Fault-tolerant MPC of redundant Permanent Magnet Synchronous Machines

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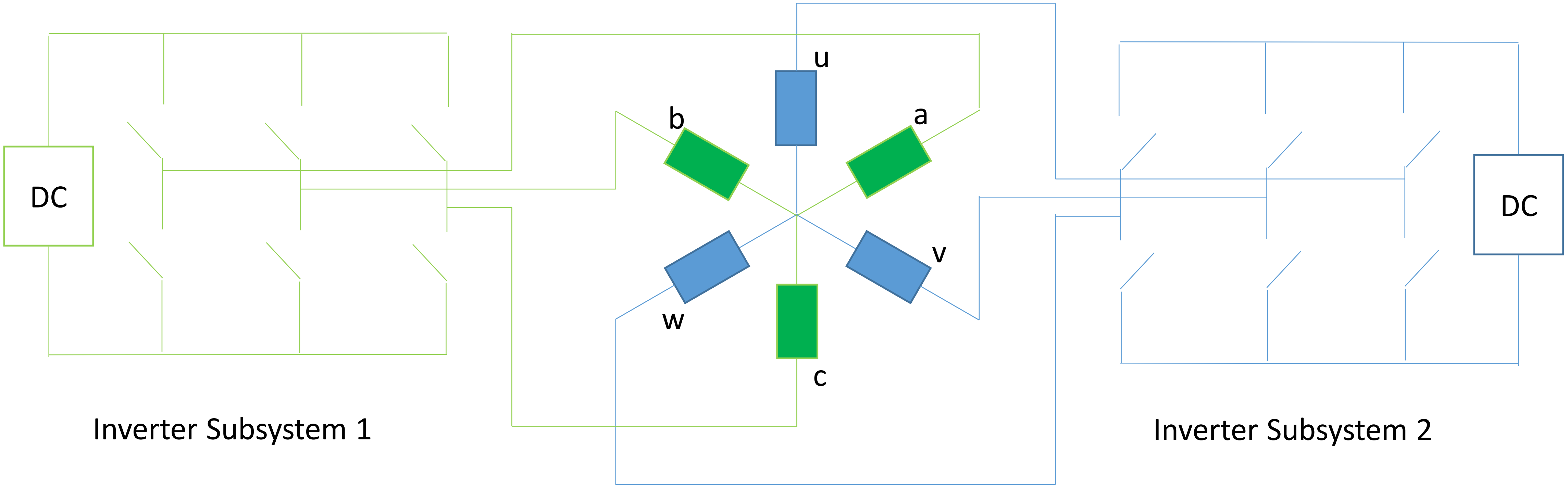
Overview

Introduction

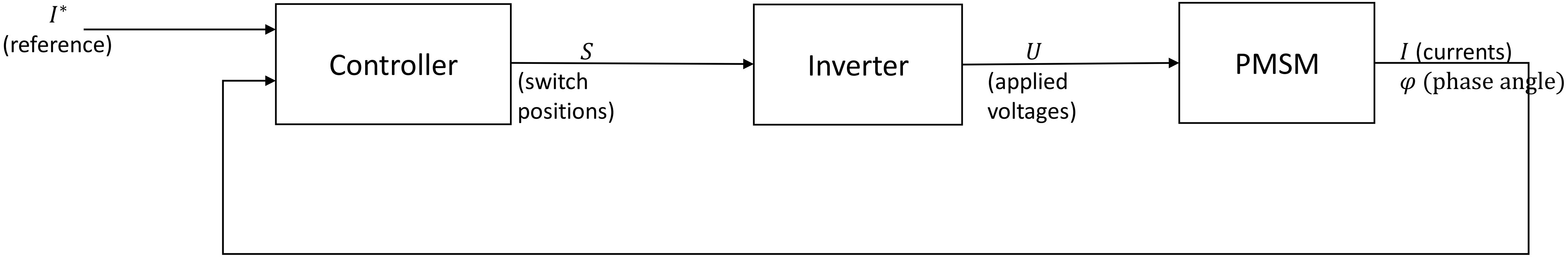
- Setup 2-redundant PMSM with 2-level inverters



Motivation - redundant PMSM

- Application: Steering drives in autonomous vehicles
- Frequent fault: Broken switch in inverter
- Use redundancy to generate desired torque

Motivation – Finite-Set MPC



Motivation – Finite-Set MPC

Common control approach for PMSM

- Computation of continuous inputs (voltages)
- Conversion of inputs to discrete switch positions by modulator

