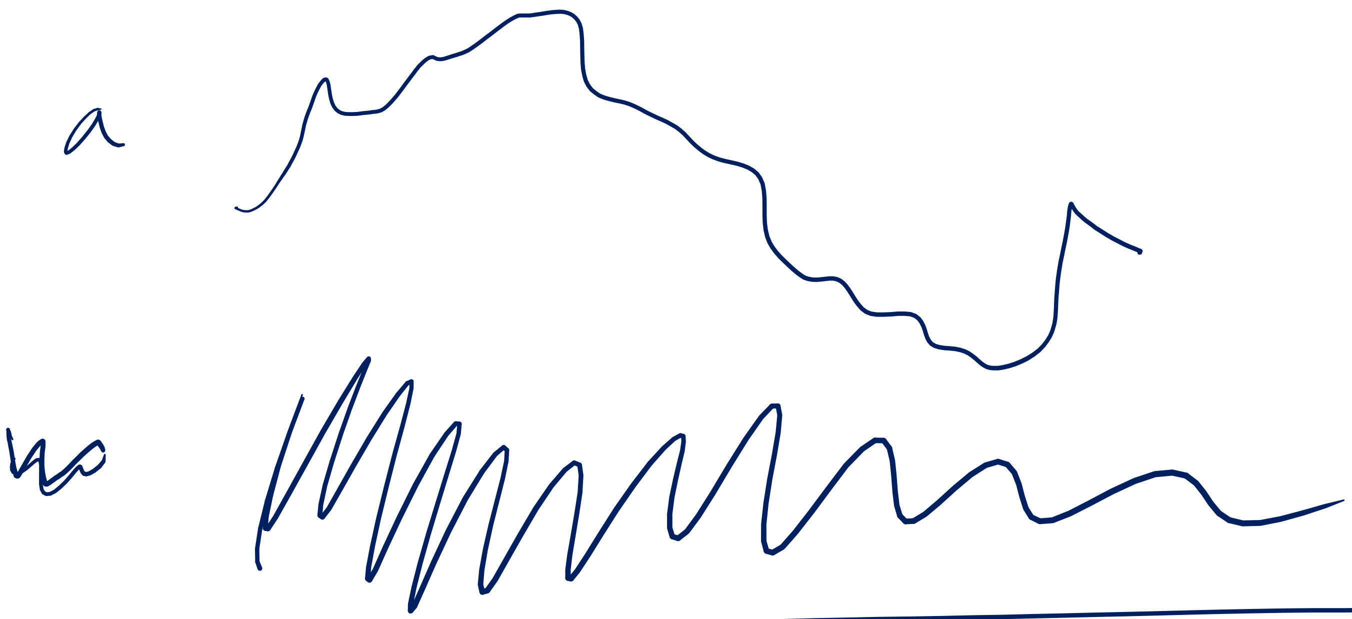
REHEARSAL LECTURE

$\dot{a} = w$



CH 8 & CH 9

CH 8 FREQ DOM. ESTIMATION

LTI - CONT. SYSTEMS

$$\boxed{\dot{y} = u - y}$$

TIME DOM

$$G(s) = \frac{Y(s)}{U(s)} = \frac{1}{1+s}$$

$$\boxed{sY = U - Y}$$

LAPL. DOMAIN

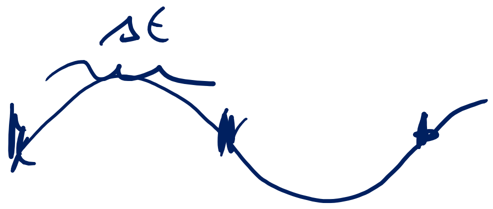
$G(j\omega)$ GIVES BODE DIAGRAM

$$(s+1)Y = U$$

$$\frac{Y}{U} = \frac{1}{1+s}$$

DISCRETE TIME SIGNALS & FOURIER TRANSFORM

NYQUIST FREQUENCY ?



