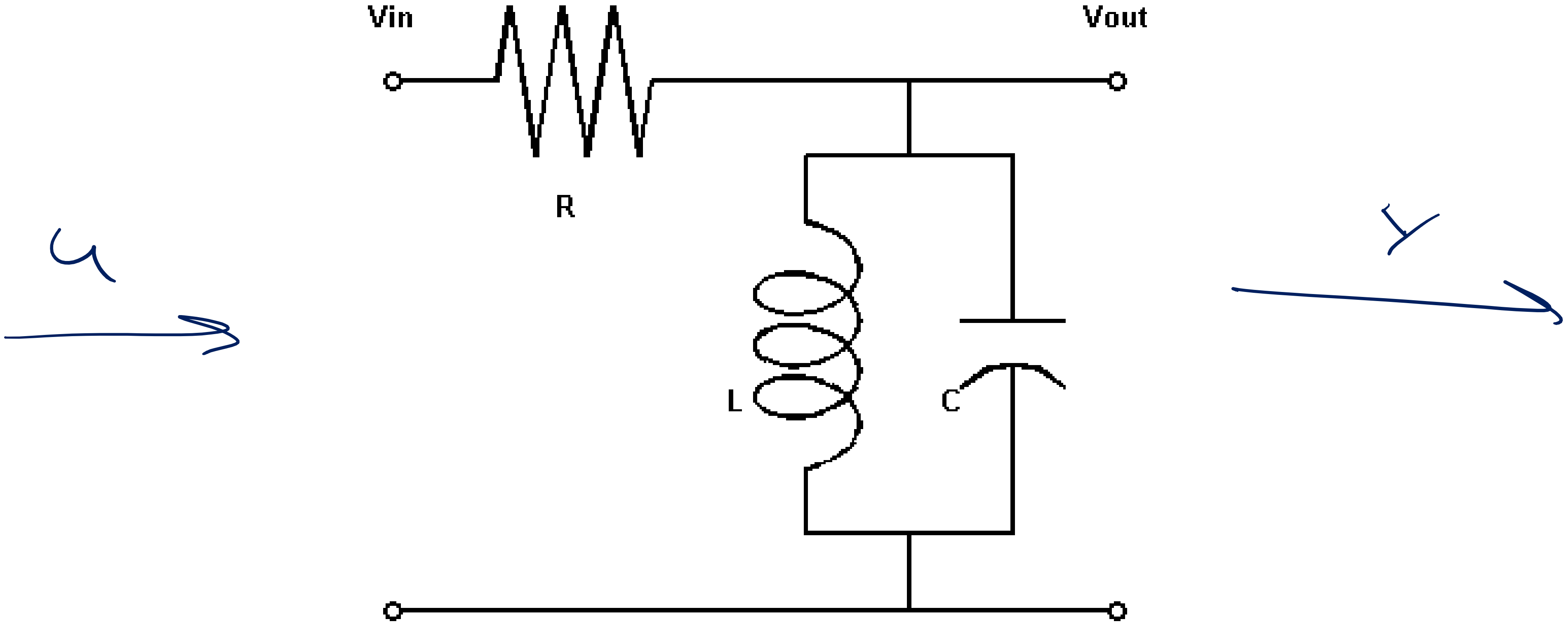
AIM OF TUTORIAL (OUTPUT ERRORS FOR DET. CONT. TIME SYSTEM).

GIVEN: 1) SYSTEM STRUCTURE

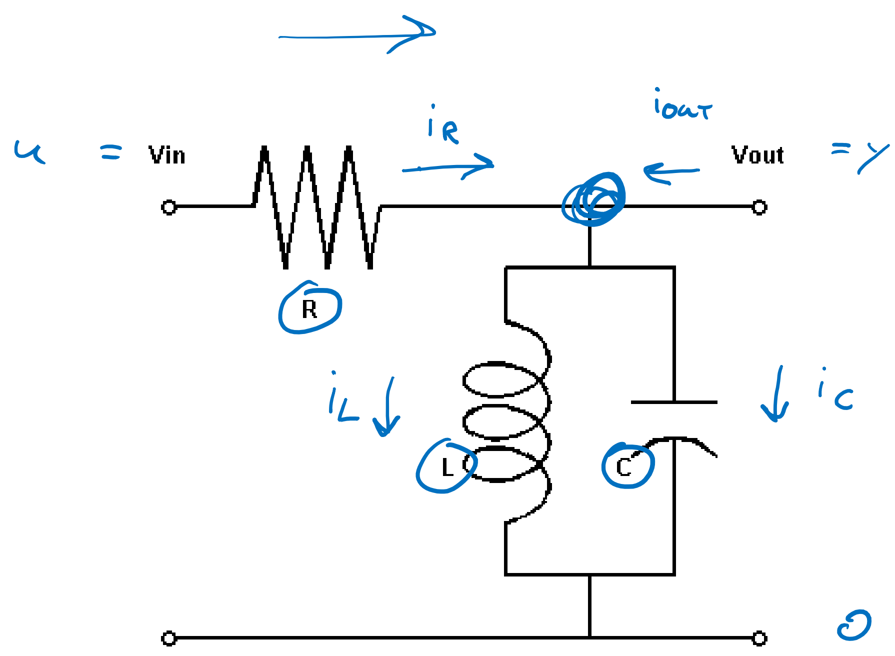



2) DATA: $\Delta t = 10^{-5}$ $N = 1000$

u

y

STEP ONE: MODEL SYSTEM




$i_R + i_{out} = i_L + i_C$ $\quad \boxed{i_{out} = 0}$

$i_R = i_L + i_C$

$R i_L + RC \cdot \dot{y} = -y + u$

$\boxed{R \frac{di_L}{dt} + RC \cdot \ddot{y} = -\dot{y} + \dot{u}}$

$R i_L + R i_C = -y + u$

$$\boxed{\ddot{y} + a_1 \dot{y} + a_0 \cdot y = b_0 \cdot u + b_1 \dot{u} + b_2 \cdot \ddot{u}}$$