Finite-Set MPC

- Direct Computation of discrete switch position
- Online Solution of Mixed-Integer Optimization Problem
- Aim: minimize switching losses in inverter

Simulation Model

- System equations in stator-fixed abc-coordinates

$$\dot{I} = L^{-1}(-(\omega\frac{\partial L}{\partial\varphi} + R)I + U - \omega\frac{\partial\Psi_m}{\partial\varphi})$$

$I(t) = [I_{1\mathrm{a}}(t) \quad I_{1\mathrm{b}}(t) \quad I_{1\mathrm{c}}(t) \quad I_{2\mathrm{a}}(t) \quad ...]^{\mathrm{T}}$	Currents of each phase and subsystem
$U(t) = [U_{1\mathrm{a}}(t) \quad U_{1\mathrm{b}}(t) \quad U_{1\mathrm{c}}(t) \quad U_{2\mathrm{a}}(t) \quad ...]^{\mathrm{T}}$	Applied Voltages
$L(\varphi(t))$	Inductance matrix (periodic)
R	Resistance matrix
$\omega(t)$	Rotational speed
$\Psi_m(\varphi(t))$	Permanent magnet flux (periodic)

Prediction Model

- System equations in rotor-fixed dq-coordinates (N subsystems)

$$\dot{x} = A(\omega)x + Bu + B_z(\omega)\Psi_{\mathrm{m,dq}}$$

$$T(x) = C\sum_{l=1}^{N}[\Psi_{m,2l-1}x_{2l} - \Psi_{m,2l}x_{2l-1} + \sum_{k=1}^{2N} L_{2l-1,k}x_k x_{2l} - L_{2l,k}x_k x_{2l-1}]$$

$x = I_{\mathrm{dq}}(t) = [I_{\mathrm{1d}} \quad I_{\mathrm{1q}} \quad I_{\mathrm{2d}} \quad \ldots]^{\mathrm{T}}$	Currents of each phase and subsystem
$u = T_{\mathrm{dq}}(\varphi(t)) \cdot U_{\mathrm{abc}}$	Applied Voltages
T	Torque
$A(\omega) = -L_{\mathrm{dq}}^{-1}(R_{\mathrm{dq}} + \omega \cdot Y \cdot L_{\mathrm{dq}})$	System matrix
$B = L_{\mathrm{dq}}^{-1}$	Input matrix
$B_z(\omega) = -\omega L_{\mathrm{dq}}^{-1} Y$	Disturbance matrix

Prediction Model

- Assumption: constant rotational speed within the prediction horizon
- Linear system dynamics
- Time discretization: explicit Euler discretization

Finite-Set MPC

- System inputs are switch positions
- Optimization Problem is a Mixed-Integer Problem
- Number of possible solutions grow exponentially with prediction horizon
- Desired Sampling time: 25 μs

Current Control - L^1-Norm

- Discrete OCP

$$\underset{U,X}{\text{minimize}} \quad \sum_{l=k}^{k+M-1} \|Q(x(l+1) - x^\star(l+1))\|_1 + \|R(u(l) - u(l-1))\|_1$$

$$\begin{aligned} \text{subject to} \quad & x(l+1) = Ax(l) + \tilde{B}(\varphi(l))u(l) + C && l = k, ..., k+M-1 && \text{System dynamics} \\ & \underline{\mathrm{x}} \le x(l+1) \le \bar{x} && l = k, ..., k+M-1 && \text{State constraints} \\ & u(l) \in \{0,1\}^{3N} && l = k, ..., k+M-1 && \text{Input constraints} \end{aligned}$$

$x(k)$: current state, $\quad u(k-1)$: current switch position

Q, R : weight matrices

Current Control - L^1-Norm

- Condensing leads to MILP

$$\min_z f^T z$$

s.t.

$$y \geq X(k) - X^\star(k)$$
$$y \geq -(X(k) - X^\star(k))$$
$$v \geq U(k) - U(k-1)$$
$$v \geq -(U(k) - U(k-1))$$

