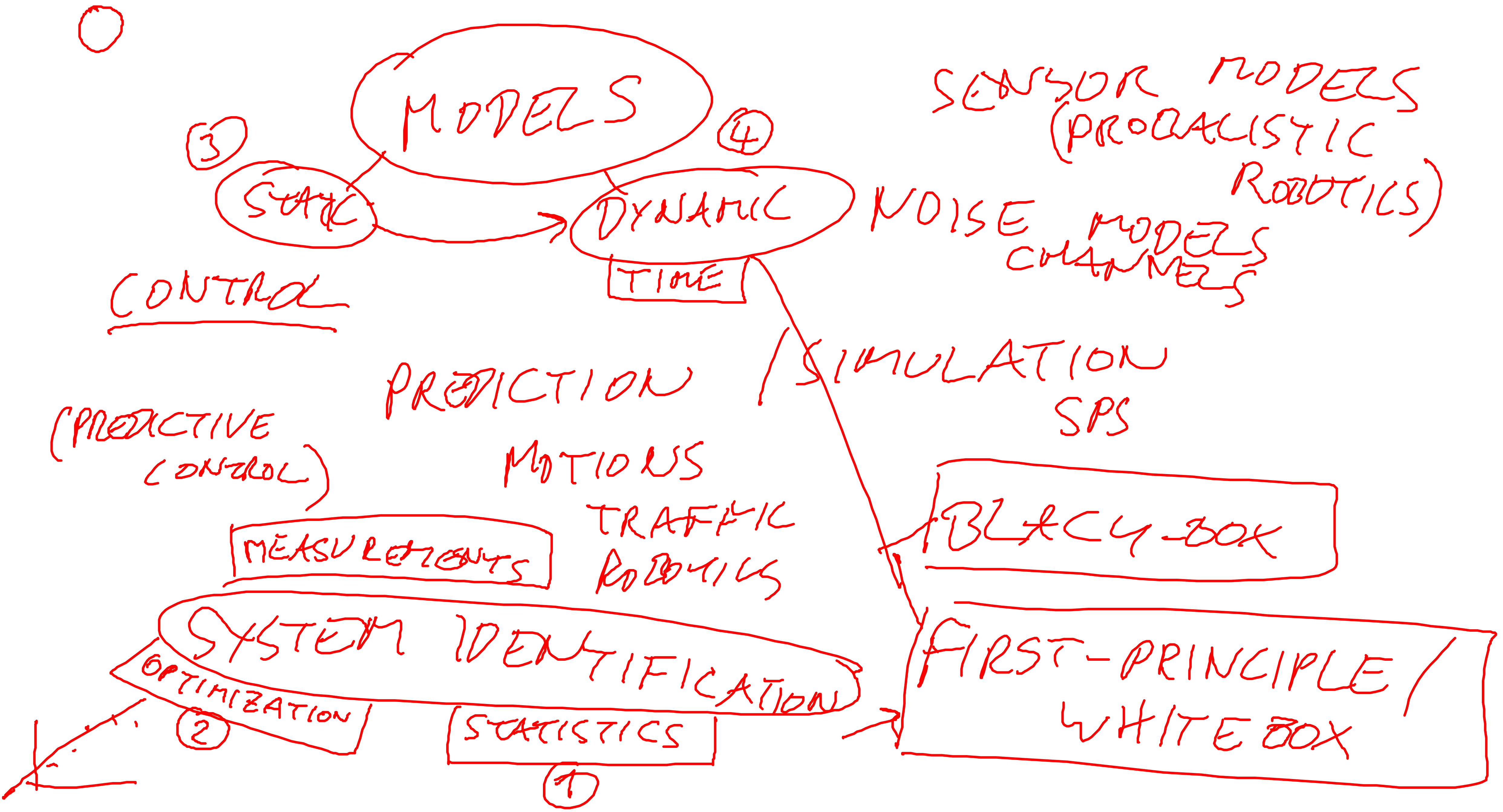
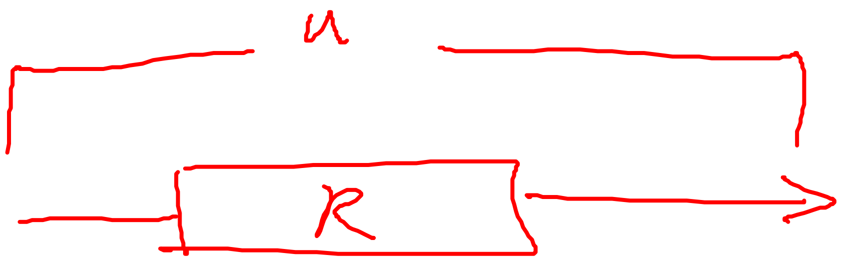
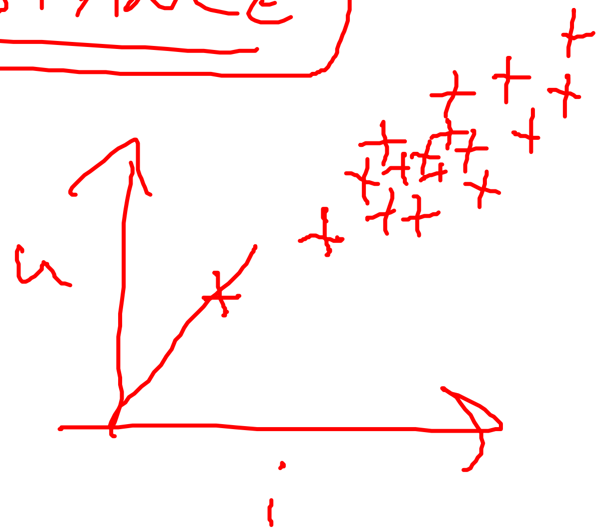

