# Real-time Optimal Control of an NPC Inverter via an RCP System

Master Thesis Presentation

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*Done in cooperation with IMTEK and Fraunhofer ISE.*

University of Freiburg

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# Overview

# Wind and photovoltaic power plants

Figure: Solar and wind plants. Source: iDEAL Energies

## Growth trend

- Solar- & wind energy sources experiencing exponential growth.

## Challenge to integration

- Need to support complex power conversion requirements to maintain grid stability.

# Motivation for optimisation techniques

Strict grid requirements demand use of advanced control/signal processing methods.

## Why use optimisation methods

Can handle problems involving

- static and dynamic constraints.
- MIMO Systems.

## Drawbacks

- Generally computationally intensive algorithms.
- Challenging for fast-sampled applications.

# Objectives & Hardware Setup

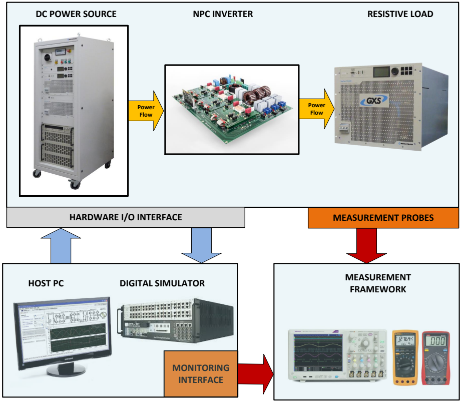
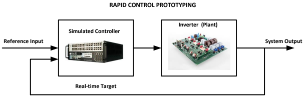



Figure: Hardware Integration for RCP.

## Main Objectives

- Prototyping environment setup.
- Design/evaluate optimal control/estimation methods
- for sampling time $T_s < 100\ \mu s$.



# Analog & Digital I/O Signal Integration

Integration of communication and measurement signals between the inverter and digital simulator.

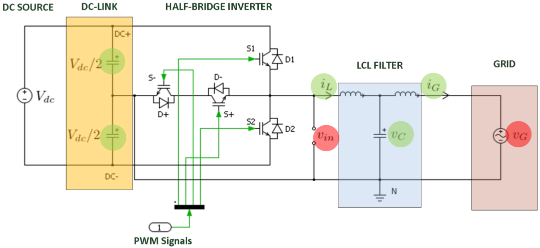
Figure: I/O Interfaces of inverter and simulator

# Grid Voltage Model

**Continuous-time grid voltage model**

$$\begin{bmatrix} \dot{\tilde{v}}_G \\ \dot{v}_G \end{bmatrix} = \begin{bmatrix} 0 & -w_0 \\ w_0 & 0 \end{bmatrix} \begin{bmatrix} \tilde{v}_G \\ v_G \end{bmatrix}$$

$$y_{\text{grid}} = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} \tilde{v}_G \\ v_G \end{bmatrix}$$




| | |
|---|---|
| $\tilde{v}_G$ | Quadrature grid voltage state |
| $v_G$ | Grid voltage state |
| $y_{\text{grid}}$ | Grid voltage output |
| $w_0$ | Angular grid frequency (assumed constant) |

# LCL Filter Model

## Continuous-time LCL-filter model

$$\begin{bmatrix} \dot{i}_L \\ \dot{v}_C \\ \dot{i}_G \end{bmatrix} = \begin{bmatrix} 0 & \frac{-1}{L_M} & 0 \\ \frac{1}{C_F} & 0 & \frac{-1}{C_F} \\ 0 & \frac{1}{L_G} & 0 \end{bmatrix} \begin{bmatrix} i_L \\ v_C \\ i_G \end{bmatrix} + \begin{bmatrix} \frac{1}{L_M} & 0 \\ 0 & 0 \\ 0 & \frac{-1}{L_G} \end{bmatrix} \begin{bmatrix} v_{in} \\ v_G \end{bmatrix}$$

$$y_{\mathrm{LCL}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} i_L \\ v_C \\ i_G \end{bmatrix}$$

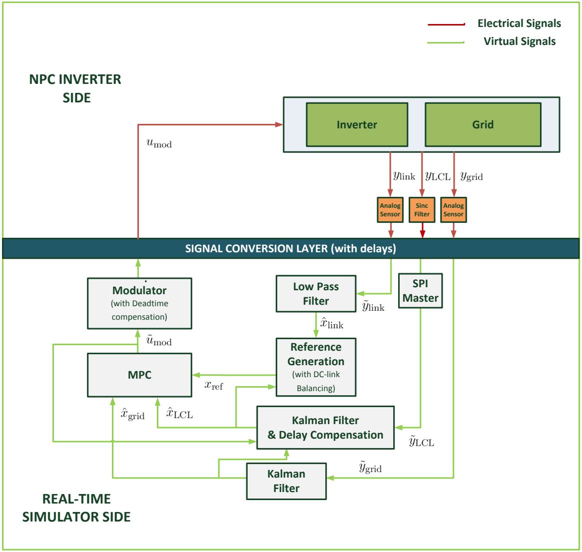
States:

| | |
|---|---|
| $i_L$ | Main inductor current |
| $v_C$ | Filter capacitor voltage |
| $i_G$ | Grid-side inductor current |

Controls:

| | |
|---|---|
| $v_{\mathrm{in}}$ | Bridge input voltage |
| $v_G$ | Grid voltage |

# Closed-loop System Overview



# SPI Interface Design

Figure: SPI Master Module with *EventDetector* and *EventGenerator* I/O blocks.