$$f_{NY} = \frac{1}{2\Delta t}$$

$2\Delta t$

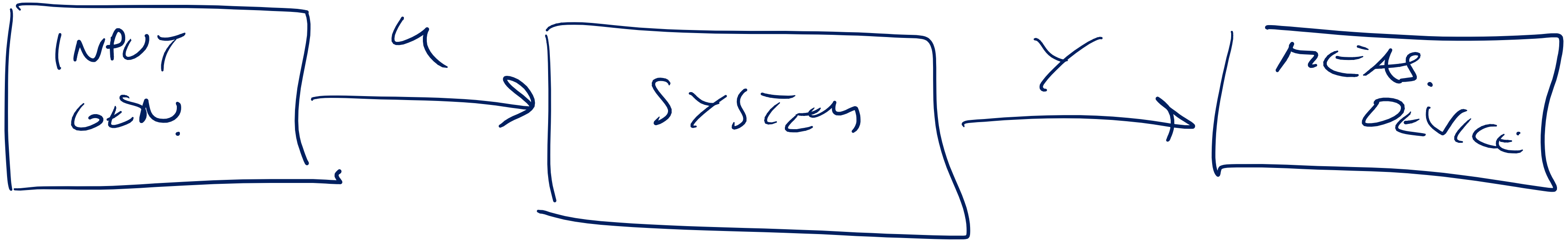
$$\omega_{NY} = 2\pi \cdot f_{NY} = \frac{\pi}{\Delta t}$$

IN PRACTICE, SAMPLE ONLY SIGNALS UP TO HALF NYQUIST

IF WE ARE CLOSE OR ABOVE NYQUIST, AN ERROR OCCURS:

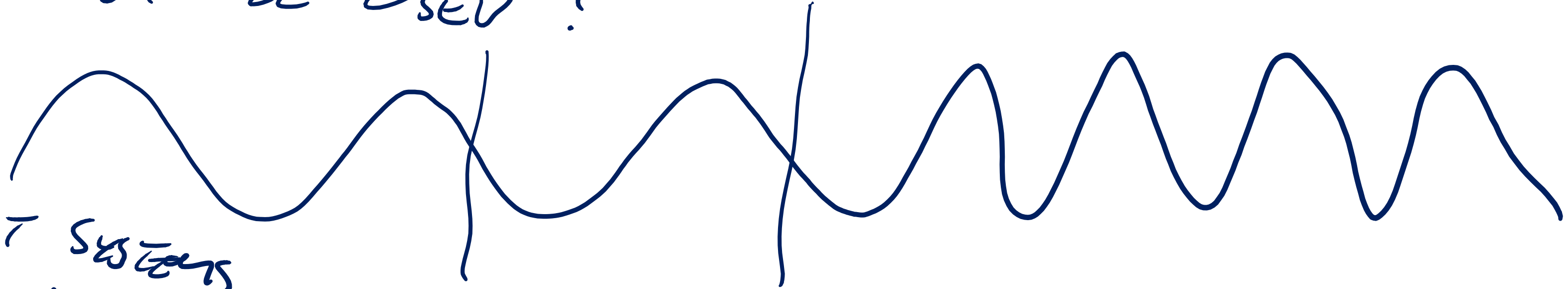
ALIASING

LTI, stable



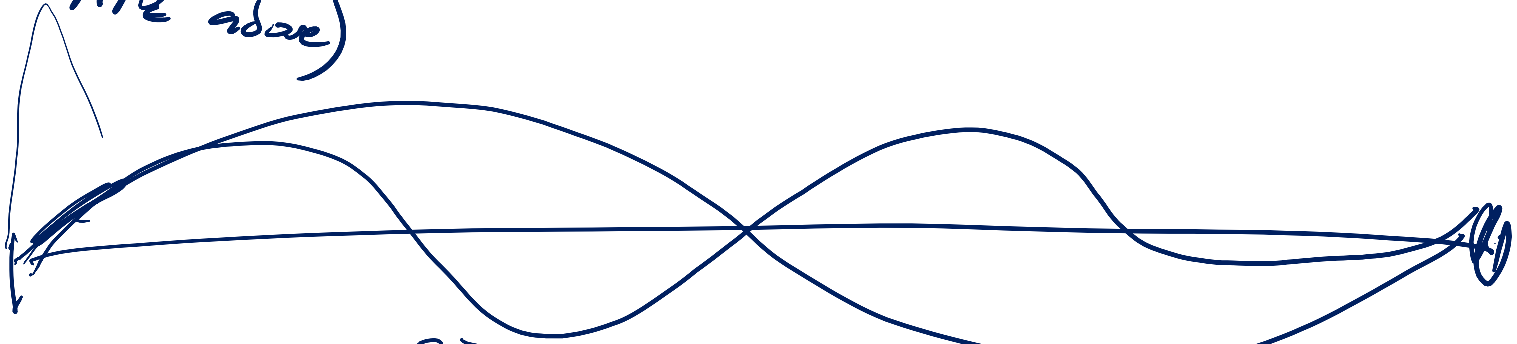
WHAT INPUT SIGNALS MIGHT BE USED ?