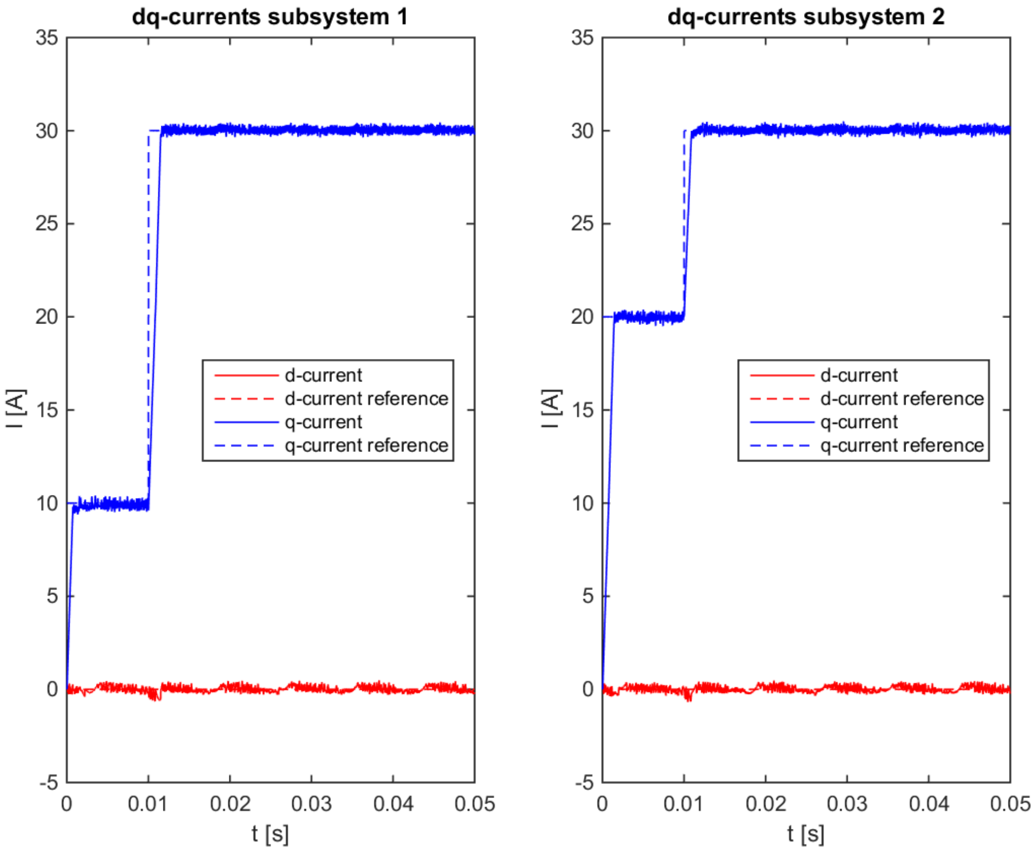
$$X(k) = \begin{pmatrix} x(k+1) \\ . \\ . \\ . \\ x(k+M) \end{pmatrix}, \; U(k) = \begin{pmatrix} u(k) \\ . \\ . \\ . \\ u(k+M-1) \end{pmatrix}$$

$$f^T = (Q \; ... \; Q \; R \; ... \; R \; 0 \; ... \; 0)$$
$$z^T = (y \; v \; u) = (y(k+1) \; ... \; y(k+M) \; v(k) \; ... \; v(k+M-1) \; u(k) \; ... \; u(k+M-1))$$

