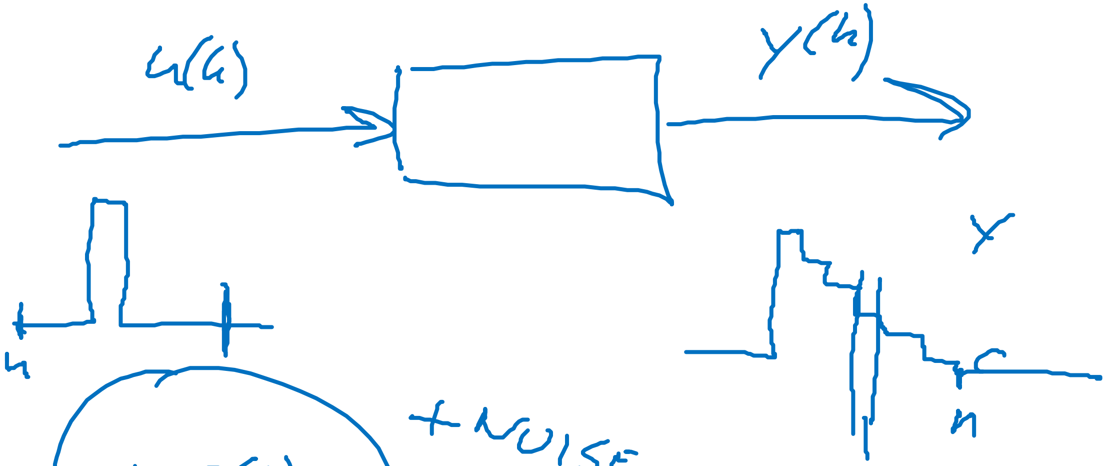
6.5 / 6.6

FIR DET

$k-i$

$$y(k) = \sum_{i=0}^{n} b_i \, u(k-i)$$



+ OUTPUT ERRORS

$$y(k) = \sum_{i=0}^{n} b_i \, u(k-i) + \varepsilon(k)$$

+ NOISE

(ε i.i.d. Gaussian) $\Downarrow$ LLS

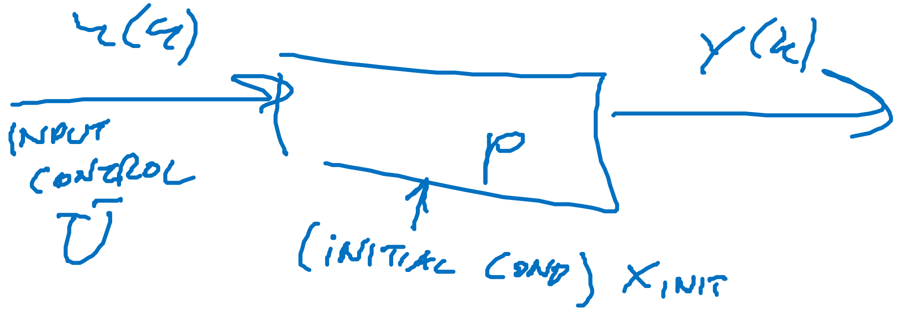
$$\arg\min_{b} \sum_{k=n+1}^{N} \left(y(k) - \sum_{i=0}^{n} b_i \, u(k-i) \right)^2$$

$\boxed{u(1), \dots}, u(N)$

$y(1), \dots, y(N)$

$$\| Y_N - \Phi_N \theta \|_2^2$$

6.6 DETERMINISTIC MODELS



$y(1), y(2), \ldots$

EX: $\boxed{1, 1}$, 2, 3, 5, 8, 13,

$$y(k) = \boxed{y(k-1) + y(k-2)}$$

$\boxed{2, 0}$, 2, 2, 4, 6, 10, 16

INITIAL CONDITIONS: ALL KNOWLEDGE NEEDED TO PREDICT FUTURE

SO FAR, NO INPUTS

$$\boxed{y(k) = y(k-1) + y(k-2) + u(k-1)}$$

$k=$		1	2	3	4	5		
$u=$		-1	-1	-1	3	3	3	
$y=$	3, 1	3	3	9	15	27		

$=: \underline{U} = (u(1), u(2), \ldots, \quad)^T$

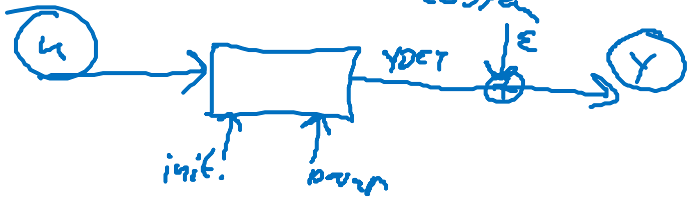
DEFINE: $\boxed{y(k) = M(k; \underline{U}, \underline{x_{INIT}}, \underline{p})}$

GENERAL DEF. FOR OUTPUT OF DYNAMIC SYSTEM.