1. FREQUENCY SWEEP



GOOD FOR VERY FAST SYSTEMS (kHz - MHz above)

2. MULTISINE EXCITATION



WINDOW LENGTH T

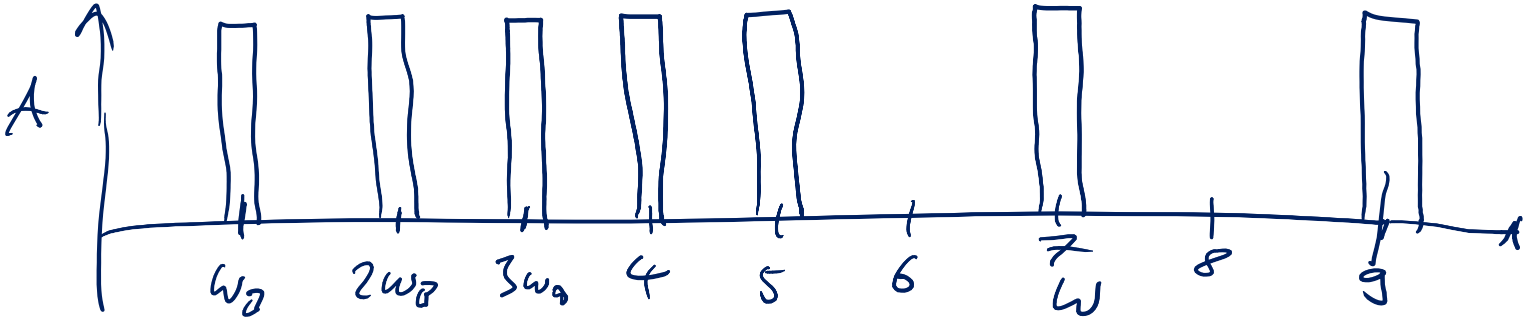
BASE FREQUENCY: $f_B = \frac{1}{T}$, $\omega_B = \frac{2\pi}{T}$

& MULTIPLES

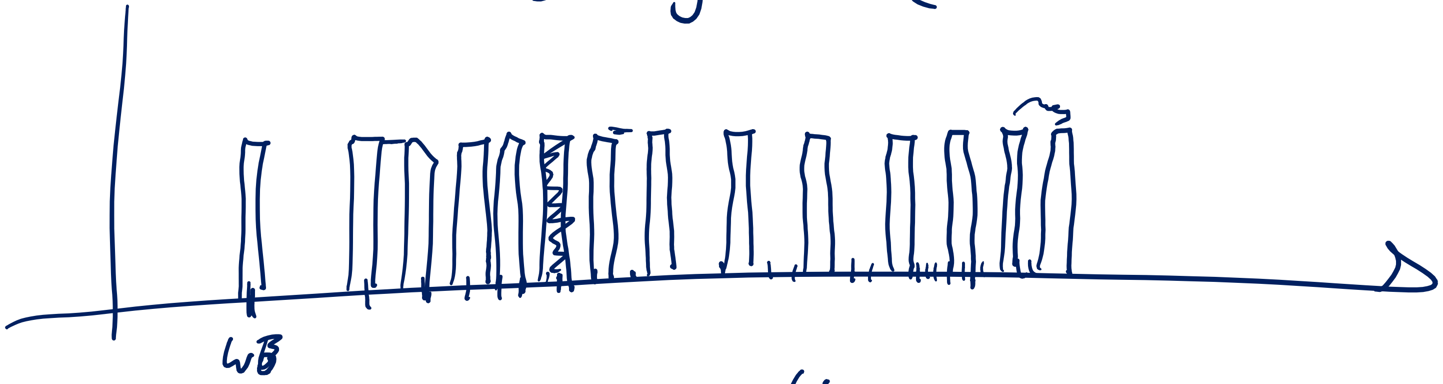
$T = N \cdot \Delta t$ (TO AVOID "LEAKAGE" ERRORS)

WHICH FREQUENCIES?

AMPLITUDE



Log Scale



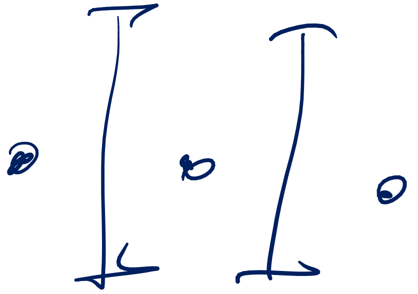
$$\frac{\omega_{k+1}}{\omega_k} \approx 1.05$$

BUT $\omega_k = \omega_B \cdot p(k)$

↑ INTEGER.