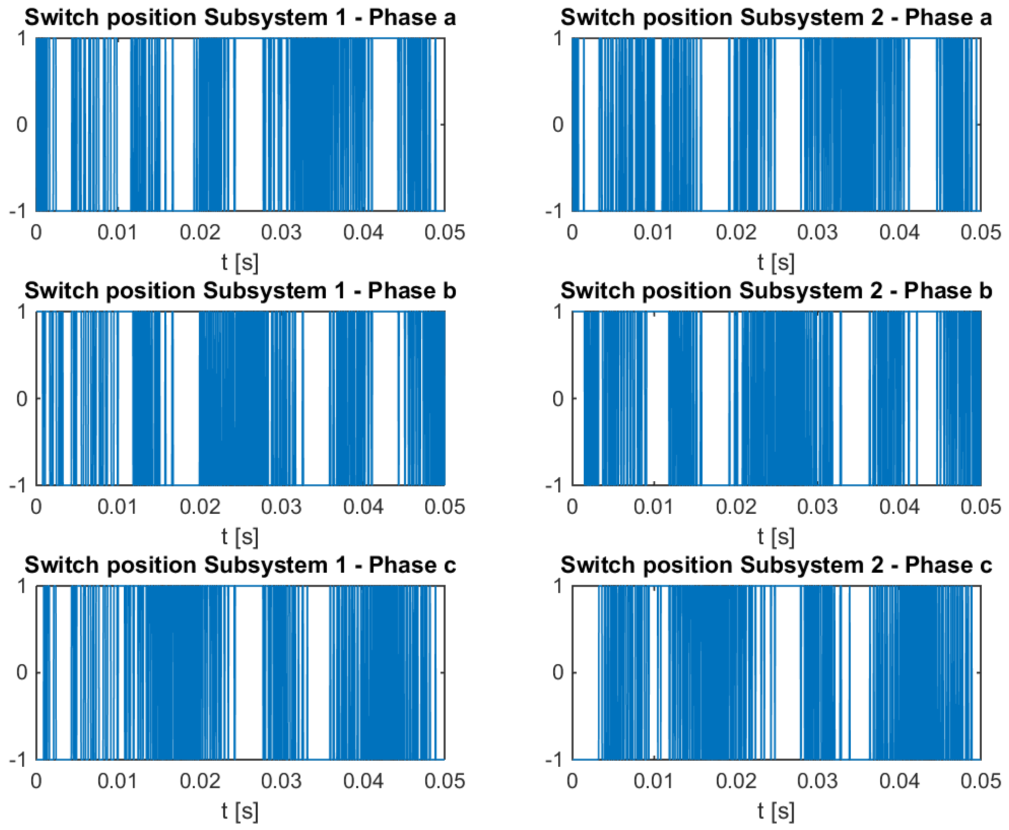
Current Control - L^1-Norm - Results

- Solver: „intlinprog“, Matlab Optimization Toolbox
- Rotational Speed: 300 rpm
- Prediction Horizon: 2
- Reference Step at 0.01 s

Current Control - L^1-Norm - Results



Current Control - L^1-Norm - Results



Current Control - L^2-Norm

- Discrete OCP

$$\underset{U,X}{\text{minimize}} \quad \sum_{l=k}^{k+M-1} \|Q(x(l+1) - x^\star(l+1))\|_2^2 + \|R(u(l) - u(l-1))\|_2^2$$

$$\begin{aligned} \text{subject to} \quad & x(l+1) = Ax(l) + \tilde{B}(\varphi(l))u(l) + C && l = k, ..., k+M-1 && \text{System dynamics} \\ & u(l) \in \{0,1\}^{3N} && l = k, ..., k+M-1 && \text{Input constraints} \end{aligned}$$

$x(k)$: current state, $\quad u(k-1)$: current switch position

Q, R : weight matrices

Current Control - L^2-Norm

- Reformulate discrete OCP as integer least-squares problem (T. Geyer, 2013)

$$\min_{U(k)} \|H \cdot U(k) - H \cdot U_{\text{unc}}(k)\|^2$$

- System dynamics, current state, reference values are contained in lower triangular matrix H
- Unconstrained solution U_{unc} can be calculated analytically