6.6.2 OUTPUT ERRORS (OE)

$$\boxed{y(k) = M(k; \bar{U}, \underbrace{x_{INIT}, p}_{\text{UNKNOWN}}) + \varepsilon(k)}$$

ε i.i.d., Gaussian



$\theta = (x_{INIT}, p)$

$$\boxed{\hat{\theta}_{ML} = \arg\min_{\theta} \sum_{k=1}^{N} \left(y(k) - M(k; \bar{U}, x_{INIT}, p)\right)^2}$$

NONLINEAR LEAST SQUARES "lsqnonlin" in MATLAB

NONLINEAR MAP M

EXAMPLE:

$$\boxed{y_{DET}(k) = p_1 \cdot y_{DET}(k-1) + p_2 u(k-1)}$$ DETERMINISTIC

$$\boxed{y(k) = y_{DET}(k) + \varepsilon(k)}$$ OUTPUT ERROR

$x_{INIT} = (y_{DET}(1))$ $\quad U = (u(1), \ldots)$

$M(1, \bar{U}, x_{INIT}, p) = x_{INIT}$

$M(2; U, x_{INIT}, p) = p_1 \cdot x_{INIT} + p_2 u(1) \; (= y_{DET}(2))$

$M(3; \bar{U}, x_{INIT}, p) = p_1 y_{DET}(2) + p_2 u(2)$

$= \underline{\underline{p_1^2}} \cdot \underline{\underline{x_{INIT}}} + \underline{\underline{p_1 \cdot p_2}} \cdot \underline{\underline{u(1)}} + \underline{\underline{p_2}}\, \underline{\underline{u(2)}}$

NONLINEAR MAP

FOR FIR

$$M(k;\cancel{x_{init}},U,p) = \sum_{i=0}^{n} b_i\, u(k-i) \qquad p=(b_0,\dots,b_n)^T$$

THUS $\boxed{M(k;U,p)=\varphi(k)^T\cdot p}$ WITH $\varphi(k)=\begin{pmatrix} u(k)\\ u(k-1)\\ \vdots \\ u(k-n)\end{pmatrix}$

$\boxed{\text{LINEAR IN PARAMETERS}}$ (LIP)

ML IS LLS

"AUTO REGRESSION" IS MORE GENERAL, $y(k)$ DEP ON PAST y

6.6.3 STOCHASTIC DISTURBANCES $\boxed{\text{EQUATION ERRORS}}$ AUTO REGRESSIVE WITH EXOGENOUS INPUTS (ARX)

$$\boxed{y(k)=h(u(k),u(k-1),\dots,u(k-n),y(k-1),\dots,y(k-n))+\varepsilon(k)}$$

EX: $h(u(k-1),y(k-1))=p_1\,y(k-1)+p_2\,u(k-1)$ $\boxed{y(k)=\underline{p_1\,y(k-1)+p_2 u(k-1)}+\varepsilon(k)}$

ε i.i.d. Gaussian $\theta=\begin{pmatrix}p_1\\p_2\end{pmatrix}$ $\boxed{\hat\theta_{ML}=\arg\min_{\theta}\sum_{k=2}^{N}(y(k)-p_1 y(k-1)-p_2 u(k-1))^2}$ LLS!

$$\| Y_N - \Phi_N \cdot \theta \|_2^2$$

$$y(k) = \theta_1 \underline{\underline{y(k-1)}} + \theta_2 u(k-1) + \varepsilon(k)$$

$$Y_N = \begin{bmatrix} y(2) \\ \vdots \\ \vdots \\ y(N) \end{bmatrix} \qquad \Phi_N = \begin{bmatrix} y(1) & u(1) \\ \vdots & \vdots \\ \vdots & \vdots \\ y(N-1) & u(N-1) \end{bmatrix}$$

