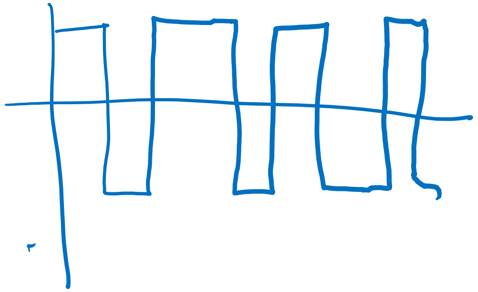
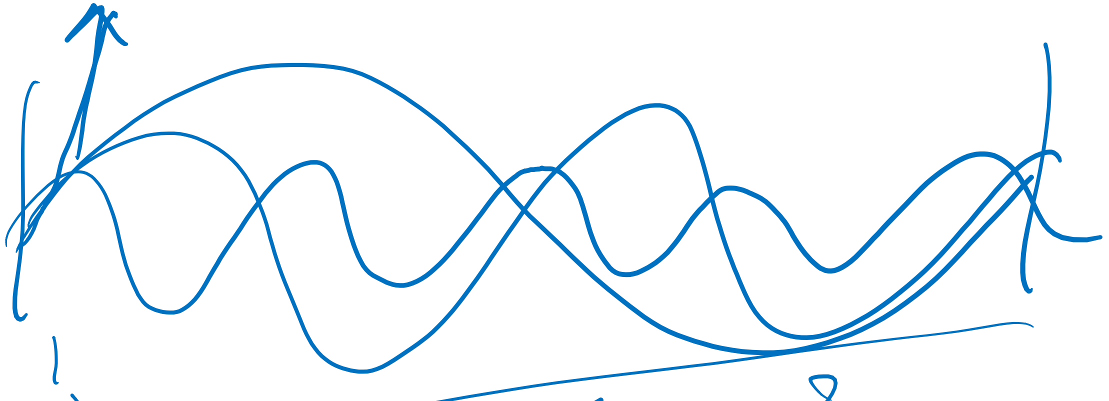
MULTI SINES

$\omega_0 \cdot k(p)$





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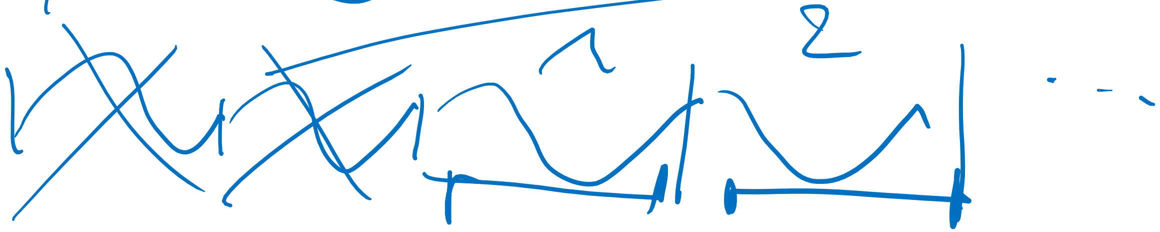
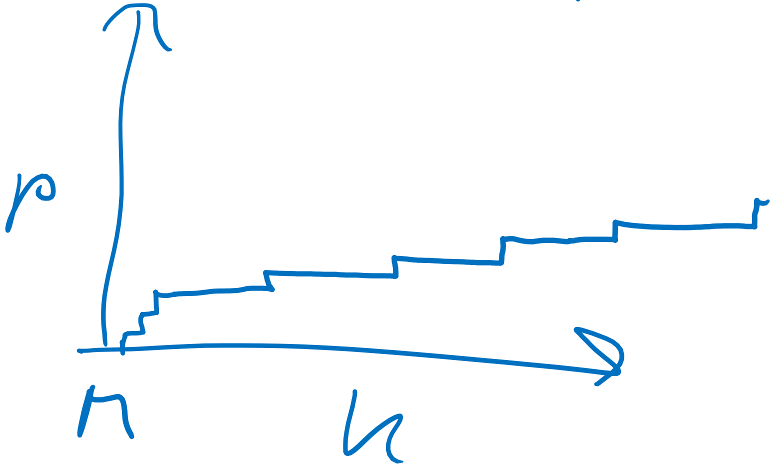
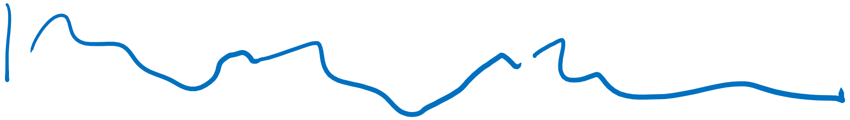
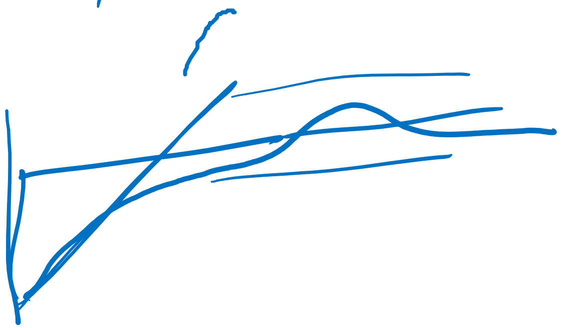
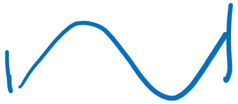