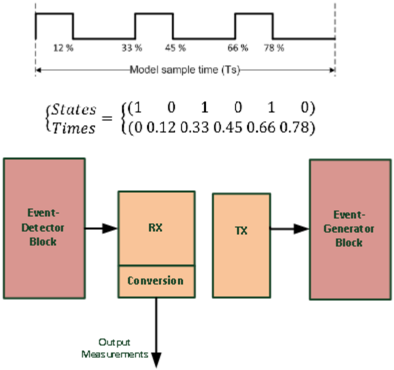
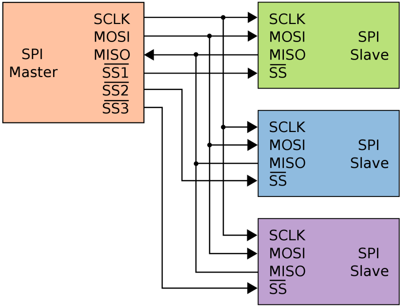



Figure: Single-master Multiple Slave Topology Source: WiKIPEDIA.



# SPI Interface Results

Figure: SPI Communication Results.

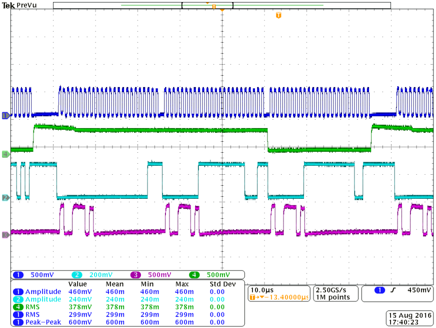
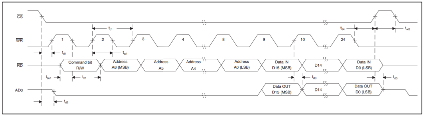



Figure: AMC1210 SPI Timing Specification. Source: Texas Instruments.



# Kalman Filter Design (Output Current)

## Output Current Estimation Problem

$$[\hat{x}_0, \dots, \hat{x}_N] = \arg \min_{[x_0, \dots, x_N]} \|x_0 - \bar{x}_0\|^2_{P_0^{-1}} + \sum_{k=0}^{N} \|y_k - C_{\text{dis}} x_k\|^2_{R^{-1}}$$

$$+ \sum_{k=0}^{N-1} \|x_{k+1} - A_{\text{dis}} x_k - B_{\text{dis}} u_k\|^2_{Q^{-1}}$$

where $x_k = \begin{bmatrix} i_L(k) \\ v_C(k) \\ i_G(k) \end{bmatrix}$, and $\hat{i}_G(N) = [0 \quad 0 \quad 1]\hat{x}_N$ is sought.

## Discrete-time LCL-filter model

$$x_{k+1} = A_{\text{dis}} x_k + B_{\text{dis}} u_k + w_k$$
$$y_k = C_{\text{dis}} x_k + v_k$$

$w_k \sim \mathcal{N}(0, Q)$ process noise

$v_k \sim \mathcal{N}(0, R)$ measurement noise

$p_0 \sim \mathcal{N}(\bar{x}_0, P_0)$ initial state noise

# Kalman Filter Design (Output Current)

## Solution Model

**Prediction**

$$\hat{x}_{[k|k-1]} = A_{\text{dis}}\hat{x}_{[k-1|k-1]} + B_{\text{dis}}u_k$$

$$P_{[k|k-1]} = A_{\text{dis}}P_{[k-1|k-1]}A_{\text{dis}}^T + Q$$

**Correction**

$$\begin{aligned}\hat{x}_{[k|k]} &= \hat{x}_{[k|k-1]} + K_{\text{gain}}^*(y_k - C_{\text{dis}}\hat{x}_{[k|k-1]}) \\ &= (I - K_{\text{gain}}^* C_{\text{dis}})\hat{x}_{[k|k-1]} + K_{\text{gain}}^* y_k\end{aligned}$$

