In CH 1, RECALL

$$x_{k+1} = A_k \cdot x_k + w_k \qquad w_k \sim N(0, W_k)$$

$$y_k = C_k \cdot x_k + v_k \qquad v_k \sim N(0, V_k)$$

KALMAN FILTER : ~~GET~~ TAKE $y_0, y_1, y_2, \ldots \Rightarrow$ COMP. $\hat{x}_0, \hat{x}_1, \hat{x}_2, \ldots$

PRED. $\hat{x}_{[k|k-1]} = A_{k-1} \cdot \hat{x}_{[k-1|k-1]}$

$$P_{[k|k-1]} = A_{k-1} \cdot P_{[k-1|k-1]} \cdot A_{k-1}^T + W_k$$

INNOV.

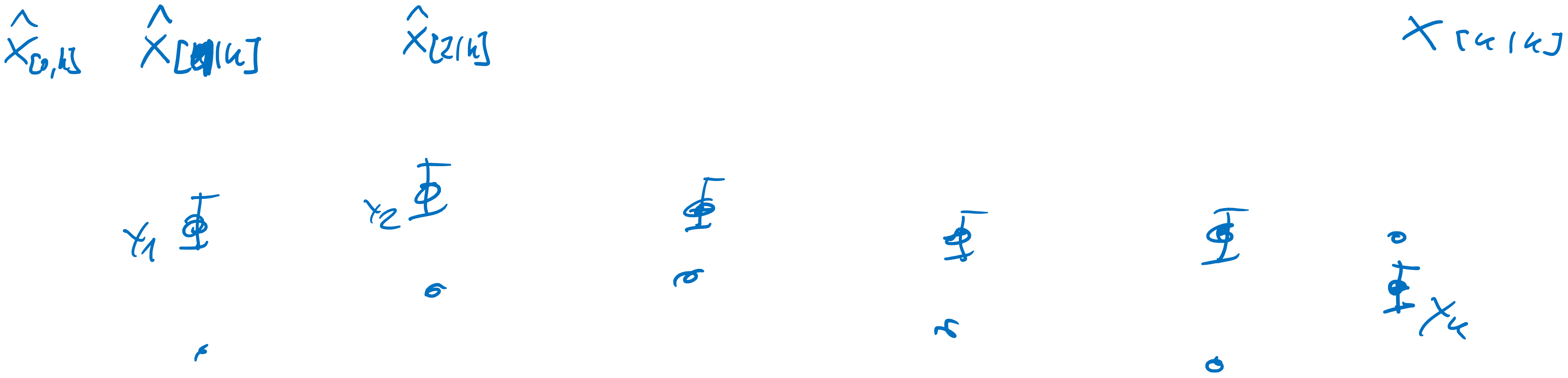
$$P_{[k|k]} = \left(P_{[k|k-1]}^{-1} + C_k^T V_k^{-1} C_k \right)^{-1}$$

$$\hat{x}_{[k|k]} = \hat{x}_{[k|k-1]} + P_{[k|k]} \cdot C_k^T V_k^{-1} \left(y_k - C_k \cdot \hat{x}_{[k|k-1]} \right)$$

	DISCR.	CONT. TIME
LIN. (KALMAN FILTER)	$x_{k+1} = A_k \cdot x_k + w_k$ $y_k = C_k \cdot x_k + v_k$ 1	$\dot{x} = A(t)\, x(t) + w(t)$ $y(t) = C(t) \cdot x(t) + v(t)$ 3
NONLIN. (EXTENDED KALMAN FILTER)	2 $x_{k+1} = f_k(x_k) + w_k$ $y_k = h_k(x_k) + v_k$	4 $\dot{x} = f(x,t) + w(t)$ $y(t) = h(x,t) + v(t)$

$A_c^c = \frac{\partial f}{\partial x}$ $\qquad$ $C^c = \frac{\partial h}{\partial x}$

WHICH OPT. PROBLEM DOES THE KF SOLVE?