MODELS
3
STATIC
4
DYNAMIC
TIME
CONTROL
SENSOR MODELS
(PROBALISTIC ROBOTICS)
NOISE MODELS
CHANNELS
PREDICTION
SIMULATION
SPS
(PREDICTIVE CONTROL)
MOTIONS
TRAFFIC
ROBOTICS
MEASUREMENTS
BLACK-BOX
SYSTEM IDENTIFICATION
OPTIMIZATION
2
STATISTICS
1
FIRST-PRINCIPLE /
WHITE BOX

STATIC ESTIMATION OF RESISTANCE





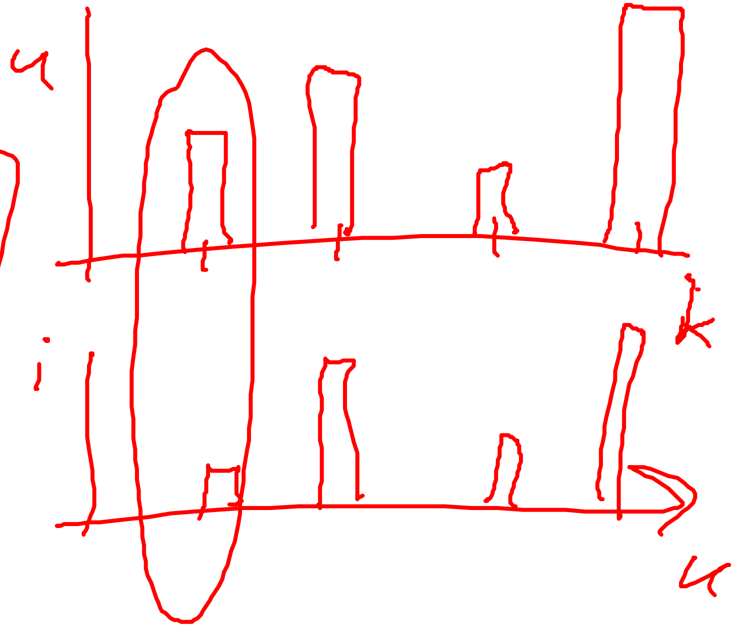
$u = R \cdot i$ — MODEL

$R(k) = \frac{u(k)}{i(k)}$

DATA

$u(1),\ u(2),\ u(3),\ \ldots$

$i(1),\ i(2),\ i(3),\ \ldots$



HOW TO ESTIMATE R ? $\hat{R}(N)$

PARAMETER → R

SIMPLE APPROACH

$$\hat{R}_{SA}(N) = \frac{1}{N} \sum_{k=1}^{N} \frac{u(k)}{i(k)}$$

(THINK)

MATLAB

NO EXERCISE THIS WEEK!