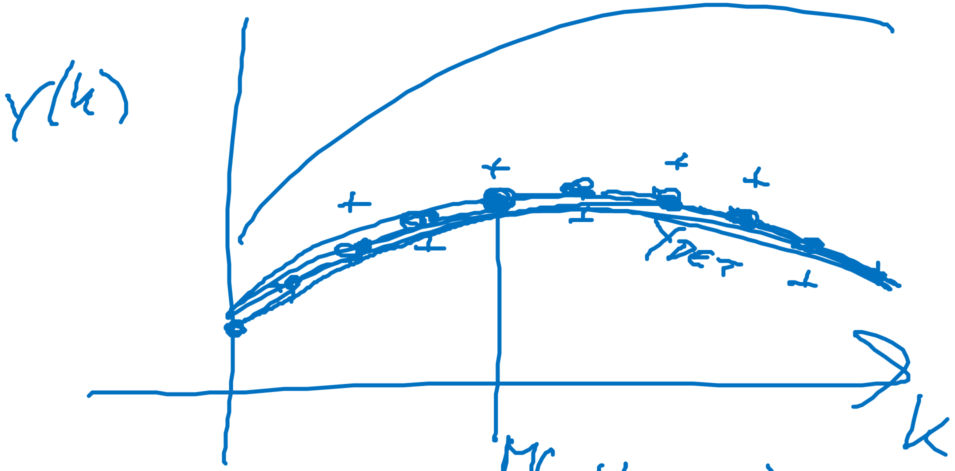
$$\hat{\theta}_{ML} = \Phi_N^{+} \cdot Y_N$$

(E.E. ~~EE~~ IS CONVEX,
FINDS GLOB. MIN.,
O.E., IN GENERAL, NOT CONVEX.

CAN USE E.E. TO INITIALIZE
O.E. TO GET BETTER RESULTS)

DIFF. OE & E.E.

O.E.

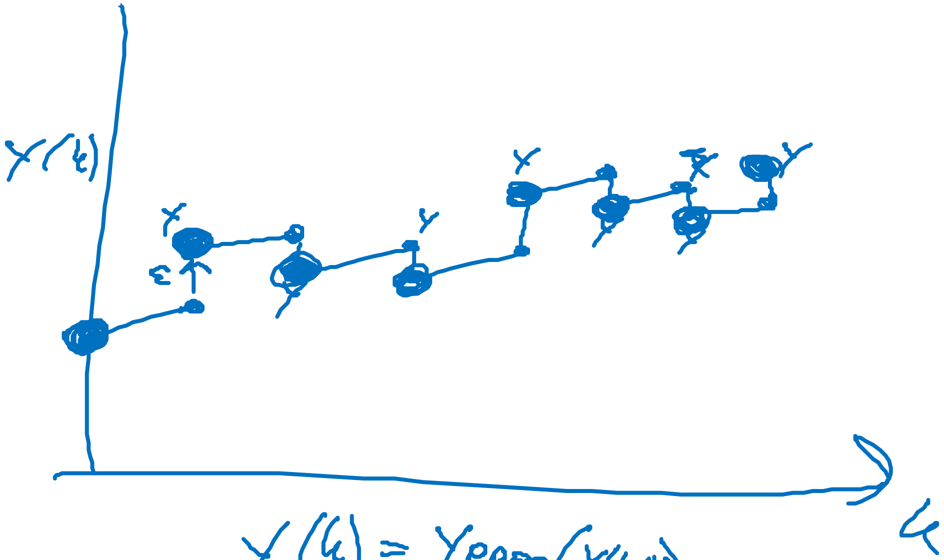


$M(k; U, x_{init}, p)$ DETERMINISTIC

\+ NOISE

$$y(k) = \underline{M(k; U, x_{init}, p)} + \varepsilon(k)$$

E.E. (

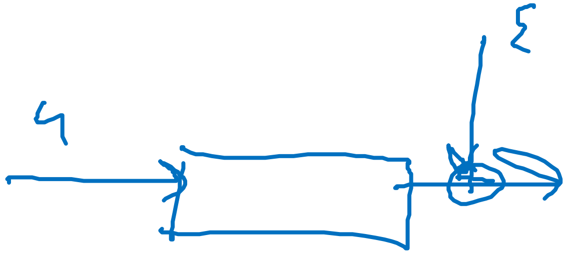


$$y(k) = \underline{Y_{PRED}(y(k-1), \ldots)} + \varepsilon(k)$$

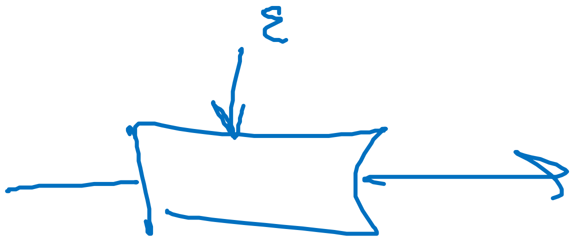
ONE STEP AHEAD PREDICTOR

PREDICTION ERROR MINIMIZATION (PEM)

O.E.