**Estimation model**

$$\hat{x}_{[k|k]} = \mathcal{A}\hat{x}_{[k-1|k-1]} + \mathcal{B}u_k + K_{\text{gain}}^* y_k$$

$$\hat{i}_{G[k|k]} = \hat{y}_{[k|k]} = [0 \quad 0 \quad 1]\hat{x}_{[k|k]}$$

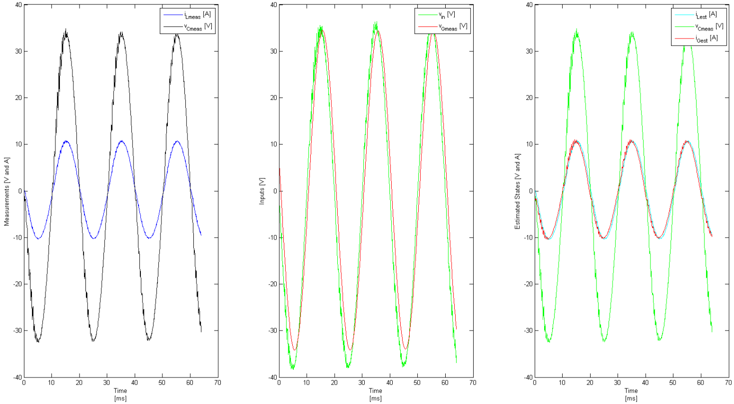
where $\mathcal{A}$ and $\mathcal{B}$ are constant Matrices

For LTI model, if $(A_{\text{dis}}, C_{\text{dis}})$ is observable and $(A_{\text{dis}}, Q^{\frac{1}{2}})$ is stabilizable, then $\lim_{k\to\infty} P_{[k+1|k]} = P_\infty \geq 0$ for any $P_0 \geq 0$.

$$K_{\text{gain}}^* = \frac{P_\infty C_{\text{dis}}^T}{R + C_{\text{dis}}P_\infty C_{\text{dis}}^T}$$

# Output Current Estimation Results

Figure: Plot showing the LCL filter measurements, inputs and estimated states.



# Kalman Filter Design (Grid Voltage)

## Grid Voltage Estimation Problem

$$[\hat{x}_0, \dots, \hat{x}_N,] = \arg \min_{[x_0, \dots, x_N]} \|x_0 - \bar{x}_0\|^2_{P_0^{-1}} + \sum_{k=0}^{N} \|y_k - C_d x_k\|^2_{R^{-1}} + \sum_{k=0}^{N-1} \|x_{k+1} - A_d x_k\|^2_{Q^{-1}}$$

where $x_k = \begin{bmatrix} \tilde{v}_G(k) \\ v_G(k) \end{bmatrix}$ and $\hat{v}_G(N) = [0 \quad 1]\hat{x}_N$ is sought.

## Discrete-time grid voltage model

$$x_{k+1} = A_d x_k + w_k$$
$$y_k = C_d x_k + v_k$$

$w_k \sim \mathcal{N}(0, Q)$ process noise

$v_k \sim \mathcal{N}(0, R)$ measurement noise

$p_0 \sim \mathcal{N}(\bar{x}_0, P_0)$ initial state noise

# Kalman Filter Design (Grid Voltage)

## Solution Model

**Prediction**

$$\hat{x}_{[k|k-1]} = A_d \hat{x}_{[k-1|k-1]}$$

$$P_{[k|k-1]} = A_d P_{[k-1|k-1]} A_d^T + Q$$

**Correction**

$$\hat{x}_{[k|k]} = \hat{x}_{[k|k-1]} + K^*_{\text{gain}}(y_k - C_d \hat{x}_{[k|k-1]})$$

**Estimation model**

$$\hat{x}_{[k|k]} = \mathcal{A}\hat{x}_{[k-1|k-1]} + K^*_{\text{gain}} y_k$$

$$\hat{v}_{G[k|k]} = \hat{y}_{[k|k]} = [0 \quad 1]\hat{x}_{[k|k]}$$

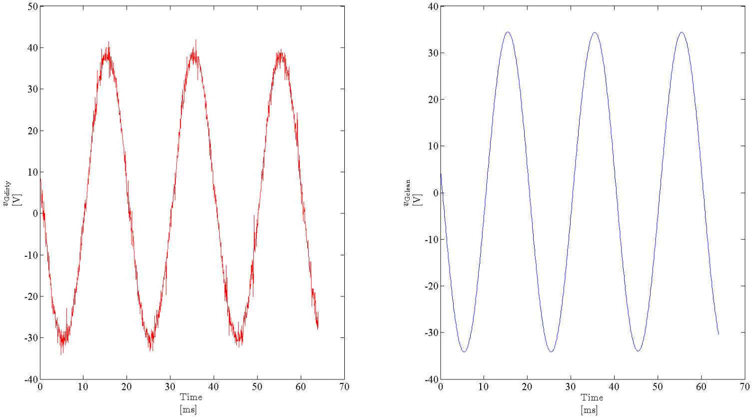
where $\mathcal{A}$ is a constant Matrice

