University of Pisa

Periodic Optimal Control of a Bipedal Walking Robot

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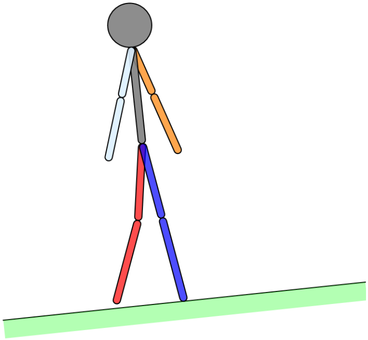
Class of problems

Multibody systems with intermittent contacts, problems such as robotic locomotion or manipulation.

Issues

- High dimensional systems
- Nonlinear dynamics
- Discontinuities due to intermittent contact forces
- Holonomic constraints (Index 3 DAE)

Planar humanoid robot model



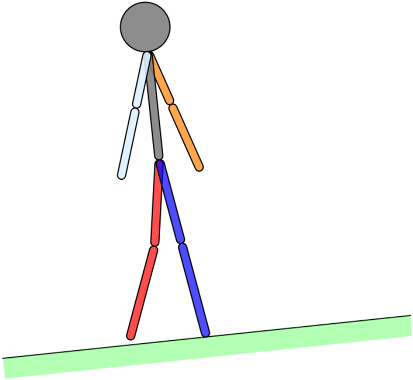
Objective

We want to find a periodic walking behaviour in order to make the system climb up the slope in an optimal way.

Scheduled sequence of phases

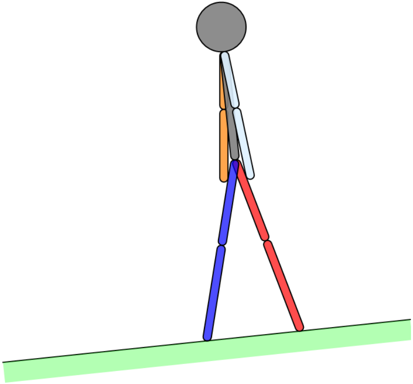
TRANSITION

Single support phase



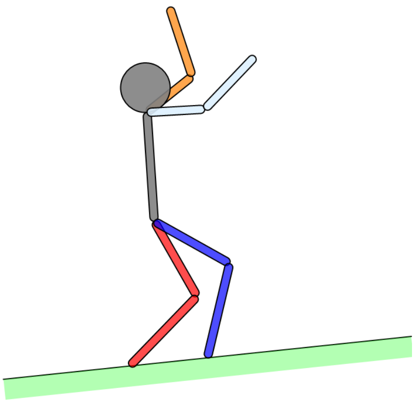
Only the right foot touches the ground

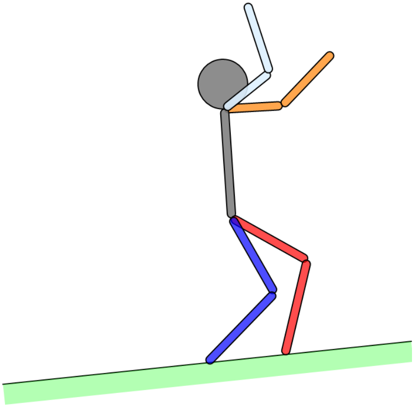
Double support phase



Both feet in contact with the ground

Periodicity of motion





We are optimizing only half of a cycle!

Continuous time optimal control problem

$$
\begin{aligned}
\min_{x(\cdot),u(\cdot),T} \quad & \int_0^{\frac{T}{2}} L(x(t), u(t))dt \\
\text{subject to} \quad & x_{red}(0) = \pi^1 x_{red}\left(\frac{T}{2}\right) && \text{periodicity constraints} \\
& F(x, \dot{x}, z, u) = 0 && \text{dynamics} \\
& f_{impulsive}(x^+, x^-, \Psi) = 0 && \text{impulsive equations in transition} \\
& h(x(t)) \geq 0 && \text{path constraints}
\end{aligned}
$$

States and controls

$$
x = \begin{pmatrix} q \\ \dot{q} \end{pmatrix} \in \mathbb{R}^{22} \qquad z = \begin{pmatrix} \ddot{q} \\ F_C \end{pmatrix} \in \mathbb{R}^{15} \qquad u = \tau \in \mathbb{R}^8
$$

[1] π is a permutation matrix

Discrete time optimal control problem

NLP

$$
\begin{aligned}
\min_{w} \quad & f(w) \\
\text{s.t.} \quad & g(w) = 0 \\
& h(w) \geq 0 \\
& w_{\text{min}} \leq w \leq w_{\text{max}}
\end{aligned}
$$

Constraints

- System dynamics, discretized using direct collocation (Lagrange polynomial of order 2, collocation at Radau points)
- Continuity of the differential states on the time grid
- Impulsive dynamics equations at transition between different phases
- No collision during single support phase
- Friction
- Periodicity on all the states, except for x

Multiphase dynamics...

$t \in [0, t_{TRANS}]$

$$\begin{cases} \dot{x} = f(x, z) \\ g_{dyn}(x, z, u) = 0 \\ g_S(x, z) = 0 \end{cases}$$

$t \in [t_{TRANS}, t_{END}]$

$$\begin{cases} \dot{x} = f(x, z) \\ g_{dyn}(x, z, u) = 0 \\ g_D(x, z) = 0 \end{cases}$$

Multiphase dynamics...