TUTORS:

TU	10-12
THU	12-14

VOME

ROBIN VERSCHUEREN

FABIAN GIRRBACH

JESUS LAGO GARCIA

OTHER ESTIMATION APPROACHES?

DATA / MEASUREMENTS: $u(1), \ldots, u(N)$

$i(1), \ldots, i(N)$

$$Ri = u$$

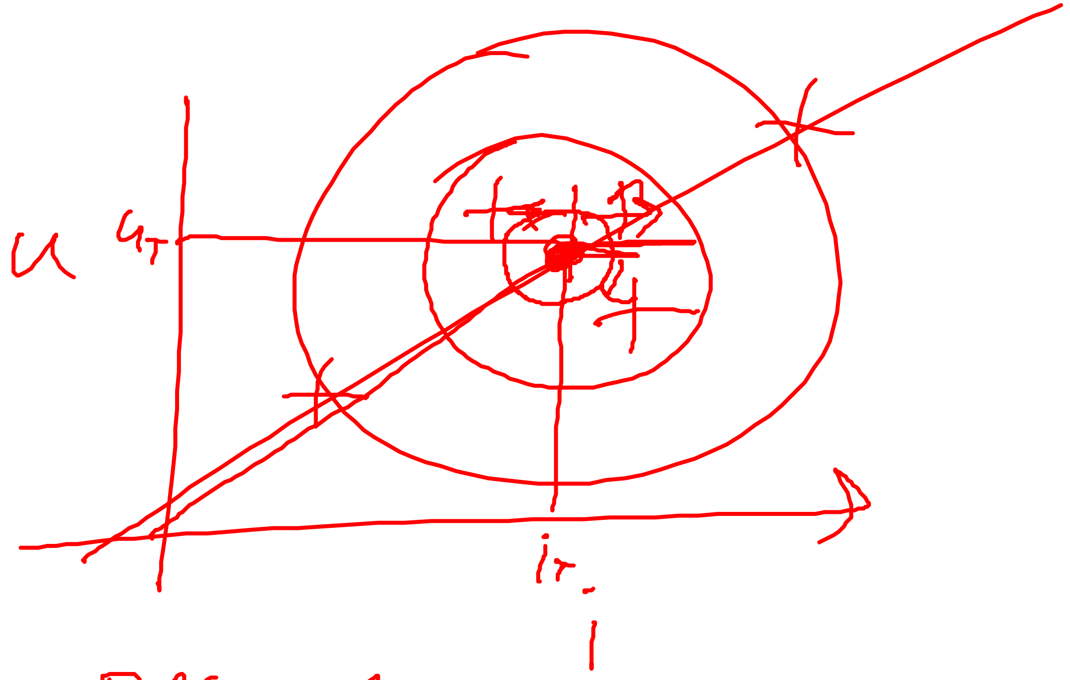
(A) $\hat{R}$ = MEDIAN OF $\frac{u(k)}{i(k)}$

(B) $\hat{R}_{EV} = \frac{u_T}{i_T}$ WITH u_T, i_T =

$$\arg\min_{u,i} \sum_{k=1}^{N} \left(u(k)-u\right)^2 + \left(i(k)-i\right)^2$$

$\hat{R}_{EV}$

$$u_T = \frac{1}{N}\sum_{k=1}^{N} u(k) \qquad i_T = \frac{1}{N}\sum_{k=1}^{N} i(k)$$