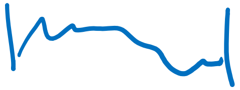


y





u  $\xrightarrow{\text{FFT}}$ $U(j\omega_0 \cdot k(p))$

y  $\rightarrow$ $Y(j\omega_0 \cdot k(p))$

$$G(j\omega_0 \cdot k(p)) = \frac{Y(\dots)}{U(\dots)}$$

PRBN

CH 9 ONLINE ESTIMATION (OF STATE)

TIME DOMAIN

DISCRETE TIME

LTI

(DETERMINISTIC MODEL)

MEASUREMENT NOISE

$$\boxed{\begin{aligned} x_{k+1} &= A_k \cdot x_k \\ y_k &= C_k \cdot x_k + v_k \end{aligned}}$$

$$x_k = A_{k-1} \cdot A_{k-2} \cdots A_0 \cdot x_0$$

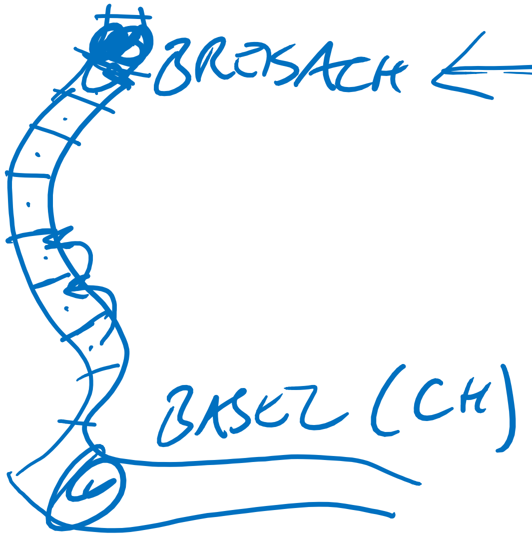
GIVEN $y_0, y_1, \ldots y_k$

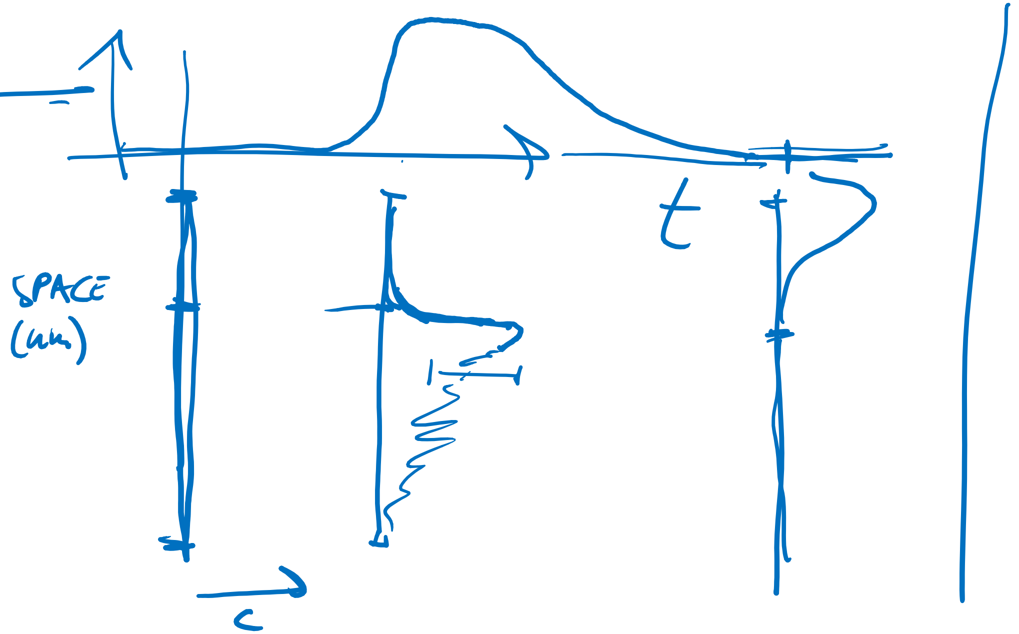
FIND x_k

(RECURSIVELY)