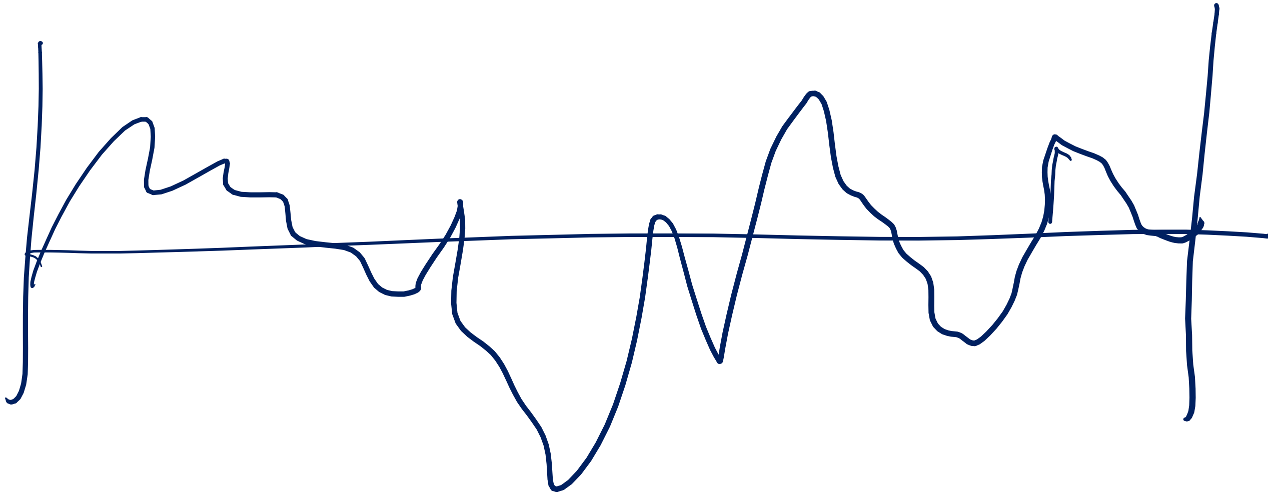
PHASE?

- NOT ALIGNED
- RANDOM
- SCHRÖDER PHASES
- OPTIMIZED FOR CREST FACTOR

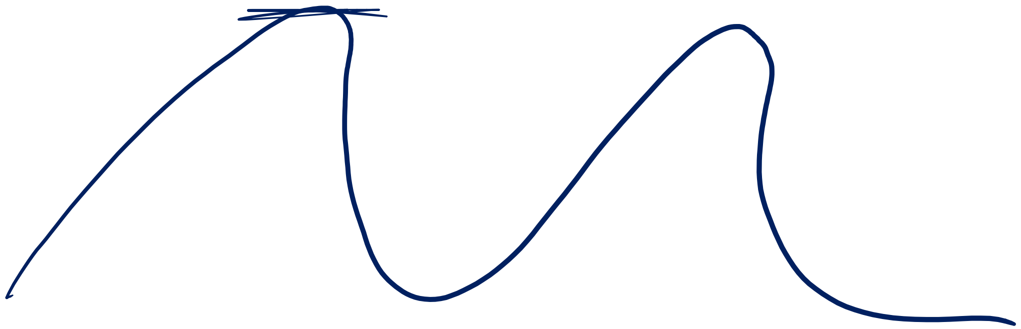


CREST FACTOR

$$CF = \frac{MAX}{RMS} = \frac{\max_{t\in[0,T]} |u(t)|}{\frac{1}{T}\int_0^T |u(t)|^2 dt}$$



$$\left(= \frac{\max |u(t)|}{\text{ROOT (AVERAGE } \cancel{\text{RB}} \text{ AMPLITUDE ON FREQUENCY DOMAIN)}} \right)$$



~~RA~~ PARSEVAL'S THEOREM

$$u(t) = \sum_{n=1}^{N} A \cdot \sin(\omega_0 \cdot kt + \varphi)$$

$$\boxed{\int_0^T |u(t)|^2 dt = \sum A^2 \cdot \int_0^{T} \sin^2(\omega_0 k)\, dt}$$

(DUE TO ORTHOGONALITY OF BASIS FUNCTIONS)

DISCRETE TIME FOURIER

$$\begin{bmatrix} u(0) \\ \vdots \\ u(N-1) \end{bmatrix} \longrightarrow \begin{bmatrix} \bar{U}(0) \\ \vdots \\ \bar{U}(N-1) \end{bmatrix} = \begin{bmatrix} \text{"DFT MATRIX"} \end{bmatrix} \cdot \begin{bmatrix} u(0) \\ \vdots \\ u(N-1) \end{bmatrix}$$

$$\bar{U} = \overset{\text{ORTHOGONAL}}{M} \cdot \bar{u}$$

$$\bar{U}^T \cdot \bar{U} = \bar{u}^T \big(\underbrace{M^T M}_{\text{(DIAGONAL)}}\big) \bar{u} = \sum u(k)^2$$

$$\bar U = \underline{fft}(\bar u) = \boxed{M \cdot \bar u} \qquad M \in \mathbb{C}^{N \times N}$$