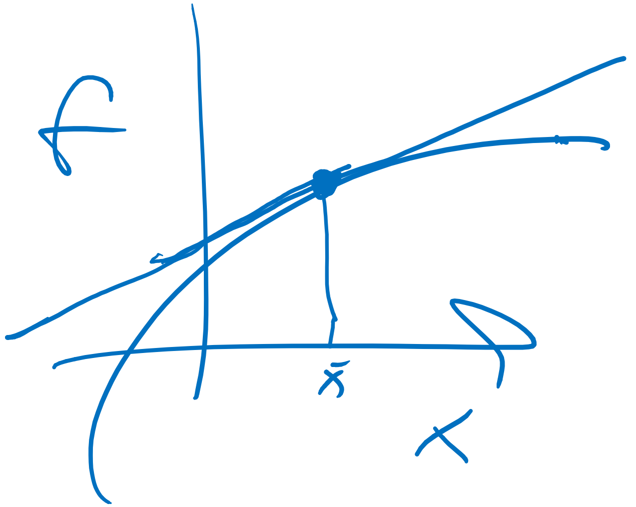


$$\boxed{(\hat{x}_{[0|k]}, \dots, \underline{\hat{x}_{[k|k]}}) = \arg\min_{x_0,\dots,x_k} (x_0-\hat{x}_0)^T P_0^{-1}(x_0-\hat{x}_0) + \sum_{i=1}^{k} \|y_i - C_i x_i\|^2_{V_i^{-1}} + \sum_{i=1}^{k} \|x_i - A_{i-1} \cdot x_{i-1}\|^2_{W_{i-1}^{-1}}}$$

TRAJECTORY ESTIMATION BASED ON ML / BAYESIAN / MAP

KF FIND <u>ONLY</u> $\hat{x}_{[k|k]}$, I.E. LAST STATE OF THE TRAJECTORY, VERY EFFICIENTLY

EXTENDED KF (DISCRETE TIME)



$$\boxed{\begin{aligned} x_{k+1} &= f_k(x_k) + w_k \\ y_k &= h_k(x_k) + v_k \end{aligned}}$$

IDEA: APPROX. f_k, h_k BY FIRST ORDER TAYLOR SERIES (LINEARIZE)

$$f_k(x) \approx f_k(\bar{x}) + \underbrace{\frac{\partial f_k}{\partial x}(\bar{x})}_{= A_k} \cdot (x - \bar{x})$$

AT BEST AVAILABLE ESTIMATE SO FAR

$$h_k(x) \approx h_k(\bar{x}) + \underbrace{\frac{\partial h_k}{\partial x}(\bar{x})}_{C_k} (x - \bar{x})$$

KF

EKF

PRED

$$\hat{x}_{[k|k-1]} = A_{k-1} \cdot \hat{x}_{[k-1|k-1]}$$

$$x_{[k|k-1]} = \boxed{f_{k-1}\left(x_{[k-1|k-1]}\right)}$$

$$P_{[k|k-1]} = A_{k-1} P_{[k-1|k-1]} A_{k-1}^T + W_k$$

SAME, WITH $\boxed{A_{k-1} := \frac{\partial f_{k-1}}{\partial x}(\,,\,)}$

INNOV.

$$P_{[k|k]} = \left(P_{[k|k-1]}^{-1} + C_k^T V_k^{-1} C_k \right)^{-1}$$

SAME, WITH $\boxed{C_k = \frac{\partial h_k}{\partial x}\left(x_{[k|k-1]}\right)}$

. . . . .

$$\boxed{\hat{x}_{[k|k]} = \hat{x}_{[k|k-1]} + P_{[k|k]} \cdot C_k^T \cdot V_k^{-1} \cdot \left(y_k - \boxed{h_k\left(\hat{x}_{[k|k-1]}\right)}\right)}$$

NOTE: EVEN FOR TI f AND h, $A_k = \frac{\partial f}{\partial x}(\hat{x}_{[k|k]}) \neq A_{k+1} = \frac{\partial f}{\partial x}(\hat{x}_{[k+1|k+1]})$

BECAUSE WE LINEARIZE AT DIFFERENT POINTS.

MEMORY OF EKF: SAME AS KF

$\hat{x}[k|k]$, $P[k|k]$

9.4 KF IN CONT. TIME

$$\boxed{\begin{aligned} \dot{x}(t) &= A^c(t) \cdot x(t) + w^c(t) \\ y(t) &= C^c(t) \cdot x(t) + v^c(t) \end{aligned}}$$

Ⓐ

DISCRETE TIME, WITH SAMPLING TIME Δt

Ⓑ

$\lim_{\Delta t \to 0}$ . ..

$$\dot{\hat{x}} = \ldots$$

$$\dot{p} = \ldots$$

$\mathrm{cov}(w^c(t_1), w^c(t_2))$

$= \delta(t_1 - t_2) \cdot W^c$

$[W^c] = \frac{[x]^2}{[t]}$

WHITE NOISE IN CONT. TIME.