C.E.



ARX MODELS TYPICALLY ARE LINEAR (LTI) AND (LIP)

$$\boxed{y(k)=\sum_{i=0}^{n} u(k-i)\cdot b_i + \sum_{i=1}^{n} y(k-i)\cdot(-a_i) + \varepsilon(k)}$$

$$\theta = (a_1,\dots,a_n,b_0,b_1,\dots b_n)^T$$

EX FOR LIP, BUT NOT LTI

$$y(k)=\theta_1\cdot u(k)^2$$

$$y(k)=\theta_1 u(k)+\theta_2 y(k-1)\cdot u(k-1)$$

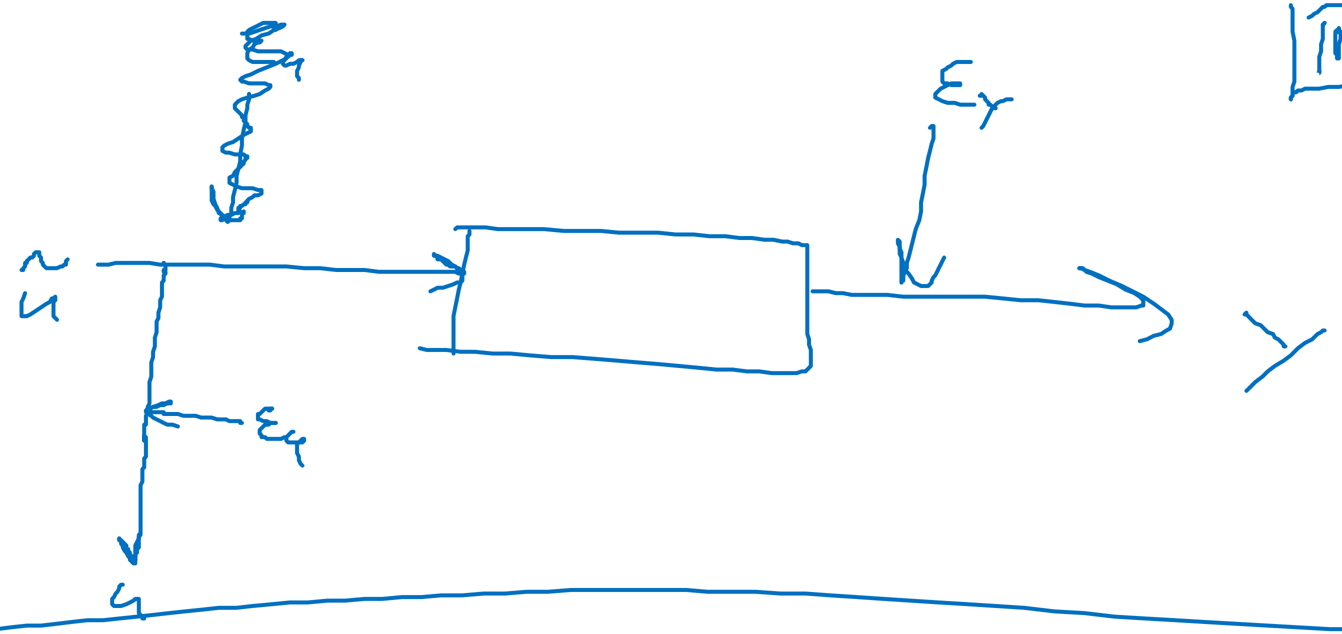
EX FOR LTI, BUT NOT LIP

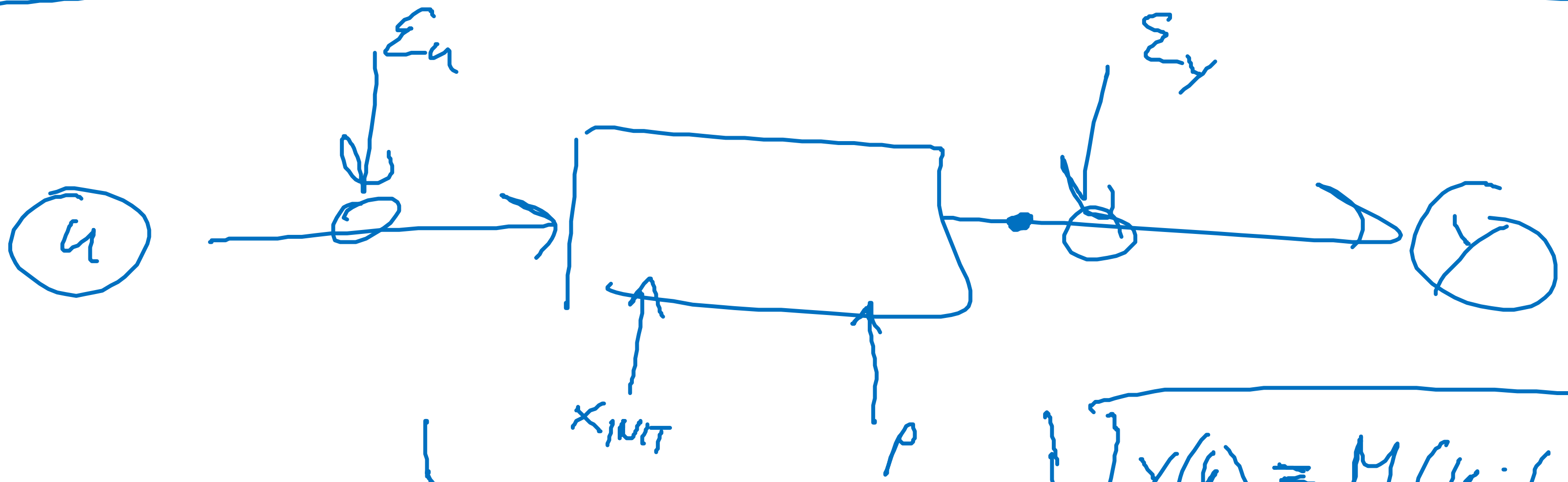
$$y(k)=\theta_1\left(y(k-1)+\theta_2 u(k-1)\right)$$

