CH 2 PROBALITY & STATISTICS

RANDOM VARIABLE X, CAN TAKE VALUES x IN A SET, E.G. $\mathbb{R}$

$$X \in [5,6] = A$$

EVENT ARE THE MAIN OBJECT E.G. A

PROBALITY OF A $\quad P(A) \in [0,1]$

AND: $\wedge$

OR: $\vee$

JOINT PROBALITY OF TWO EVENTS

$$P(A \wedge B) = P(A \& B) = \underline{\underline{P(A,B)}}$$

$A = (X \leq 5)$ $\qquad B = (X \geq 7)$

$P(A, B) = 0$

EVENTS CAN BE MUTUALLY EXCLUSIVE

$P(A \vee B) = P(A) + P(B)$ (E.G. ABOVE)

EVENTS CAN BE INDEPENDENT

$P(A, B) = P(A) \cdot P(B)$

E.G. MALE = A

DARK HAIR = B

OTHERWISE WE HAVE DEPENDENCE

CONDITIONAL PROBABILITY $P(A|B)$ "A GIVEN B"

$$\boxed{P(A|B) = \frac{P(A,B)}{P(B)}}$$

LONG HAIR = B

MALE = A

	A MALE	FEMALE	
B LONG HAIR	10%	20%	30%
SHORT HAIR	40%	30%	70%
	50%	50%	

$P(A, B) = 10\%$

$P(B) = 30\%$

$P(A|B) = \frac{10}{30} = \frac{1}{3} = 33.3\%$

BAYES' THM:

$$\boxed{P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}}$$

A : TRUE BUT UNKNOWN

B : MEASUREMENT

$P(A)$: PRIOR BELIEF

$P(B)$: (NOT SO IMPORTANT)

$P(B|A)$: COND. PROB. OF MEASUREMENT GIVEN TRUTH

$P(A|B)$: OUR MAIN INTEREST:

2.2 PROBABILITY DENSITY FUNCTIONS (PDF)

SCALAR RANDOM VARIABLE $X \in \mathbb{R}$

$p_X(x)$ PDF $\qquad p_X : \mathbb{R} \to \mathbb{R}$

$p_X(x) \geq 0 \quad \forall x \in \mathbb{R} \qquad \int_{-\infty}^{\infty} p_X(x)\,dx = 1$

$$\boxed{P(X \in [a,b]) = \int_a^b p_X(x)\,dx}$$

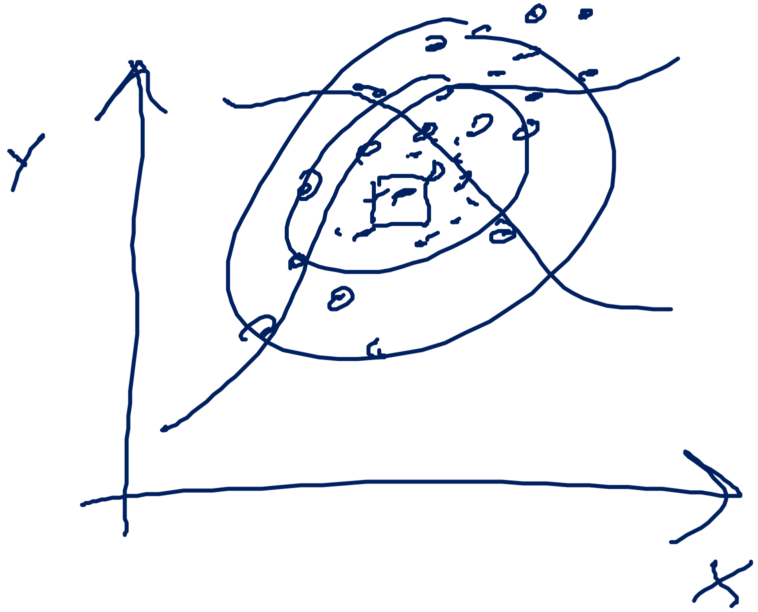
FOR ALL $a \leq b$

UNIT: $\frac{1}{[X]}$

JOINT PDF $p_{X,Y}(x,y) = p(x,y)$ (SLOPPY)

CONDITIONAL PDF $p(x|y) = \frac{p(x,y)}{p(y)}$

INDEP. $p(x,y) = p(x) \cdot p(y)$



2.2.1 MEAN & VARIANCE

$f(X)$ $\qquad f: \mathbb{R} \to \mathbb{R}$

$$\mathbb{E}\{f(X)\} = \int_{-\infty}^{\infty} f(x) \cdot p_X(x)\, dx$$

"EXPECTATION" OR "MEAN"