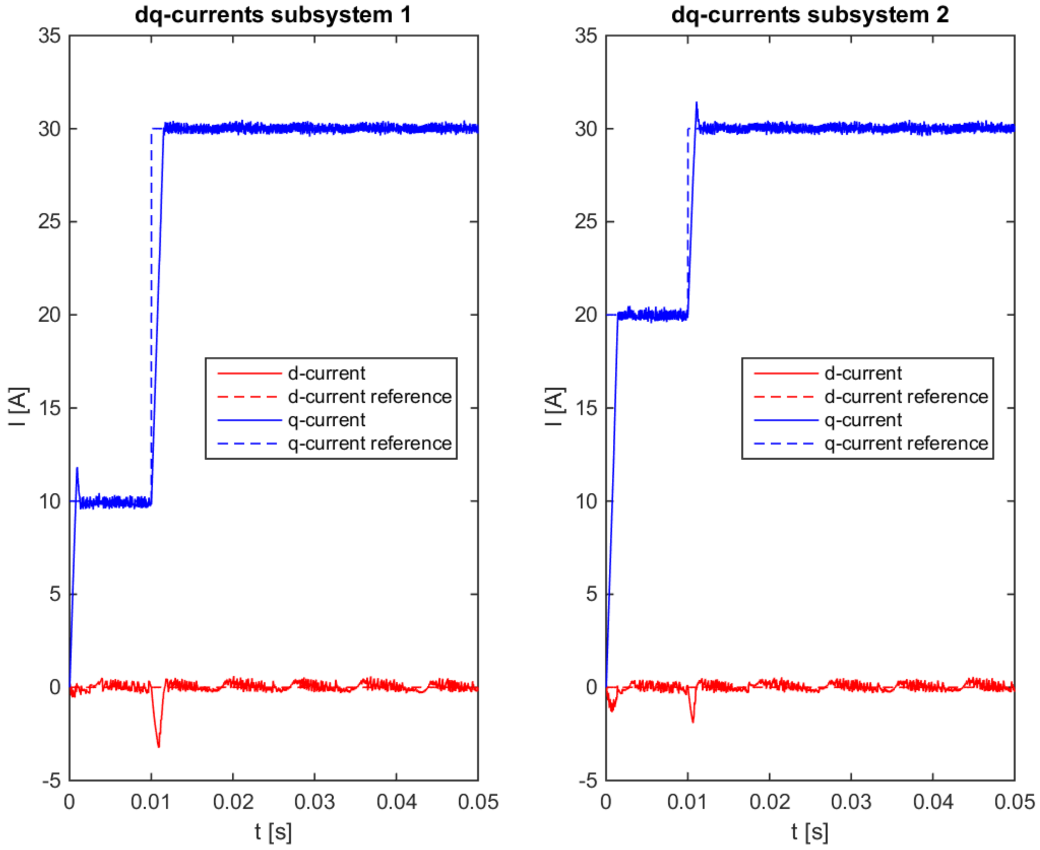
Current Control - L^2-Norm

- Use Sphere Decoding Algorithm to solve integer least squares problem
- based on Branch&Bound

Current Control - L^2-Norm - Results

- Rotational Speed: 300 rpm
- Prediction Horizon: 2
- Reference step at 0.01 s
- Average computation time: 200 μs

Current Control - L^2-Norm - Results



Torque Control

- Discrete OCP

$$\begin{aligned}
\underset{U,X}{\text{minimize}} \quad & \sum_{l=k}^{k+M-1} \|T(x(l+1)) - T^\star(l+1)\|_2^2 + \lambda_u \|u(l) - u(l-1)\|_2^2 + \lambda_i \|x(l+1)\|_2^2 \\
\text{subject to} \quad & x(l+1) = Ax(l) + \tilde{B}(\varphi(l))u(l) + C \qquad l = k, ..., k+M-1 \qquad \text{System dynamics} \\
& u(l) \in \{0,1\}^{3N} \qquad l = k, ..., k+M-1 \qquad \text{Input constraints}
\end{aligned}$$

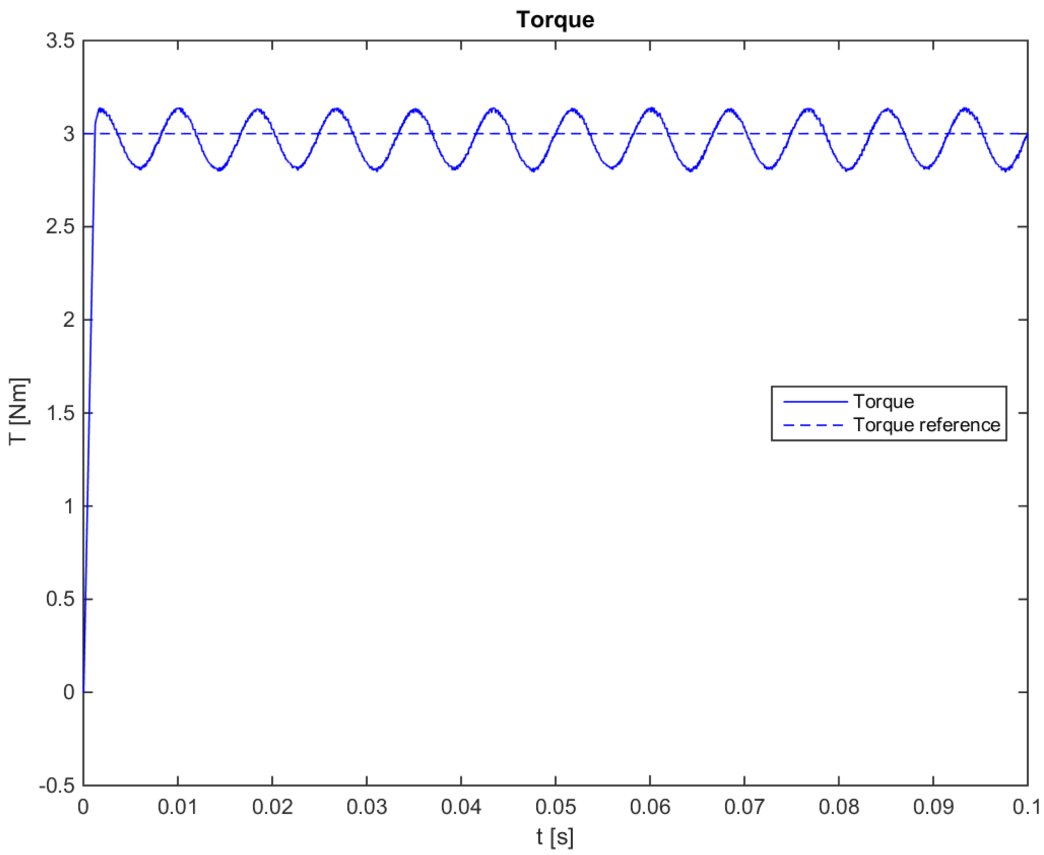
$x(k)$: current state, $\quad u(k-1)$: current switch position

