Exercise 4: Maximum Likelihood Estimation

(to be returned on Dec 1, 2015, 8:15 in HS 26, or before in building 102, 1st floor, 'Anbau')

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Your MATLAB solution has to run from a main script called `main.m`, which can call other functions/scripts, but when running this script all the necessary results and plots should be clearly visible. In order to submit your code, include all the necessary files in a single folder using a folder format name `Ex03_Surname1` up to the four possible surnames of the group members. Compress the folder in a `.zip` file and send it to `jesuslagogarcia@gmail.com`. Please state also your name and the names of your team members in the e-mail.

Exercise Tasks

1. **Maximum likelihood estimation on wind distribution** (10 points)

   In wind energy, the wind speed is an important quantity that varies according to the weather condition and location Often, one is interested in the average power that can be produced at a specific location with a specific wind turbine.

Figure 1: Wind farm
Source: http://cleantechnica.com/2012/01/25/wind-transport-news-5-stories/

Due to the high complexity, a deterministic wind model would be very difficult to obtain. However, it has been observed that the magnitude of the wind speed in a wind farm throughout a year follows a distribution given by the following conditioned probability density function:

$$p(v|\lambda, k) = \begin{cases} \frac{k}{\lambda} \left(\frac{v}{\lambda}\right)^{k-1} e^{-(v/\lambda)^k} & v \geq 0 \\ 0 & v \leq 0 \end{cases} \quad (1)$$

This distribution is known as the Weibull distribution, where $\lambda > 0$ and $k > 0$ are the parameters of the distribution and v is the magnitude of the wind speed at the turbine location. The parameters λ and k depend on the location. The task of this sheet will be to estimate these parameters λ and k starting from measurements of the wind speed at a given location.

The main motivation on estimating these parameters is the study of the average power that a certain turbine will produce in a certain location. In order to do that, the expected value of the power profile $P_{\text{Power}}(v)$ of the turbine is calculated using Eq. (1). As an illustration, a typical power profile of a wind turbine is given in Figure 2.
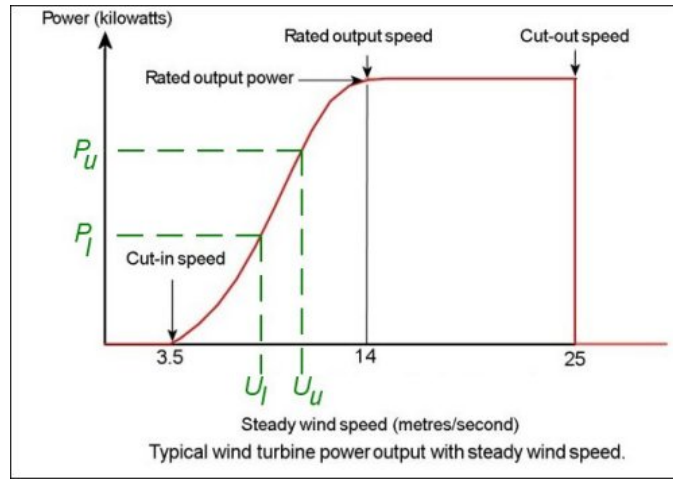
Tasks:

Figure 2: Wind turbine power profile
Source: http://www.wind-power-program.com/wind_statistics.htm

(a) Formulate the negative log-likelihood function given N measurements of wind speed $v_1, \ldots, v_N$ from the same same turbine throughout the year. Simplify this function as much as possible. (3 points)

(b) State the minimisation problem and simplify again the objective function. *Hint: constant terms in the objective function do not alter the solution of a minimisation problem.* (1 point)

(c) Once the minimisation problem has been obtained, it can be solved using appropriate optimisation software e.g. in MATLAB. Import the wind speed data from the website and plot it using the histogram function. (1 point)

(d) Solve the minimisation problem obtained in Task (a) to estimate λ and k. To do that you should use the function fmincon of MATLAB. Take a look to the following website to learn how to use fmincon: http://www.mathworks.com/help/optim/ug/fmincon.html Keep in mind that the minimisation has the objective function that you derived in Task (b) but also a constraints on the parameters λ and k. (3 points)

(e) Using the data of a power curve given in Table 1, compute the expected value of the turbine power on the studied location. Use the trapezoidal rule to integrate and compute the expected value for the power. (2 point)

Table 1: Power curve data

Wind speed (m/s)	0	1	2	3	4	5	6	7	8
Output power (kW)	0	0	3	25	82	174	321	532	815

Wind speed (m/s)	9	10	11	12	13	14	15	16	17
Output power (kW)	1180	1580	1900	2200	2480	2700	2850	2950	3020

Wind speed (m/s)	18	19	20	21	22	23	24	25	> 25
Output power (kW)	3020	3020	3020	3020	3020	3020	3020	3020	0

This sheet gives in total 10 points