(RESULT: KALMAN FILTER)

MOTIVATING EXAMPLE: POLL. EST. IN RHINE





RECALL: RECURSIVE LIN. L.S. (RLS)

$y_1, y_2, \ldots$ $\quad (y(k) \rightarrow y_k)$

Θ

$y_k = \phi_k^T \Theta + \varepsilon_k$

AIM: $\Theta \equiv x_0$

$\phi_k^T \Theta \equiv C_k \cdot x_k$

$= C_k A_{k-1} \cdot A_{k-2} \cdots A_0 \cdot x_0$

$\varepsilon_k \equiv v_k$

START: A-PRIORI INFORMATION ~~$\Theta_0 \sim \mathcal{N}$~~ $\Theta \sim \mathcal{N}(\hat{\Theta}_0, Q_0^{-1})$

$\uparrow$ COV

Q_0: INF.

IT:

$$\boxed{Q_k = Q_{k-1} + \phi_k \phi_k^T}$$

$$\boxed{\hat{\Theta}_k = \hat{\Theta}_{k-1} + Q_k^{-1} \cdot \phi_k (y_k - \phi_k^T \Theta)}$$

$$\hat{\Theta}_k = \arg\min_{\Theta} \; (\Theta - \hat{\Theta}_0)^T Q_0 (\Theta - \hat{\Theta}_0) + \sum_{i=1}^{k} (y_i - \phi_i^T \Theta)^2$$

$$Q_k = Q_{k-1} + \phi_k \phi_k^T \qquad \phi_k^T = C_k \cdot A_{k-1} \cdots A_0$$

$$= Q_{k-1} + (C_k A_{k-1} \cdots A_0)^T (C_k A_{k-1} \cdots A_0)$$

$$\boxed{\hat{\Theta}_k = \hat{\Theta}_{k-1} + Q_k^{-1} (C_k A_{k-1} \cdots A_0)^T (Y_k - C_k A_{k-1} A_{k-2} \cdots A_0 \hat{\Theta}_{k-1})}$$

$$\hat{\Theta}_k \equiv \hat{X}_{[0|k]}$$

"AT TIME k"

NEW NOTATION: $\hat{X}_{[k|m]}$: ESTIM. OF STATE AT TIME k GIVEN ALL MEASUREMENTS UP TO TIME m

$$\hat{X}_{[k|m]} = A_{k-1} \cdot A_{k-2} \cdots A_0 \hat{\Theta}_m$$

$$= A_{k-1} \cdots A_0 \cdot \hat{X}_{[0|m]}$$

$$\underbrace{\mathrm{Cov}(\hat{X}_{[k|m]})}_{=: P_{[k|m]}} = (A_{k-1} \cdots A_0)\, Q_m^{-1} \cdot (A_0^T \cdots A_{k-1}^T)$$

$$\underline{\hat{X}_{[k|k]}} = A_{k-1} \cdots A_0 \cdot \hat{\Theta}_k = \underbrace{A_{k-1} \cdot A_{k-2} \cdots A_0 \hat{\Theta}_{k-1}}_{\hat{X}_{[k|k-1]}} + \overbrace{(A_{k-1} \cdots A_0)\, Q_k^{-1} (A_0^T \cdots A_{k-1}^T)}^{P_{[k|k]}} C_k^T (Y_k - C_k \cdot \underline{A_{k-1} \cdots A_0 \hat{\Theta}_{k-1}})$$

(3) $$\boxed{\hat{x}_{[k|k]} = \hat{x}_{[k|k-1]} + P_{[k|k]} \cdot C_k^T \left(y_k - C_k \cdot \hat{x}_{[k|k-1]} \right)}$$

(1) $$\boxed{\hat{x}_{[k|k-1]} = A_{k-1} \cdot \hat{x}_{[k-1|k-1]}}$$

$$\boxed{\begin{matrix} \hat{x}_{[k-1|k-1]} \\ P_{[k-1|k-1]} \end{matrix}} \rightarrow \boxed{\begin{matrix} \hat{x}_{[k|k]} \\ P_{[k|k]} \end{matrix}}$$

$$P_{[k|k]} = \quad \underline{A_{k-1} \cdots A_0 \, Q_k^{-1} A_0^T \cdots A_{k-1}^T}$$

$$(\;)^{-T} \, \underline{Q_k = Q_{k-1} + (A_0^T \cdots A_{k-1}^T) \, C_k^T C_k \, (A_{k-1} \cdots A_0)} \cdot (\;)^{-1}$$