For LTI model, if $(A_d, C_d)$ is observable and $(A_d, Q^{\frac{1}{2}})$ is stabilizable, then $\lim_{k\to\infty} P_{[k+1|k]} = P_\infty \geq 0$ for any $P_0 \geq 0$.

$$K^*_{\text{gain}} = \frac{P_\infty C_d^T}{R + C_d P_\infty C_d^T}$$

# Grid Voltage Filtering Results

Figure: Plot showing the filtered and unfiltered grid voltage signals, $v_G$



# Low-pass Filter Design (DC-Link Voltage)

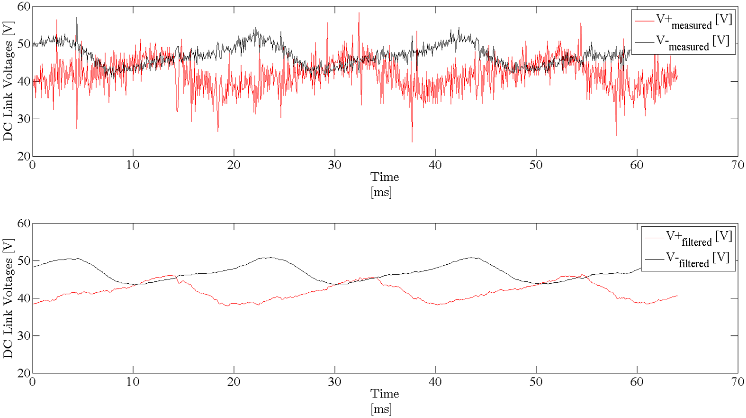
$$H_{\mathrm{LPF}}(s) = \frac{1}{\frac{s}{w_c} + 1} = \frac{w_c}{s + w_c} \text{ (s-domain)}$$

where $f_c = 500\,\mathrm{Hz}$ and $w_c = 2 \times \pi \times 500 = 1000\pi\,\mathrm{rad\,s^{-1}}$ and $s = \frac{T_s(z-1)}{2(z+1)}$ by the bilinear transform.

$$\begin{aligned} H_{\mathrm{LPF}}(z) &= \frac{w_c T_s(z+1)}{2(z-1) + w_c T_s(z+1))} \text{ (z-domain)} \\ &= \frac{w_c T_s(z+1)}{(w_c T_s + 2)z + (w_c T_s - 1)} \end{aligned}$$

# DC-link Filtering Results

Figure: DC-link filtering during openloop operation.



# Controller Design: Optimal Control Problem Formulation

$$\underset{x(.),\, u(.)}{\text{minimise}} \quad \int_0^{NT_s} \|x(t) - x_{\mathrm{ref}}(t)\|_Q^2 + \|u(t) - u_{\mathrm{ref}}(t)\|_R^2 \, dx$$

subject to

$$\begin{aligned}
& x(0) = x_0 && \text{(fixed initial state)},\\
& \dot{x}(t) = Ax(t) + Bu(t), \quad t \in [0, NT_s] && \text{(dynamic constraints)},\\
& \begin{bmatrix} x_{\mathrm{min}} \\ u_{\mathrm{min}} \end{bmatrix} \leq \begin{bmatrix} x(t) \\ u(t) \end{bmatrix} \leq \begin{bmatrix} x_{\mathrm{max}} \\ u_{\mathrm{max}} \end{bmatrix}, \; t \in [0, NT_s] && \text{(path constraints)}
\end{aligned}$$

# Controller Design: qpOASES

## Discretised Formulation for qpOASES

$$
\begin{aligned}
\underset{v}{\text{minimise}} \quad & \frac{1}{2}v^T H v - v^T H v_{\text{ref}} \\
\text{subject to} \quad & l_A \leq A_{\text{eq}} v \leq u_A, \\
& l_v \leq v \leq u_v
\end{aligned}
$$

where the cost function

$$
\begin{aligned}
F_{\text{obj}}(v - v_{\text{ref}}) &= \frac{1}{2}(v - v_{\text{ref}})^T H (v - v_{\text{ref}}) \\
&= \frac{1}{2}(v^T H v - 2v^T H v_{\text{ref}} + v_{\text{ref}}^T H v_{\text{ref}})
\end{aligned}
$$

$$
v = \begin{bmatrix} x_0 \\ u_0 \\ x_1 \\ u_1 \\ \vdots \\ x_{N-1} \\ u_{N-1} \\ x_N \end{bmatrix}, \quad
v_{\text{ref}} = \begin{bmatrix} \bar{x}_0 \\ \bar{u}_0 \\ \bar{x}_1 \\ \bar{u}_1 \\ \vdots \\ \bar{x}_{N-1} \\ \bar{u}_{N-1} \\ \bar{x}_N \end{bmatrix}, \quad
H = \begin{bmatrix} Q_0 & & & & & & & \\ & R_0 & & & & & & \\ & & Q_1 & & & & & \\ & & & R_1 & & & & \\ & & & & \ddots & & & \\ & & & & & Q_{N-1} & & \\ & & & & & & R_{N-1} & \\ & & & & & & & Q_N \end{bmatrix}
$$