- Last term necessary to guarantee energy optimality

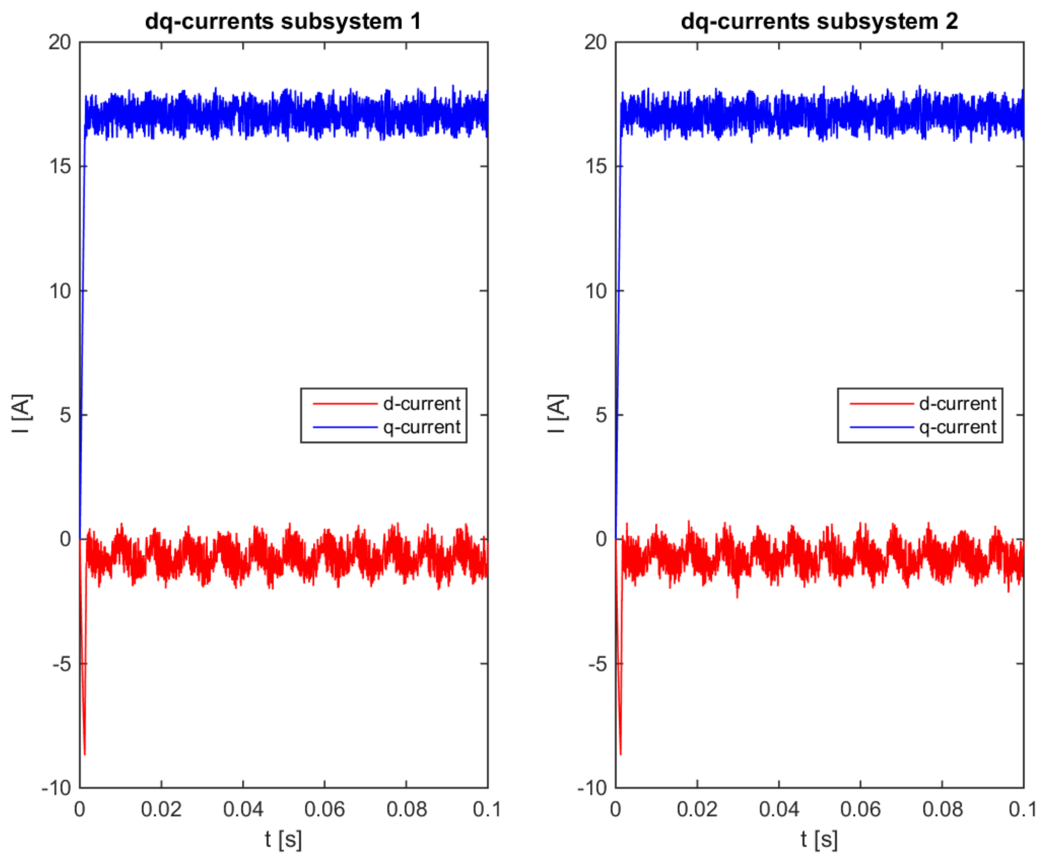
Torque Control - Results

- Rotational Speed: 300 rpm
- Control method: Full enumeration
- Prediction horizon: 2