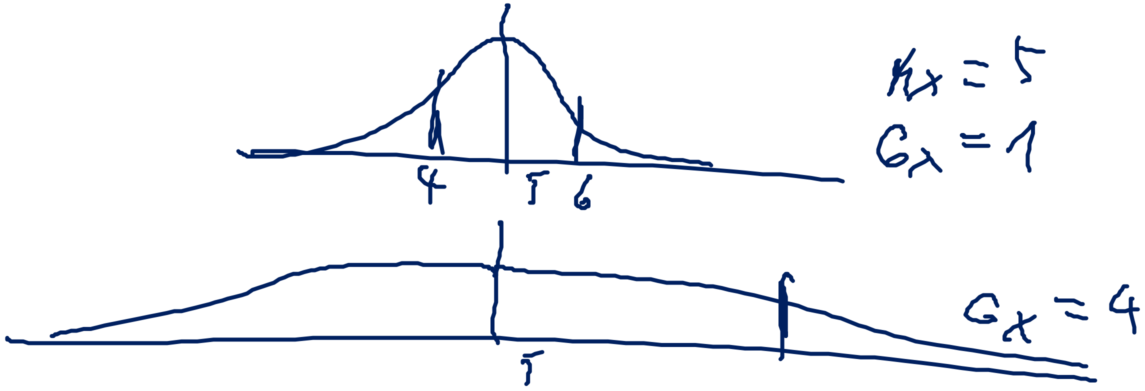
$f(X) = a + b \cdot X$ WITH FIXED a, b

$\mathbb{E}(a+bX) = a + b \cdot \mathbb{E}\{X\}$ BUT NOT:

$$\mathbb{E}\{X^2\} \neq (\mathbb{E}\{X\})^2$$

SHORTHAND:

$$\mu_X := \mathbb{E}\{X\}$$



VARIANCE

$$\sigma_X^2 := \mathbb{E}\{(X-\mu_X)^2\} = \mathbb{E}\{X^2 - 2X\mu_X + \mu_X^2\}$$

$$= \mathbb{E}\{X^2\} - 2\mu_X^2 + \mu_X^2$$

$$= \mathbb{E}\{X^2\} - \mu_X^2$$

STANDARD-DEVIATION

$$\sigma_X := \sqrt{\sigma_X^2}$$

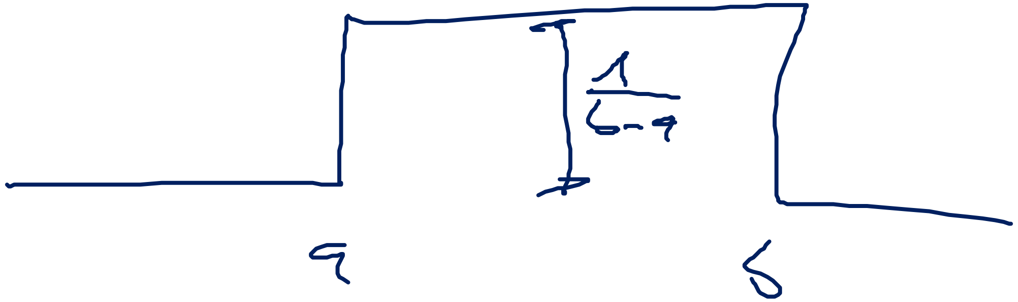
UNIT: $[\mu_X] = [X]$

$[\sigma_X] = [X]$

EXAMPLES:

UNIFORM DISTRIBUTION / PDF BETWEEN a AND b

$$p(x) = \begin{cases} \frac{1}{b-a} & \text{IF } x \in [a, b] \\ 0 & \text{ELSE} \end{cases}$$



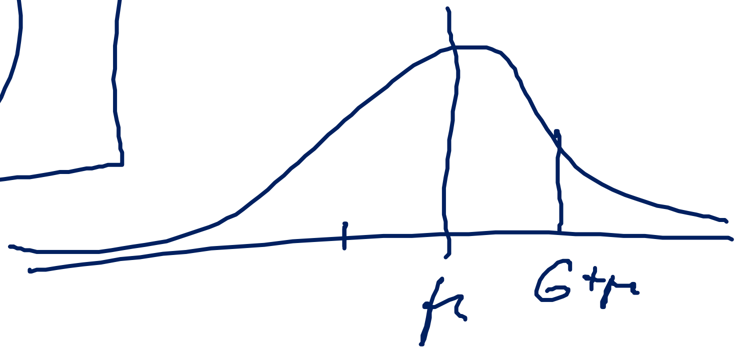
$$\int_{-\infty}^{\infty} p(x) = 1$$

$$\mu_x = \frac{a+b}{2}$$

$$\sigma_x^2 = \frac{(b-a)^2}{12}$$

GAUSSIAN - PDF, NORMAL DISTRIBUTION

$$\boxed{p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)}$$



$$\int_{-\infty}^{\infty} p(x) = 1$$

$$\mu_X = \mu$$

$$\sigma_X^2 = \sigma^2$$

2.3 MULTIDIMENSIONAL RANDOM VARIABLES