$$A_{\text{eq}} = \begin{bmatrix} I & & & & & & & & \\ A_{\text{dis}} & B_{\text{dis}} & -I & & & & & & \\ & & \ddots & \ddots & \ddots & & & & \\ & & & & A_{\text{dis}} & B_{\text{dis}} & -I & & \\ & & & & & & A_{\text{dis}} & B_{\text{dis}} & -I \end{bmatrix}, \; l_A = u_A = \begin{bmatrix} x(0) \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \\ 0 \end{bmatrix}, \; u_v = \begin{bmatrix} x_{\max} \\ u_{\max} \\ x_{\max} \\ u_{\max} \\ \vdots \\ x_{\max} \\ u_{\max} \\ x_{\max} \end{bmatrix};$$

and $l_v = -u_v$

$$\begin{bmatrix} \dot{i}_L \\ \dot{v}_C \\ \dot{i}_G \\ \dot{\tilde{v}}_G \\ \dot{v}_G \end{bmatrix} = \overbrace{\begin{bmatrix} 0 & \frac{-1}{L_M} & 0 & 0 & 0 \\ \frac{1}{C_F} & 0 & \frac{-1}{C_F} & 0 & 0 \\ 0 & \frac{1}{L_G} & 0 & 0 & \frac{-1}{L_G} \\ 0 & 0 & 0 & 0 & -w_0 \\ 0 & 0 & 0 & w_0 & 0 \end{bmatrix}}^{A_{\text{cont}}} \begin{bmatrix} i_L \\ v_C \\ i_G \\ \tilde{v}_G \\ v_G \end{bmatrix} + \overbrace{\begin{bmatrix} \frac{1}{L_M} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}}^{B_{\text{cont}}} \begin{bmatrix} v_{\text{in}} \end{bmatrix}$$

# Controller Design: MPT Toolbox

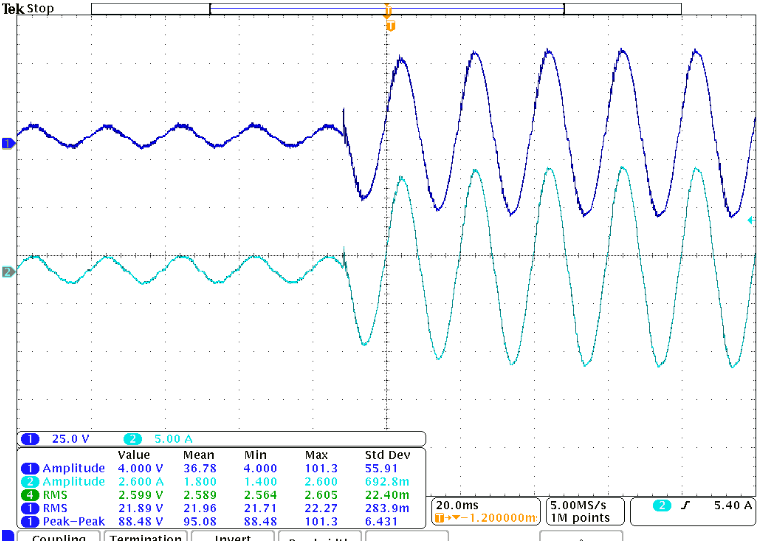
## Discretised Formulation for Explicit MPC

$$\begin{aligned}
\underset{x_0, u_0, x_1, \ldots, u_{N-1}, x_N}{\text{minimise}} \quad & \sum_{k=0}^{N-1} \|x_k - \bar{x}_{\text{ref}}\|_Q^2 + \|u_k\|_R^2 + \|x_N - \bar{x}_{\text{ref}}\|_{Q_N}^2 \\
\text{subject to} \quad & x_0 = x(0), \\
& x_{k+1} = A_d x_k + B_d u_k, \quad k = 0, \ldots, N-1 \\
& \begin{bmatrix} x_{\min} \\ u_{\min} \end{bmatrix} \leq \begin{bmatrix} x_k \\ u_k \end{bmatrix} \leq \begin{bmatrix} x_{\max} \\ u_{\max} \end{bmatrix}, \quad k = 0, \ldots, N-1
\end{aligned}$$

$$\begin{bmatrix} \dot{i}_L \\ \dot{v}_C \\ \dot{i}_G \\ \dot{v}_G \end{bmatrix} = \overbrace{\begin{bmatrix} 0 & \frac{-1}{L_M} & 0 & 0 \\ \frac{1}{C_F} & 0 & \frac{-1}{C_F} & 0 \\ 0 & \frac{1}{L_G} & 0 & \frac{-1}{L_G} \\ 0 & 0 & 0 & 0 \end{bmatrix}}^{A_{\text{cont}}} \begin{bmatrix} i_L \\ v_C \\ i_G \\ v_G \end{bmatrix} + \overbrace{\begin{bmatrix} \frac{1}{L_M} \\ 0 \\ 0 \\ 0 \end{bmatrix}}^{B_{\text{cont}}} \begin{bmatrix} v_{\text{in}} \end{bmatrix}$$