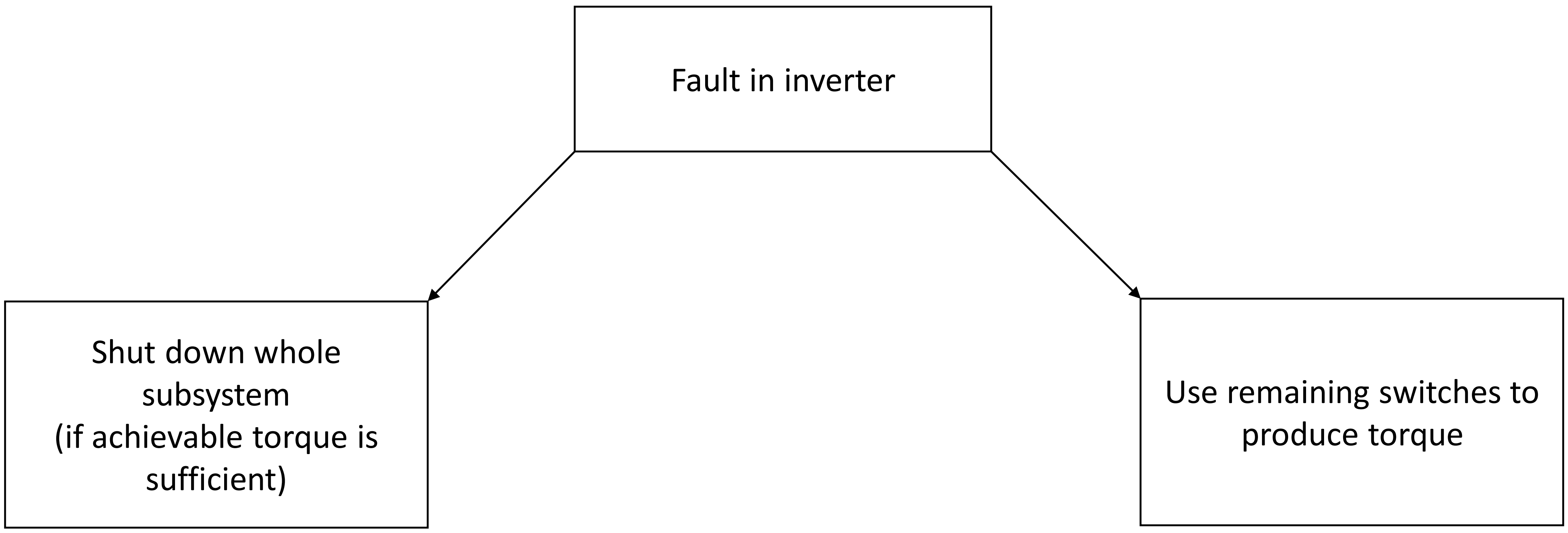
Torque Control - Results



Torque Control - Results



Fault-tolerant Torque control



Fault-tolerant Torque control

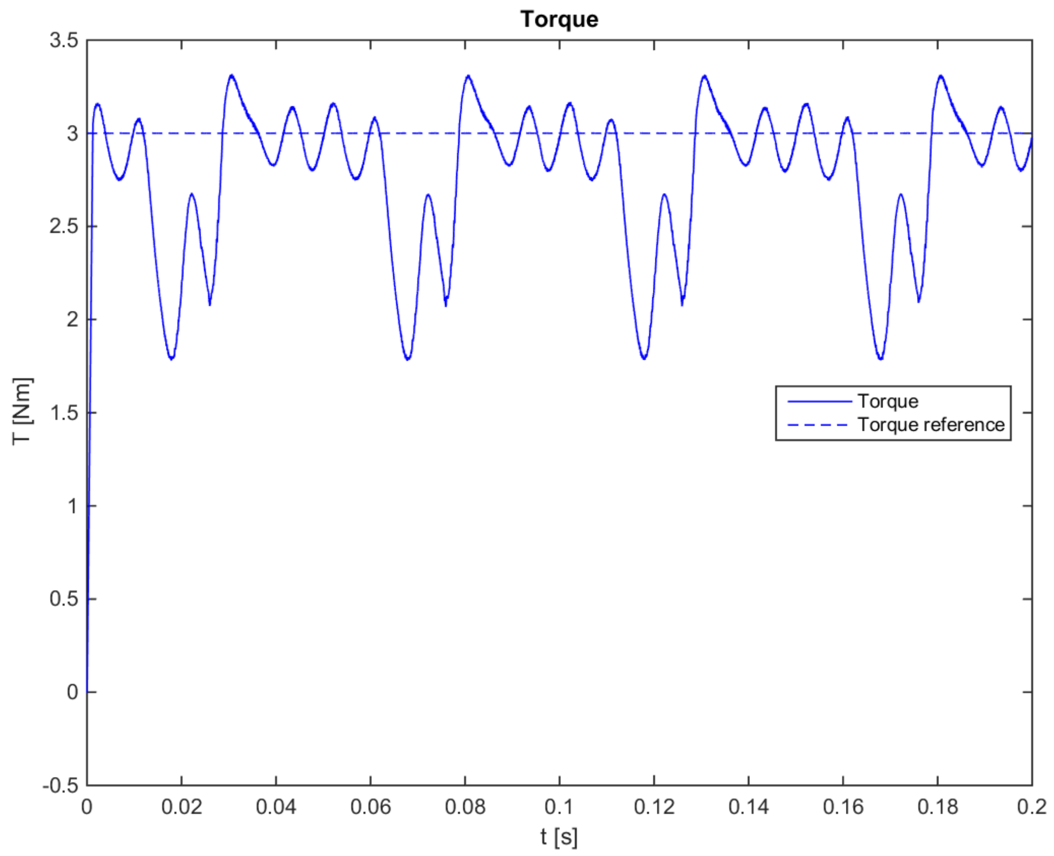
- Discrete OCP – fault in switch j

$$\begin{aligned}
\underset{U,X}{\text{minimize}} \quad & \sum_{l=k}^{k+M-1} \|T(x(l+1)) - T^\star(l+1)\|_2^2 + \lambda_u \|u(l) - u(l-1)\|_2^2 + \lambda_i \|x(l+1)\|_2^2 \\
\text{subject to} \quad & x(l+1) = Ax(l) + \tilde{B}(\varphi(l))u(l) + C \qquad l = k, ..., k+M-1 \qquad \text{System dynamics} \\