COMPUTED BY "FAST FOURIER TRANSF (FFT)"

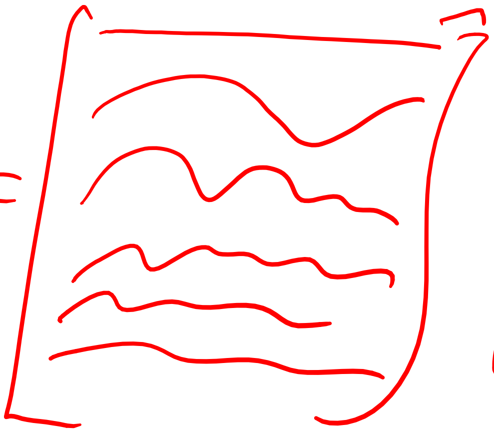
$$M^H M = I \cdot N$$

$$\bar u = ifft(\bar U) = \frac{1}{N} M^H \cdot \bar U$$

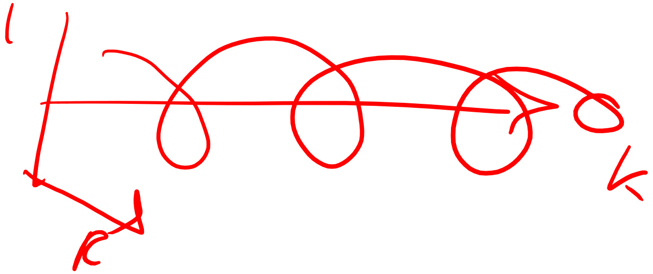
$$\bar u = ifft(fft(\bar u)) = \frac{1}{N} M^H \cdot (M \cdot \bar u) = \bar u$$

$$M = \begin{bmatrix} \alpha^0 & \alpha^1 & \alpha^2 & \alpha^3 & \cdots \\ \alpha^0 & \alpha^2 & \alpha^4 & \alpha^6 & \\ & & & & \end{bmatrix}^T =$$

(OR SIMILAR)



$$\boxed{\alpha = e^{-\frac{2\pi j}{N}}}$$



FOURIER MATRIX

AFTER RECORDING THE MULTISINE OUTPUT SIGNAL,
DISCARDING FIRST PERIODS (TRANS.)
AVERAGING "GOOD" PERIODS
GET ~~$\hat{Y}(k)$ AND $\hat{U}(k)$~~ $\hat{y}(t)$

$\hat{Y}(k)$ FROM iFFT of $\hat{y}$

$$\text{TIME} \underset{\text{ifft}}{\overset{\text{fft}}{\rightleftarrows}} \text{FRE}$$

$$\boxed{G(j\omega_B \cdot k) = \frac{\hat{Y}(k)}{\hat{U}(k)}}$$

FOR k IN INPUT SIGNAL

$|G|$

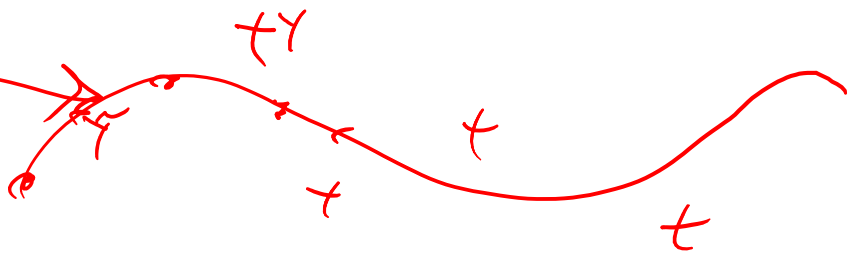
$\arg G$

OUTPUT ERROR vs EQUATION ERROR

PURE ~~OUTPUT~~ EQ. ERRORS

$$y(k) = \underbrace{\sum_{i=1}^{n} a_i y(k-i) + b_i u(k-i)}_{\text{PRED}} + \varepsilon(k)$$

TRUE VALUE & MEASUREMENT



$$\boxed{\tilde{y}(k) = \underbrace{\sum a_i \tilde{y}(\ ) + u(\ )}_{\text{PRED}}}$$

$$y(k) = \tilde{y}(k) + \varepsilon(k)$$