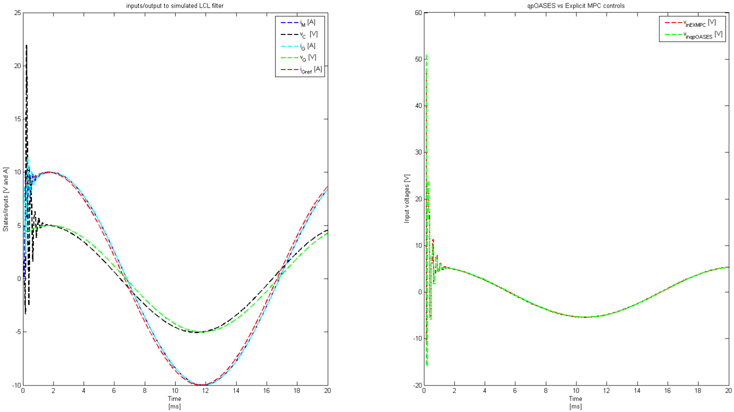
# Real-time Reference Tracking Results (Offline MPC)

Figure: Reference current set point change from 1 A to 8 A RMS.



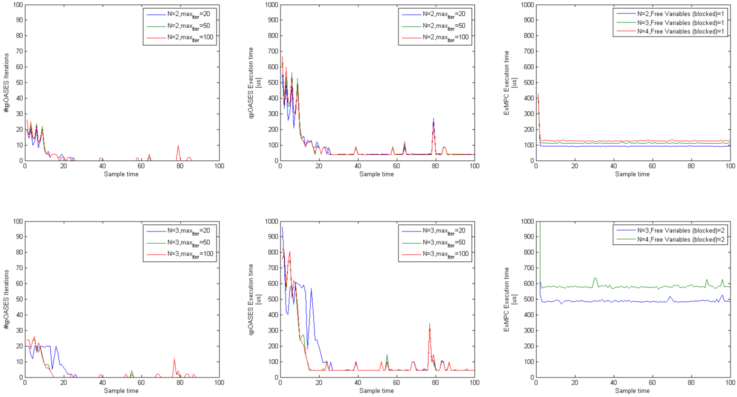
# Simulation Results: Online vs Offline Control Inputs

Figure: A comparison of qpOASES and Explicit MPC inputs for a set AC reference current.



# Simulation: qpOASES vs Explicit MPC Performance

Figure: Plot comparing performance of qpOASES and EXMPC during simulation.



# PWM Generator Design: Complementary Signals

Gate $G_x$

$$t_1(k) = 0, \qquad S_1(k) = 0$$

$$t_2(k) = \overline{t_2}(k) + t_{\mathrm{dt}}, \qquad S_2(k) = 1$$

$$t_3(k) = \frac{1 + d(k)}{2}, \qquad S_3(k) = 0$$

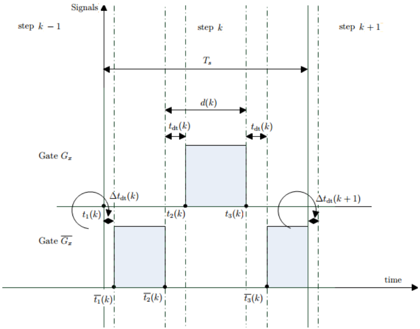
Gate $\overline{G_x}$

$$\overline{t_1}(k) = \Delta t_{\mathrm{dt}}(k), \qquad \overline{S_1}(k) = 1$$

$$\overline{t_2}(k) = \frac{1 - d(k)}{2}, \qquad \overline{S_2}(k) = 0$$

$$\overline{t_3}(k) = t_3(k) + t_{\mathrm{dt}}, \qquad \overline{S_3}(k) = 1$$

Figure: Centered complementary signals with deadtime



# PWM Generator Results

Figure: Four PWM signals driving one phase leg of the NPC inverter

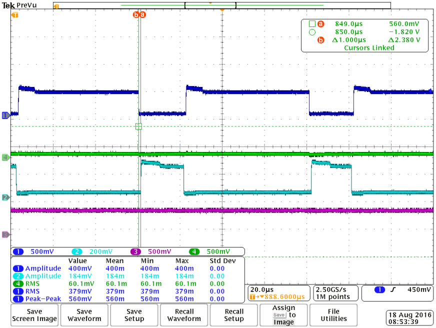
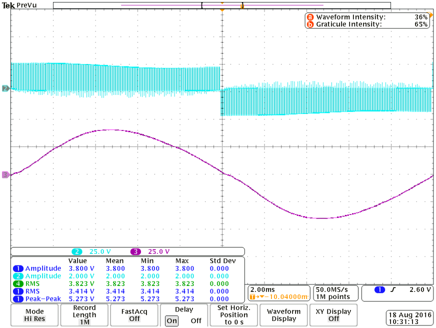


Figure: LCL filter input $v_in$ and output $v_G$ voltages



# Conclusions

The following core modules developed for RCP:

1. A PWM Generator to actuate the inverter.
2. An SPI master module to read digital LCL filter measurements.
3. Kalman and low pass filters.
4. Online and offline-based MPC controllers for current control.