$R \cdot i_R = -(y - u)$

$L \frac{d}{dt}(i_L) = -(0 - y) = y$

$i_C = -\frac{d}{dt}(0 - y)\, C = \dot{y} \cdot C$

$$\boxed{\begin{aligned} &R\, i_R = -y + u \\ &\boxed{L \frac{di_L}{dt} = y} \\ &i_C = \dot{y} \cdot C \end{aligned}}$$

$\frac{R}{L} y + RC \cdot \ddot{y} = -\dot{y} + \dot{u}$

$\boxed{RC \cdot \ddot{y} + \dot{y} + \frac{R}{L} y = \dot{u}}$

$\boxed{a \ddot{y} + \dot{y} + b y = \dot{u}}$

ONLY TWO PARS CAN BE IDENTIFIED!

$\boxed{a = RC}$ $\quad \boxed{b = \frac{R}{L}}$

$[a] = s \quad [b] = \frac{1}{s}$

$$a\ddot{y} + \dot{y} + by = \dot{u}$$

$$as^2Y + sY + bY = sU$$

$$\frac{Y}{U} = \frac{s}{as^2+s+b} = G(s)$$

START SIMULATION IN MATLAB

$a = \cancel{1}\ 3$, $\quad b = 2$

POLES OF SYSTEM ?

$$as^2 + s + b \overset{!}{=} 0$$

$$s^2 + \frac{1}{a}s + \frac{b}{a} = 0$$

$$s_{1,2} = -\frac{1}{2}\frac{1}{a} \pm \sqrt{\underbrace{\frac{1}{4a^2} - \frac{b}{a}}_{\text{NEGATIVE}}}$$

NEGATIVE = OSCILLATION

3

RECALL:

GIVEN:

$u, y$ $\qquad y \in \mathbb{R}^N, \quad \theta \in \mathbb{R}^d$

OBTAINED PREDICTION MODEL $M(\theta; u)$ (using lsim)

GEN. LS FITTING FCN $M(\underline{\underline{\theta}}; u) - y =: res(\theta)$

$$\hat{\theta}_{ML} = \arg\min_{\theta} \|res(\theta)\|_2^2$$

(using lsqnonlin)

ATTENTION: INITIAL GUESS NEEDED FOR N-LS

HOW CERTAIN IS $\hat{\theta}_{ML}$ ?

DESIRED $\Sigma_{\theta} := cov(\hat{\theta}_{ML}) \approx \sigma_y^2 \cdot \underline{\underline{\left( J(\hat{\theta})^T \cdot \underline{\underline{J(\hat{\theta})}} \right)^{-1}}}$

$$\sigma_y^2 \approx \left( \frac{\|res(\hat{\theta}_{ML})\|_2^2}{(N-d)} \right)$$

$$J(\hat{\theta}) = \frac{\partial M}{\partial \theta}(\hat{\theta}) = \underbrace{\boxed{\phantom{xx}}}_{\dim \theta} \dim y$$

IDEA: GET $J(\vec{\theta})$ BY FINITE DIFFERENCES

$$\boxed{\frac{\partial M}{\partial \theta_1} \approx \frac{M\left(\theta + \begin{pmatrix}\Delta\theta_1 \\ 0 \\ \vdots\end{pmatrix}\right) - M(\theta)}{\Delta\theta_1}}$$

$$J(\theta) = \frac{\partial M}{\partial \theta}(\vec{\theta}) = N\left\{\underbrace{\begin{pmatrix} \frac{\partial M_1}{\partial \theta_1} & \cdots & \frac{\partial M_1}{\partial \theta_d} \\ \frac{\partial M_2}{\partial \theta_1} & & \vdots \\ \vdots & & \vdots \\ \frac{\partial M_N}{\partial \theta_1} & \cdots & \frac{\partial M_N}{\partial \theta_d} \end{pmatrix}}_{d}\right.$$

$$\frac{\partial M}{\partial \theta_1}$$

PERTURB EACH PARAM. BY 1%

$$\begin{pmatrix}\Delta\theta_1 \\ 0 \\ \vdots \\ 0\end{pmatrix}$$

$$M\left(\theta + \begin{pmatrix}\Delta\theta_1 \\ 0 \\ \vdots \\ 0\end{pmatrix}\right) = M(\theta) + J(\theta)\cdot\begin{pmatrix}\Delta\theta_1 \\ 0 \\ \vdots \\ 0\end{pmatrix} + O\left((\Delta\theta_1)^2\right)$$

$$M\left(\theta + \begin{pmatrix}\Delta\theta_1 \\ 0\end{pmatrix}\right) = M(\theta) + \left(\frac{\partial M}{\partial \theta_1}(\theta)\right)\cdot \Delta\theta_1 + \dots$$

COMPARISON WITH JESUS:

$L = 1\,mH$

$C = 25{,}3\,\mu F$

$R = 10\,\Omega$

$a = R \cdot C = 253\,\mu s$

$b = \frac{R}{L} = 10\,000\,\frac{\Omega}{H} = 10\,000\,\frac{1}{s}$

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WE GOT: $a = 0.00252 \pm 0.00007 = 2520 \pm 70\,\mu s$

$b = 100\,000 \pm 3000 = 100.000 \pm 3000\,\frac{1}{s}$

GUESS: WAS $R = 100\,\Omega$ ??