& u_{-j}(l) \in \{0,1\}^{3N-1} \qquad l = k, ..., k+M-1 \qquad \text{Input constraints} \\
& u_j(l) = 0 \vee 1 \qquad l = k, ..., k+M-1 \qquad \text{Fault constraints}
\end{aligned}$$

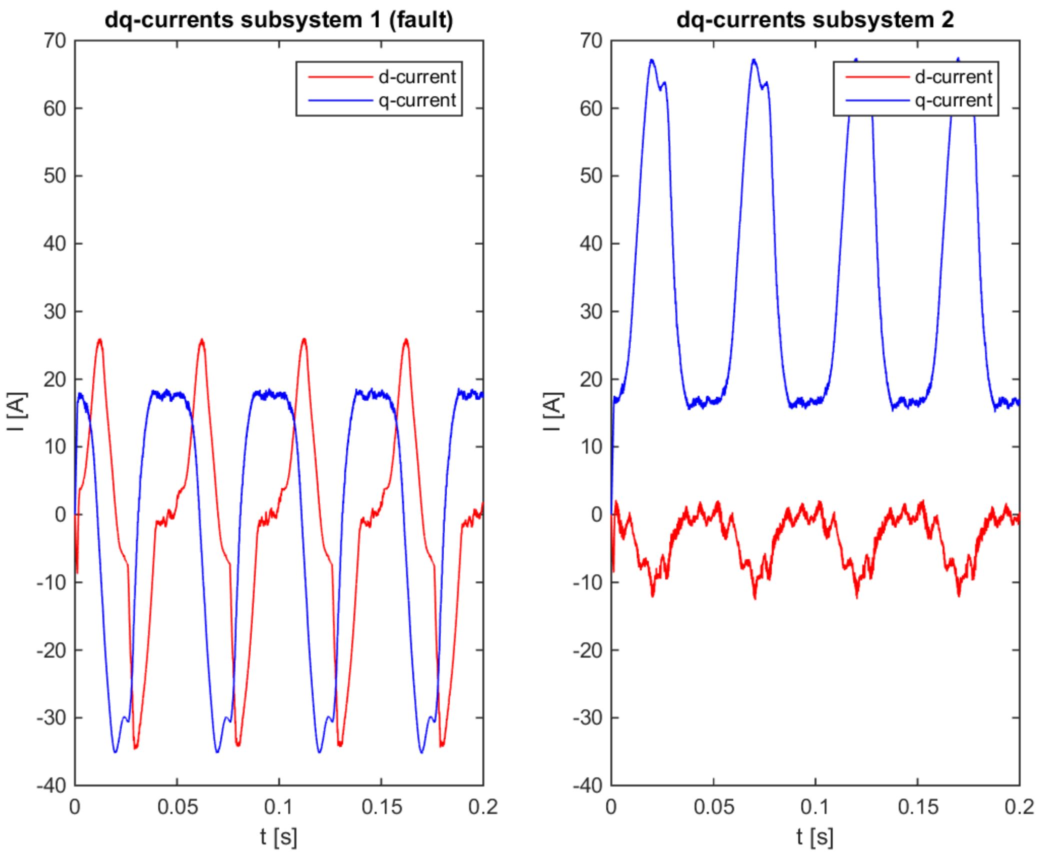
$x(k)$: current state, $\quad u(k-1)$: current switch position

- Switch j fixed at 0 or 1

Fault-tolerant Torque control - Results



Fault-tolerant Torque control - Results



Future Work

- Adapt efficient current control method on torque control
- Focus on algorithmic efficiency; computation time