$\mathrm{cov}(v^c(t_1), v^c(t_2))$

$= \delta(t_1 - t_2) \cdot V^c$

$[V^c] = [t] \cdot [y]^2$

① TRANSF. SYSTEM FROM CONT. TO DISCR TIME

IDEA: USE EULER INTEGRATION SCHEME

$$t_k = k \cdot \Delta t$$

$$\dot{x} = A^c \cdot x + w^c$$

$$x_k \equiv x(t_k)$$

$$\boxed{x_{k+1} = x_k + \Delta t \cdot A^c \cdot x_k} + \underbrace{\int_{t_k}^{t_{k+1}} w^c(t)\,dt}_{=:\,w_k}$$

$$= \underbrace{(I + \Delta t\, A^c)}_{A_k} \cdot x_k + w_k$$

$$\boxed{W_k = \Delta t \cdot W^c}$$

$$[W_k] = [x]^2$$

$$= A_k \cdot x_k + w_k$$

i.e. $\boxed{A_k = I + \Delta t \cdot A^c}$

$$y = C^c \cdot x + v^c$$

$$y_k = C^c \cdot x_k + v_k$$

$$\boxed{C_k = C^c}$$

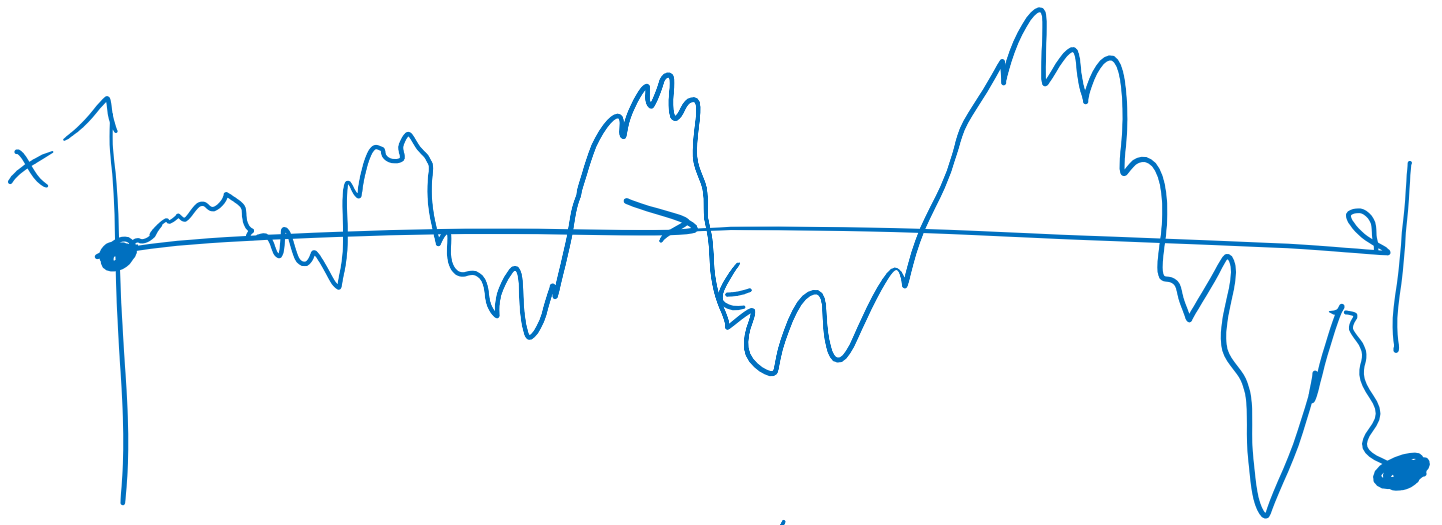
$$y_k = \left(\frac{1}{\Delta t}\right) \int_{t_k}^{t_{k+1}} y(t)\,dt$$