(ASS: $(A_{k-1} \cdots A_0)$ INVERTIBLE)

$$(A_{k-1} \cdots A_0)^{-T} Q_k \, (A_{k-1} \cdots A_0)^{-1} = (A_{k-1} \;)^{-1} Q_{k-1} (\;)^{-1} + C_k^T C_k$$

$$P_{[k|k]}^{-1} = P_{[k|k-1]}^{-1} + C_k^T C_k$$

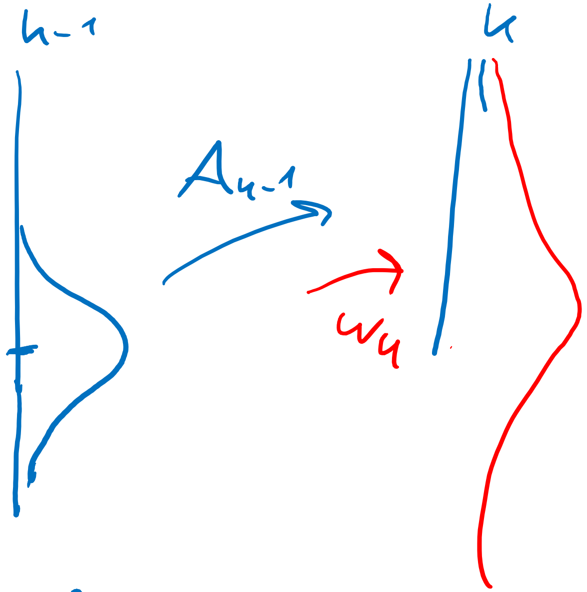
(2) $$\boxed{P_{[k|k]} = \left(P_{[k|k-1]}^{-1} + C_k^T C_k \right)^{-1}}$$

(0) $$\boxed{P_{[k|k-1]} = A_{k-1} \cdot P_{[k-1|k-1]} \cdot A_{k-1}^T}$$

Ⓐ PREDICTION STEP (BEFORE MEASUREMENT):

$$\hat{x}_{[k|k-1]} = A_{k-1} \cdot \hat{x}_{[k-1|k-1]}$$

$$P_{[k|k-1]} = A_{k-1} \cdot P_{[k-1|k-1]} A_{k-1}^{T} + W_{k-1}$$



Ⓑ INNOVATION UPDATE STEP (AFTER MEASUREMENT):

$$P_{[k|k]} = \left(P_{[k|k-1]}^{-1} + C_k^{T} C_k \right)^{-1}$$

?

$$\hat{x}_{[k|k]} = \hat{x}_{[k|k-1]} + P_{[k|k]} \cdot C_k^{T} \cdot \left(y_k - C_k \cdot \hat{x}_{[k|k-1]} \right)$$

$$x_{k+1} = A_k \cdot x_k + w_k \qquad w_k \sim N(0, W_k)$$

9.3 KALMAN FILTER

GIVEN MODEL

$$\boxed{\begin{aligned} x_{k+1} &= A_k \cdot x_k + w_k \\ y_k &= C_k \cdot x_k + v_k \end{aligned}} + b_k$$

KNOWN AFFINE DRIFT TERMS

STATE NOISE: $w_k \sim \mathcal{N}(0, W_k)$

MEASUREMENT NOISE: $v_k \sim \mathcal{N}(0, V_k)$

(A) PREDICTION STEP (AS BEFORE)

$$\hat{x}_{[k|k-1]} = A_{k-1} \hat{x}_{[k-1|k-1]} + b_{k-1}$$

$$P_{[k|k-1]} = A_{k-1} P_{[k-1|k-1]} \cdot A_{k-1}^T + W_{k-1}$$

VERY SIMPLE IF (CONSTANT IN TIME)

$A_k = A$

$C_k = C$

$W_k = W$

$V_k = V$

$P_{[k|k]} \rightarrow P$ (SOL. ALG. RICC. EQ.)

(B) INNOV.

$$P_{[k|k]} = \left(P_{[k|k-1]}^{-1} + \underline{C_k^T V_k^{-1}} C_k \right)^{-1}$$

$$\hat{x}_{[k|k]} = \hat{x}_{[k|k-1]} + \underline{P_{[k|k]} \cdot C_k^T V_k^{-1}} \left(y_k - C_k \hat{x}_{[k|k-1]} \right)$$

FOR CONST. MATRICES

$$P_{[k|k]} \overset{!}{=} P_{[k-1|k-1]} \overset{!}{=} P$$

$$P = \left(\left(APA^T + W \right)^{-1} + C^T V^{-1} C \right) \quad \text{(ALG. RICC. EQ).}$$

$$\boxed{\hat{x}_{[k|k]} = \underline{\underline{A}} \cdot \hat{x}_{[k-1|k-1]} + b_k + \underbrace{P \cdot C^T \cdot V^{-1}}_{=:L} \left(y_k - C \cdot A \cdot \hat{x}_{[k-1|k-1]} \right)}$$