$$=\theta_1 y(k-1)+\theta_1\cdot\theta_2\, u(k-1)$$

NOT LIP

INPUT & OUTPUT ERRORS (CH 7.2)





i.i.d. Gaussian

STOCHASTIC

$$\boxed{y(k) = M(k; U + E_N^u, x_{INIT}, p) + E_y(k)}$$

DETERMINISTIC

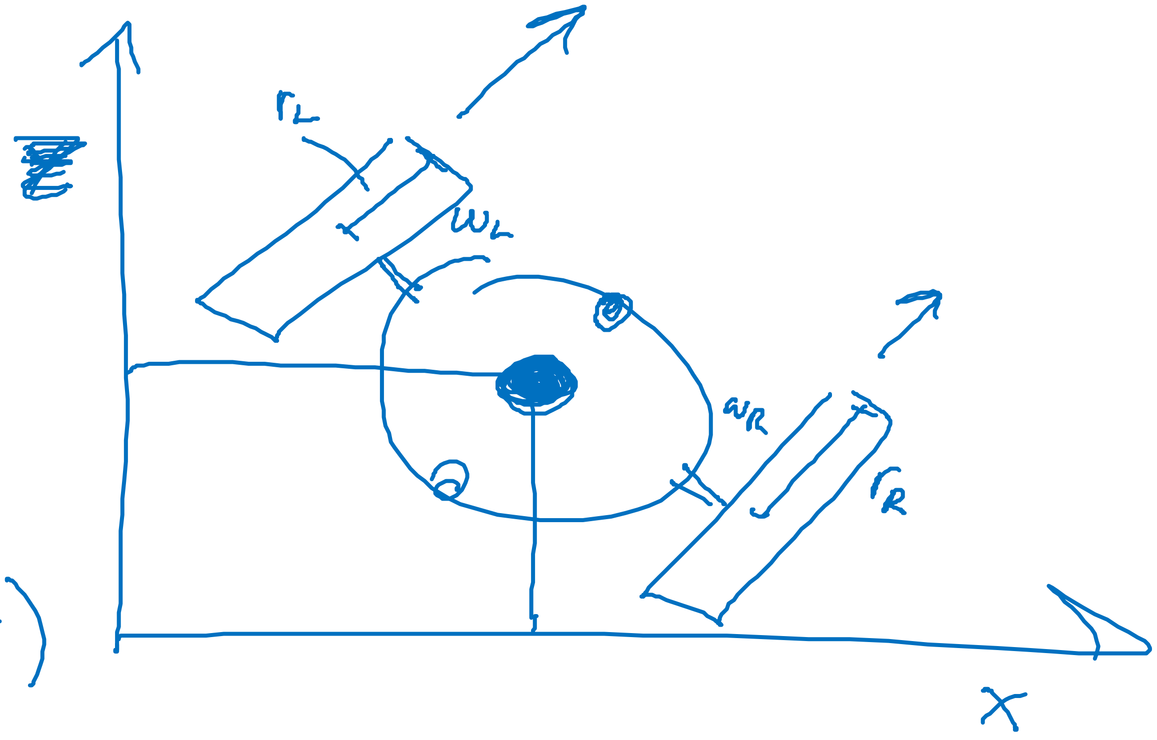
$$\boxed{\hat{\theta}_{ML} = \arg\min_{\theta} \sum_{k=1}^{N} \frac{\left(M_*(k; U+\varepsilon_N^u, x_{INIT}, p) - y(k)\right)^2}{\sigma_y^2} + \frac{(\varepsilon_u(k))^2}{\sigma_u^2}}$$