Observations

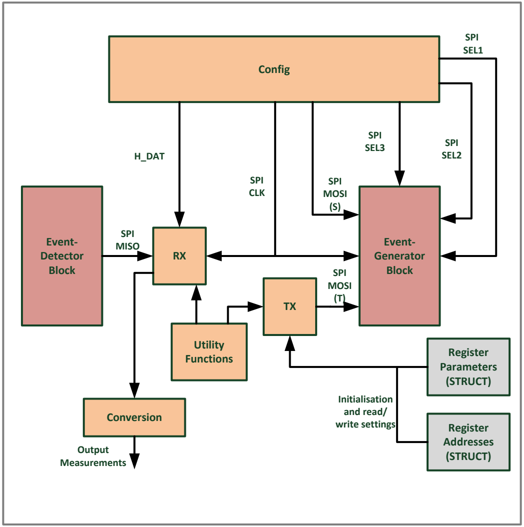
- Offline MPC more stable for fast sampled applications.
- Short horizons & decision variable dimensions => speed.
- Delays caused unsmooth results (SPI, Simulator, Sincs).
- qpOASES scales well with prediction horizons.
- qpOASES shows good performance after initialisation.

# Future Work

1. Real-time execution with qpOASES.
2. Condensing to improve execution time.
3. Parallel measurement/readout architecture.
4. Model verification for PWM Generator.
5. MPC for three-phase control applications.
6. Grid Synchronisation with MPC.


Thank you

# SPI Master Architecture



# PWM Generator Structure

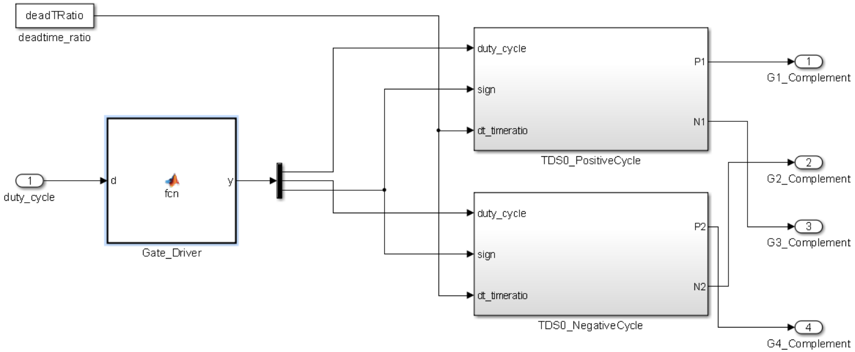
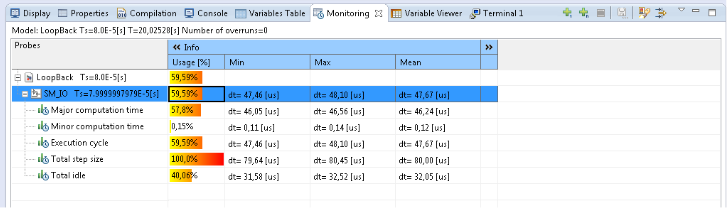