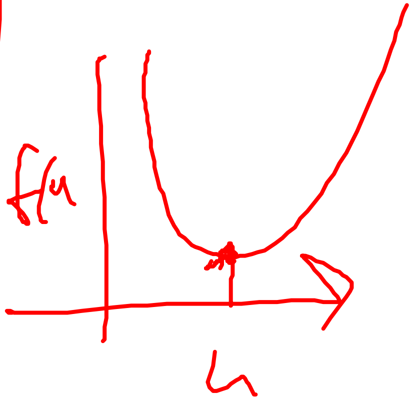


FIRST AVERAGE $u(k)$ AND $i(k)$, THEN DIVIDE

HOW TO FIND

$$(u_T, i_T) = \arg\min_{u,i} \underbrace{\sum_{k=1}^{N} (u(k) - u)^2}_{f(u)} + \underbrace{\sum_{k=1}^{N} (i(k) - i)^2}_{g(i)}$$

$$f(u) + g(i)$$



$$\arg\min_{u} f(u)$$

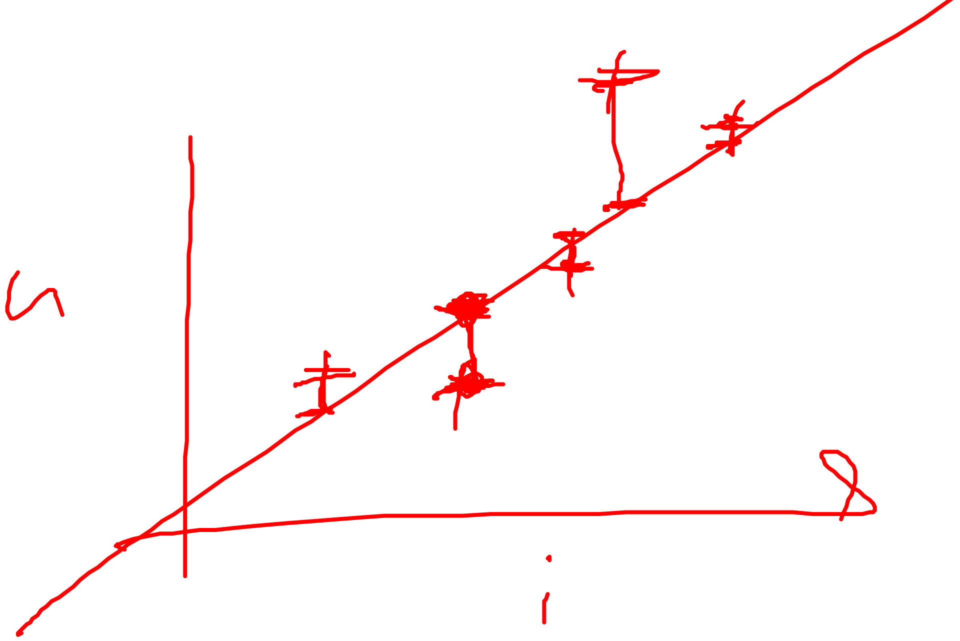
$$\frac{\partial f}{\partial u}(u) \overset{!}{=} 0$$

$$\Longleftrightarrow$$

$$u_T = \frac{\sum_{k=1}^{N} u(k)}{N}$$

$$\frac{\partial f}{\partial u} = \sum_{k=1}^{N} 2\,(u(k) - u)\cdot(-1)$$

$$= +2 \cdot N \cdot u - 2 \sum_{k=1}^{N} u(k)$$



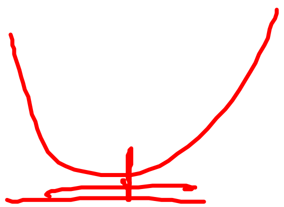
LEAST-SQUARES ESTIMATION

(c) $$\hat{R}_{LS}(N) = \arg\min_{R} \underbrace{\sum_{k=1}^{N} \left(u(k) - R\cdot i(k)\right)^2}_{f(R)}$$

$$0 \overset{!}{=} \frac{\partial f}{\partial R} = \sum_{k=1}^{N} 2\left(u(k) - R\cdot i(k)\right)\cdot(-1)\cdot\left(i(k)\right)$$

$$= R\cdot 2\cdot\sum i(k)^2 - 2\sum u(k)\cdot i(k)$$

$$\boxed{\hat{R}_{LS} = \frac{\sum u(k)\cdot i(k)}{\sum i(k)^2}}$$



$\hat{R}_{SA}$

$(\hat{R}_{MEDIAN})$

$\hat{R}_{EV}$

$\hat{R}_{LS}$

WHICH IS CORRECT ?

