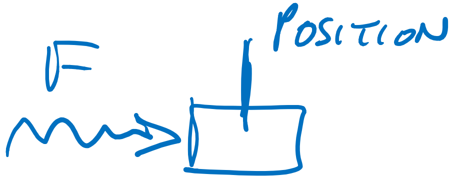
	DISCR	CONT
L	X	X
NL		

KALMAN FILTER

DATA FROM ROBIN

SAMPLING TIME 0.002 s



CONT. UNKNOWN ~~FORCE~~ ~~F~~ ACCELERATION a

MEASUREMENTS: POSITION $\boxed{p}$

$$\boxed{\ddot{p} = a}$$

QUESTION: GET ~~VELOCITY~~ SPEED $s = \dot{p}$ OR a ?

FIRST (NAIVE) WAY TO GET $s = \dot{p}$:

$$\frac{y(k+1) - y(k)}{\Delta t} \approx s(k)$$

VERY NOISY !

DIFFERENTIATING NOISE CREATES LARGER NOISE.

DO NEVER DO IT !

FOR KALMAN FILTER, WE NEED A MODEL

$$\boxed{y = \underbrace{p}_{\text{TRUE POSITION}} + \underbrace{v}_{\text{NOISE}}}$$

v "MEASUREMENT ERRORS"

$v \sim \mathcal{N}(0, \sigma_v^2)$ WITH $\sigma_v \approx 2$

$\sigma_v^2 = 4$

$\ddot{p} = a$ SECOND ORDER DIFF. EQ.

TRANSFORM TO FIRST ORDER

$\rightarrow \dot{p} = s$ + SMALL NOISE

$\rightarrow \dot{s} = a$ + "

$\dot{a} = 0$ + STATE NOISE

$\left.\begin{matrix} p \\ s \end{matrix}\right\}$ STATES

a } INPUT (ARBITRARY, BUT SLOWLY VARYING)

a BECOMES A STATE, MOVES LIKE RANDOM WALK

$$\boxed{\dot{x} = A^c \cdot x + \text{STATE NOISE}}$$

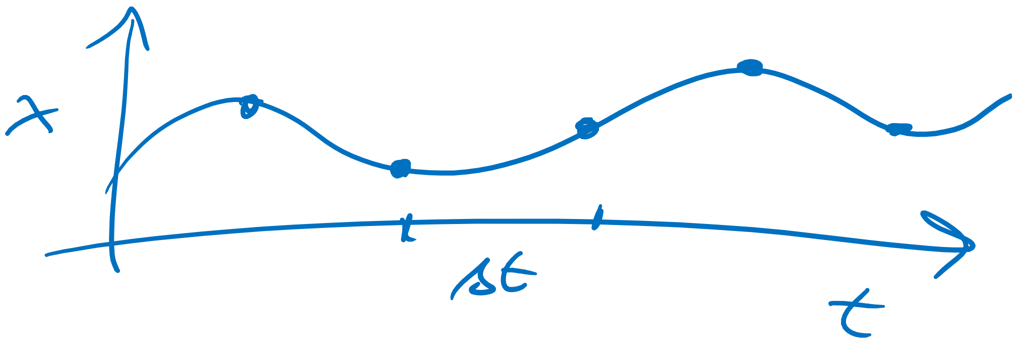
$$A^c = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

A^c: CONT. TIME LTI MODEL MATRIX

x : STATES $x = \begin{pmatrix} p \\ s \\ a \end{pmatrix}$

IDEA FOR TODAY:

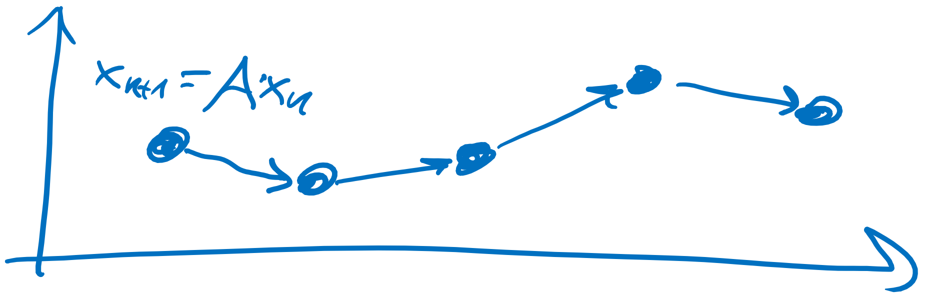
- TRANSF TO DISCR. TIME $\quad x_k \approx x(t_k) \quad$ WITH $\quad t_k = k \cdot \Delta t$
- ADD NOISE
- APPLY D.T. KF



$$\boxed{\dot{x} = A^c \cdot x}$$

$$\left[\begin{aligned} x(t) &= \exp(A^c \cdot t) \cdot x(0) \\ \dot{x}(t) &= A^c \cdot \underbrace{\exp(A^c \cdot t) \cdot x(0)} \\ &= A^c \cdot x(t) \end{aligned}\right]$$

$\Delta t = 0.002$



APPLY MATRIX EXPONENTIAL

$$A = \exp(A^c \cdot \Delta t)$$

$$\boxed{x_{k+1} = A \cdot x_k}$$

$$x_k = \begin{pmatrix} p_k \\ s_k \\ a_k \end{pmatrix}$$

NEED: STATE NOISE COVARIANCE W

IN $\boxed{x_{k+1} = A \cdot x_k + w_k}$

$$W = \begin{pmatrix} 10^{-8} & & \\ & 10^{-8} & \\ & & \alpha \end{pmatrix}$$

$w_k \sim \mathcal{N}(0, W)$

α NEEDS TO BE SPECIFIED BY EXPERIMENTATION

$$y_k = p_k + v_k$$
$$= C \cdot x_k + v_k \qquad C = (1\ 0\ 0)$$

$v_k \sim \mathcal{N}(0, \sigma_v^2) \qquad \sigma_v^2 = 4 \equiv V$

$$x_{k+1} = A \cdot x_k + w_k$$

GIVEN A, C, V, W, CAN GO TO KALMAN FILTER...

[MATRIX EXPONENTIAL]

$$\exp(x) = \underline{1} + x + \frac{x^2}{2} + \frac{x^3}{3\cdot 2} + \dots + \frac{x^k}{k!}$$

$$\exp(A) = I + A + \frac{1}{2}A^2 + \frac{1}{3!}A^3 + \dots + \frac{1}{k!}A^k + \dots$$

$$\exp(A\cdot\Delta t) = I + \Delta t\cdot A + \frac{1}{2}A^2 + \dots$$

SMALL

$$A = \begin{pmatrix} 1 & \Delta t & \frac{1}{2}\Delta t^2 \\ 0 & 1 & \Delta t \\ 0 & 0 & 1 \end{pmatrix}$$

Given:

$$x_{k+1} = A_k x_k + w_k \text{ and } y_k = C_k x_k + v_k \tag{9.19}$$

with i.i.d. zero mean noises with covariances W, V, the steps of the Kalman Filter to solve this problem are:

1. Prediction Step:

$$\hat{x}_{[k|k-1]} = A_{k-1} \cdot \hat{x}_{[k-1|k-1]} \tag{9.20}$$

$$P_{[k|k-1]} = A_{k-1} \cdot P_{[k-1|k-1]} \cdot A_{k-1}^\top + W_{k-1} \tag{9.21}$$

2. Innovation Update Step:

$$P_{[k|k]} = \left(P_{[k|k-1]}^{-1} + C_k^\top V^{-1} C_k\right)^{-1} \tag{9.22}$$

$$\hat{x}_{[k|k]} = \hat{x}_{[k|k-1]} + P_{[k|k]} \cdot C_k^\top V^{-1}(y_k - C_k \hat{x}_{[k|k-1]}) \tag{9.23}$$