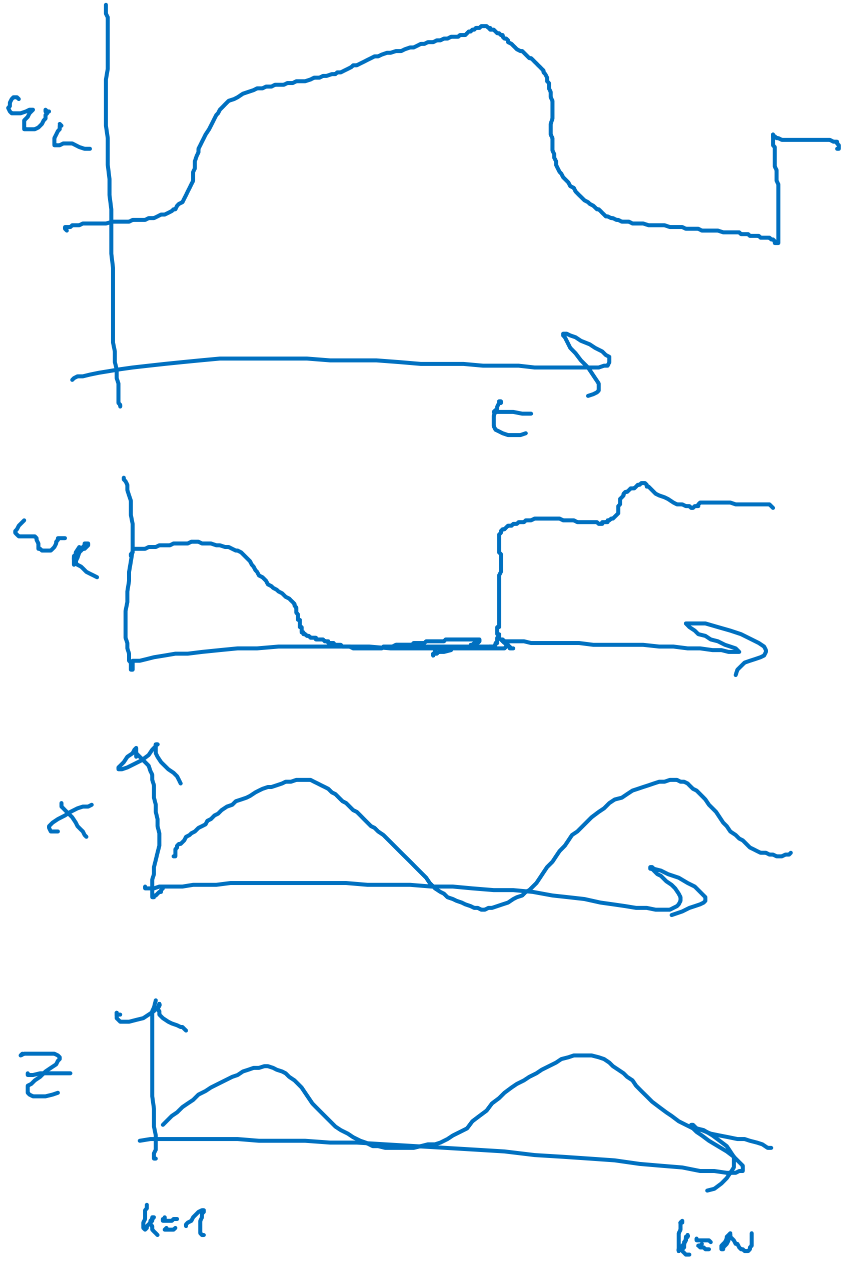
$$\varepsilon_N^u = \begin{pmatrix} \varepsilon_u(1) \\ \vdots \\ \varepsilon_u(N) \end{pmatrix} \qquad \theta = \left(p^T, x_{INIT}^T, \varepsilon_N^{uT}\right)^T$$