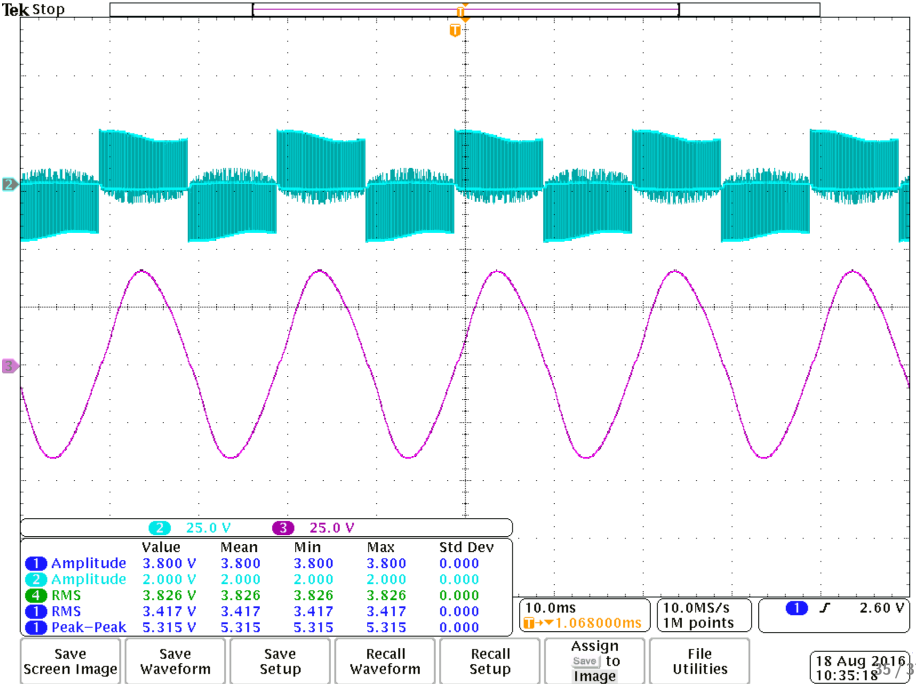

Figure: Subsystem View of Modulator Internal Structure

# Explicit MPC Real-time Performance




Model: LoopBack Ts=8.0E-5[s] T=20,02528[s] Number of overruns=0

| Probes | Usage [%] | Min | Max | Mean |
|---|---|---|---|---|
| LoopBack Ts=8.0E-5[s] | 59,59% | | | |
| SM_IO Ts=7.9999997979E-5[s] | 59,59% | dt= 47,46 [us] | dt= 48,10 [us] | dt= 47,67 [us] |
| Major computation time | 57,8% | dt= 46,05 [us] | dt= 46,56 [us] | dt= 46,24 [us] |
| Minor computation time | 0,15% | dt= 0,11 [us] | dt= 0,14 [us] | dt= 0,12 [us] |
| Execution cycle | 59,59% | dt= 47,46 [us] | dt= 48,10 [us] | dt= 47,67 [us] |
| Total step size | 100,0% | dt= 79,64 [us] | dt= 80,45 [us] | dt= 80,00 [us] |
| Total idle | 40,06% | dt= 31,58 [us] | dt= 32,52 [us] | dt= 32,05 [us] |


Tek Stop
2 25.0 V
3 25.0 V
Value Mean Min Max Std Dev
1 Amplitude 3.800 V 3.800 3.800 3.800 0.000
2 Amplitude 2.000 V 2.000 2.000 2.000 0.000
4 RMS 3.826 V 3.826 3.826 3.826 0.000
1 RMS 3.417 V 3.417 3.417 3.417 0.000
1 Peak–Peak 5.315 V 5.315 5.315 5.315 0.000
10.0ms
1.068000ms
10.0MS/s
1M points
1 2.60 V
Save Screen Image
Save Waveform
Save Setup
Recall Waveform
Recall Setup
Assign Save to Image
File Utilities
18 Aug 2016
10:35:18

## Prediction

$$\hat{x}_{[k|k-1]} = A\hat{x}_{[k-1|k-1]} \quad (1)$$

$$P_{[k|k-1]} = AP_{[k-1|k-1]}A^T + W \quad (2)$$

where $P_{[k|k-1]} \in \mathbb{R}^{n_x \times n_x}$ is the *predicted state covariance*, $\hat{x}_{[k|k]} \in \mathbb{R}^{n_x}$ is the *predicted state estimate*. It is assumed that the noise covariances are constant and thus $W_k = W$ and $V_k = V$.

## Innovation Update

$$P_{[k|k]} = (P^{-1}_{[k|k-1]} + C^T V^{-1} C)^{-1} = (1 - K_k C)P_{[k|k-1]} \quad (3)$$

$$\hat{x}_{[k|k]} = \hat{x}_{[k|k-1]} + \underbrace{P_{[k|k]} C^T V^{-1}}_{\text{Optimal Kalman Gain}} \times \overbrace{(y_k - C\hat{x}_{[k|k-1]})}^{\text{Innovation Residual}} \quad (4)$$

$$K_k = P_{[k|k]} C^T V^{-1} = \frac{P_{[k|k-1]} C^T}{\underbrace{V + CP_{[k|k-1]} C^T}_{\text{Innovation Covariance}}} \quad (5)$$

$$\hat{x}_{[k+1|k]} = A\hat{x}_{[k|k]} = K_{k+1|k}y_k + (A - K_{k+1|k}C)\hat{x}_{[k|k-1]}, \quad k \geq 0 \tag{6}$$

$$K_{k+1|k} = \frac{AP_{[k|k-1]}C^T}{V + CP_{[k|k-1]}C^T} \tag{7}$$

$$\begin{aligned} P_{[k+1|k]} &= AP_{[k|k]}A^T + W = A(1 - K_kC)P_{[k|k-1]}A^T + W \\ &= A(P_{[k|k-1]} - P_{[k|k-1]}C^T(V + CP_{[k|k-1]}C^T)^{-1}CP_{[k|k-1]})A^T + W \end{aligned} \tag{8}$$