$$\boxed{V_k = \frac{V^c}{\Delta t}}$$

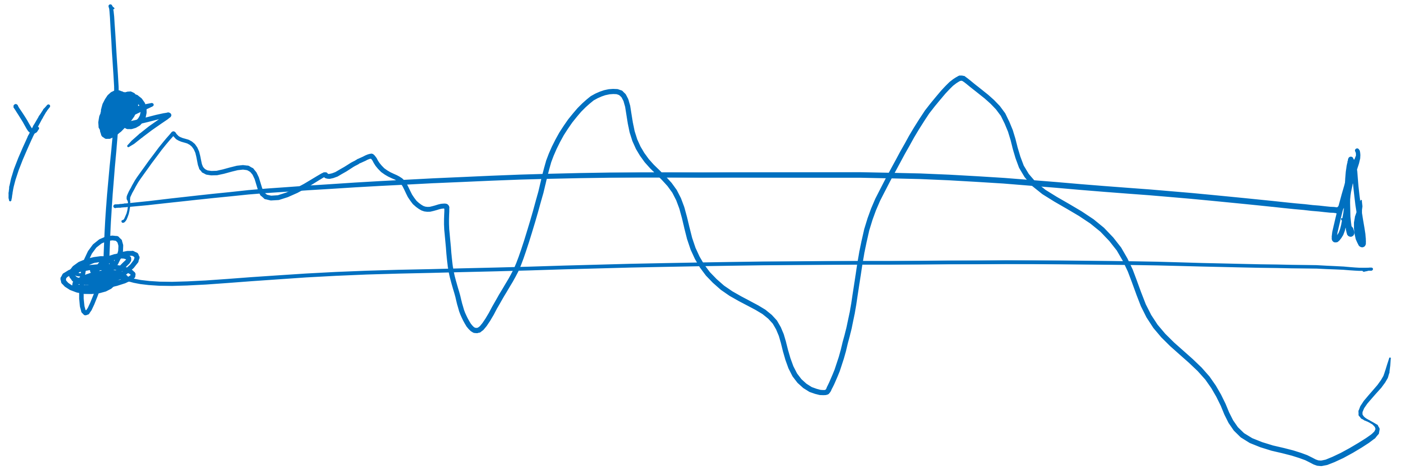
Ex:

$\dot{x} = w$

$y = x$



Δt



$$A_k = I + \Delta t A^c \qquad W_k = \Delta t \cdot W^c \qquad C_k = C^c \qquad V_k = \frac{V^c}{\Delta t}$$

Discr. KF: (pred) $x_{[k|k-1]} = (I + \Delta t A^c) \cdot x_{[k-1|k-1]} = x_{[k-1|k-1]} + \Delta t A^c \cdot x_{[k-1|k-1]}$

$$\to \quad P_{[k|k-1]} = (I + \Delta t A^c)\, P_{[k-1|k-1]}\, (I + \Delta t A^c)^T + \Delta t \cdot W^c$$

$$= P_{[k-1|k-1]} + \Delta t \cdot \underline{\underline{P_{[k-1|k-1]} \cdot A^{cT}}} + \Delta t \cdot \underline{\underline{A^c \cdot P_{[k-1|k-1]}}} + O(\Delta t^2) + \Delta t \cdot W^c$$

$$P_{[k|k]} = (P_{[}^{-1} + C_k^T V_k^{-1} C_k)^{-1} = (P_{[k|k-1]}^{-1} + C^{cT} (V^c)^{-1} C^c \cdot \Delta t)^{-1}$$

$$\overset{!!}{=} \left(P^{-1} \cdot (I + P \cdot C^{cT} V^{c-1} C^c \Delta t)\right)^{-1} P = (I + P_{[k|k-1]} \cdot C^{cT} V^{c-1} C^c \Delta t)^{-1} \cdot P_{[k|k-1]}$$

$$= (I - \Delta t\, P_{[k|k-1]} \cdot C^{cT} V^{c-1} C^c + O(\Delta t^2)) \cdot P_{[\quad]}$$

$$\to \quad = P_{[k|k-1]} - \Delta t\, \underline{\underline{P_{[\quad]} C^{cT} V^{c-1} C^c \cdot P_{[\quad]}}} + O(\Delta t^2)$$

$$x_{[k|k]} = x_{[k|k-1]} + P_{[k|k]} \cdot C^{c^T} \cdot V^{c^{-1}} \cdot \Delta t \cdot \left(y_k - C^c \cdot x_{[k|k-1]}\right)$$

$$\cancel{x_{[k|k-1]}}$$

$$x_{[k-1|k-1]} \rightarrow x_{[k|k]}$$

$$x_{[k|k]} = x_{[k-1|k-1]} + \Delta t\, A^c \cdot x_{[k-1|k-1]} + \cancel{\Delta t} P C^{c^T} V^{c^{-1}} \left(y - C^c x\right)$$

$$x_{[k|k]} = x_{[k-1|k-1]} + \Delta t \left(A^c x + P C^{c^T} V^{c^{-1}} \left(y - C^c x\right)\right)$$

$$\frac{x_{[k|k]} - x_{[k-1|k-1]}}{\Delta t} = A^c x + P C^{c^T} V^{c^{-1}} \left(y - C^c x\right) \rightarrow \boxed{\dot{\hat{x}} = A^c \hat{x} + P C^{c^T} V^{c^{-1}} \left(y - C^c \hat{x}\right)}$$

$$\boxed{\dot{P} = P A^{c^T} + A^c \cdot P - P C^{c^T} V^{c^{-1}} C^c \cdot P}$$

CONT. TIME KF.