GIVEN $U = \begin{pmatrix} u(1) \\ \vdots \\ u(N) \end{pmatrix}, \quad y_N = \begin{pmatrix} y(1) \\ \vdots \\ y(N) \end{pmatrix}$

FIND $\hat{\theta}_{ML}$ BY LARGE SCALE NONLINEAR LEAST SQUARES

LONG EXAMPLE:





INPUTS

$$u = \begin{pmatrix} \omega_L \\ \omega_R \end{pmatrix}$$

OUTPUTS

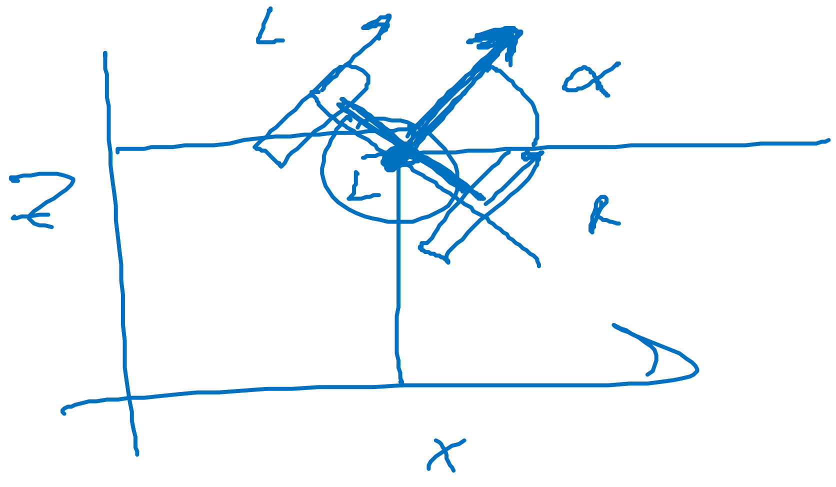
$$y = \begin{pmatrix} x \\ z \end{pmatrix}$$

SAMPLING TIME Δt

UNKNOWN PARS:

$$p = \begin{pmatrix} r_L \\ r_R \\ L \end{pmatrix}$$

TASK: GIVEN u_N, y_N, FIND p



$x_{INIT} = \begin{pmatrix} x \\ z \\ \alpha \end{pmatrix}$ AT $t = 0$

"STATE"

DERIVE CONT. TIME MODEL

GENERAL: $\dot{x} = f(x, u)$
$y = g(x, u)$

x STATE
u INPUT
y OUTPUT

$\omega_L \cdot r_L$ GROUND SPEED ON LEFT SIDE (ASSUME: NO SLIP)

$\omega_R \cdot r_R$ " RIGHT "

$$\frac{d}{dt}\begin{pmatrix} x \\ z \end{pmatrix} = \begin{pmatrix} \cos\alpha \\ \sin\alpha \end{pmatrix} \cdot \frac{(\omega_L \cdot r_L + \omega_R \cdot r_R)}{2}$$

$$\frac{d}{dt}\alpha = -\frac{(\omega_L \cdot r_L - \omega_R r_R)}{L}$$

(ASSUME L GIVEN)
r_L, r_R, L

HOW TO GENERATE $\boxed{M(u_j U, x_{INIT}, p)}$?

INTERMEDIATE STEP: HOW TO GO FROM CONT. TIME TO DISCRETE TIME?