MATHEM. NOTATION

$$x \in \mathbb{R}^n \qquad \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \qquad \begin{bmatrix} x(1) \\ x(2) \\ \vdots \\ x(m) \end{bmatrix}$$

$$A \in \mathbb{R}^{n \times m}$$

$$A^T \cdot A \in \mathbb{R}^{m \times m} \qquad A^T \in \mathbb{R}^{m \times n}$$

$$f(x)$$

$$f : \mathbb{R}^n \rightarrow \mathbb{R}^m$$

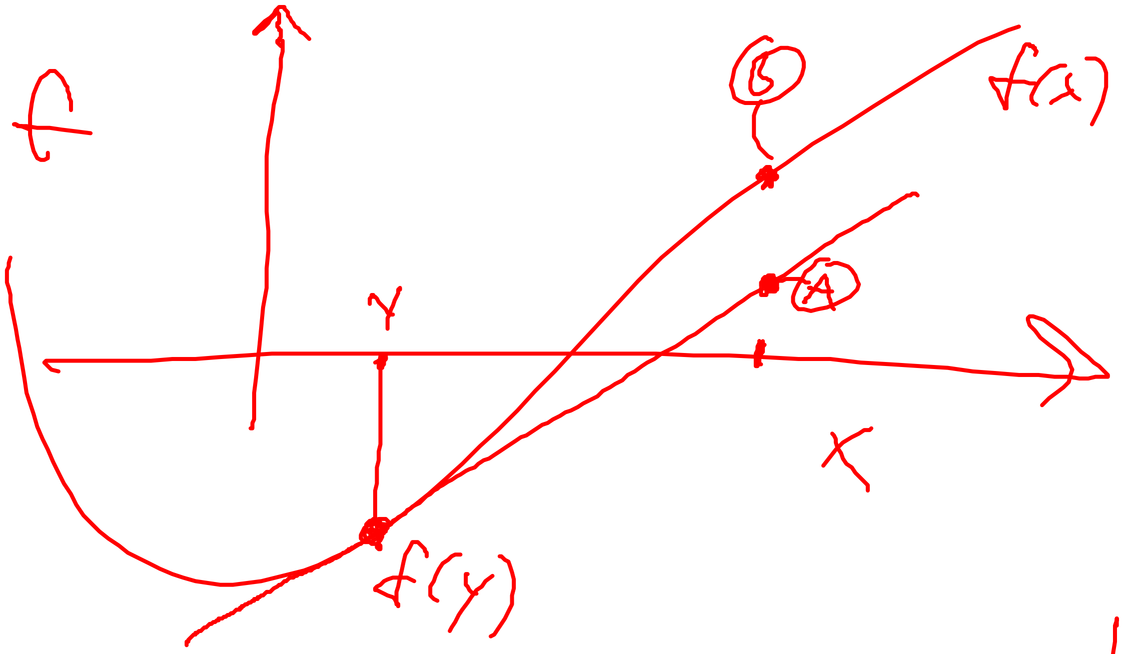
$$f(x) = \begin{bmatrix} f_1(x_1, x_2, \ldots, x_n) \\ f_2 \quad '' \\ \vdots \\ f_m(x_1, \ldots, x_n) \end{bmatrix}$$

$$\frac{\partial f}{\partial x}(x) = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \vdots & & \\ \frac{\partial f_m}{\partial x_1} & \cdots & \frac{\partial f_m}{\partial x_n} \end{bmatrix} \in \mathbb{R}^{m \times n}$$

(B)

$$f(x) \approx f(y) + \frac{\partial f}{\partial x}(y) \cdot (x - y)$$ (TAYLOR)

(A)



$$\|x\|_2^2 = \sum_{i=1}^{n} (x_i)^2$$

$$= x^T \cdot x$$

$$\|x\|_Q^2 := x^T \cdot Q \cdot x$$

WITH $Q = Q^T$, $Q \succ 0$

$(Q = Q^T)$

$Q \succ 0 \iff$ pos. definite

$\iff$ eig$(Q) > 0$