$t \in [0, t_{TRANS}]$

$$\begin{cases} \dot{x} = f(x,z) \\ g_{dyn}(x,z,u) = 0 \\ g_S(x,z) = 0 \end{cases}$$

$$\begin{cases} p_R(q) = 0 \\ F_{C_L} = 0 \end{cases}$$

$t \in [t_{TRANS}, t_{END}]$

$$\begin{cases} \dot{x} = f(x,z) \\ g_{dyn}(x,z,u) = 0 \\ g_D(x,z) = 0 \end{cases}$$

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Multiphase dynamics...

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$$t \in [t_{TRANS}, t_{END}]$$

$$\begin{cases} \dot{x} = f(x,z) \\ g_{dyn}(x,z,u) = 0 \\ p_R(x) = 0 \\ p_L(x) = 0 \end{cases}$$

index 3 DAE

Multiphase dynamics...

$t \in [0, t_{TRANS}]$

$$\begin{cases} \dot{x} = f(x, z) \\ g_{dyn}(x, z, u) = 0 \\ F_{C_L} = 0 \\ \dot{p}_R(x) = 0 \end{cases}$$

$p_R(0) = 0$

$t \in [t_{TRANS}, t_{END}]$

$$\begin{cases} \dot{x} = f(x, z) \\ g_{dyn}(x, z, u) = 0 \\ \dot{p}_R(x) = 0 \\ \dot{p}_L(x) = 0 \end{cases}$$

$$p_R\left(x(t_{TRANS})\right) = 0$$
$$p_L\left(x(t_{TRANS})\right) = 0$$

index 2 DAE

Multiphase dynamics...

$t \in [0, t_{TRANS}]$

$$\begin{cases} \dot{x} = f(x,z) \\ g_{dyn}(x,z,u) = 0 \\ F_{C_L} = 0 \\ \ddot{p}_R(x) = 0 \end{cases}$$

$t \in [t_{TRANS}, t_{END}]$

$$\begin{cases} \dot{x} = f(x,z) \\ g_{dyn}(x,z,u) = 0 \\ \ddot{p}_R(x) = 0 \\ \ddot{p}_L(x) = 0 \end{cases}$$

$$p_R\left(x(0)\right) = 0$$
$$\dot{p}_R\left(x(0)\right) = 0$$

$$p_R\left(x(t_{TRANS})\right) = 0$$
$$p_L\left(x(t_{TRANS})\right) = 0$$
$$\dot{p}_R\left(x(t_{TRANS})\right) = 0$$
$$\dot{p}_L\left(x(t_{TRANS})\right) = 0$$

index 1 DAE

Multiphase dynamics...with invariants!

$$t \in [0, t_{TRANS}] \qquad t \in [t_{TRANS}, t_{END}]$$

$$p_R(x(0)) = 0 \qquad \dot{p}_R(x(0)) = 0$$

$$\begin{cases} \dot{x} = f(x, z) \\ F_{C_L} = 0 \\ \ddot{p}_R(x) = 0 \end{cases}$$

$$p_R(x(t_{TRANS})) = 0$$
$$p_L(x(t_{TRANS})) = 0$$
$$\dot{p}_R(x(t_{TRANS})) = 0$$
$$\dot{p}_L(x(t_{TRANS})) = 0$$

$$\begin{cases} \dot{x} = f(x, z) \\ \ddot{p}_R(x) = 0 \\ \ddot{p}_L(x) = 0 \end{cases}$$

Transition

$$\begin{cases} q^+ = q^- \\ M(q)\left(\dot{q}^+ - \dot{q}^-\right) = \text{Impulse} \\ J_{C_R}(q)\dot{q}^+ = 0 \\ J_{C_L}(q)\dot{q}^+ = 0 \end{cases}$$

Multiphase dynamics...with invariants!

$$t \in [0, t_{TRANS}] \qquad\qquad t \in [t_{TRANS}, t_{END}]$$

$$p_R\left(x(0)\right) = 0$$
$$\dot{p}_R\left(x(0)\right) = 0$$

$$\begin{cases} \dot{x} = f(x, z) \\ F_{C_L} = 0 \\ \ddot{p}_R(x) = 0 \end{cases}$$

$$p_R\left(x(t_{TRANS})\right) = 0$$
$$p_L\left(x(t_{TRANS})\right) = 0$$
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$$\begin{cases} \dot{x} = f(x, z) \\ \ddot{p}_R(x) = 0 \\ \ddot{p}_L(x) = 0 \end{cases}$$

Transition

$$\begin{cases} q^+ = q^- \\ M(q)\left(\dot{q}^+ - \dot{q}^-\right) = \text{Impulse} \\ J_{C_R}(q)\dot{q}^+ = 0 \\ J_{C_L}(q)\dot{q}^+ = 0 \end{cases}$$

Multiphase dynamics...with invariants!

$$t \in [0, t_{TRANS}] \qquad t \in [t_{TRANS}, t_{END}]$$

$$p_R\left(x(0)\right) = 0 \qquad \begin{cases} \dot{x} = f(x, z) \\ F_{C_L} = 0 \\ \ddot{p}_R(x) = 0 \end{cases} \qquad p_L\left(x(t_{TRANS})\right) = 0 \qquad \begin{cases} \dot{x} = f(x, z) \\ \ddot{p}_R(x) = 0 \\ \ddot{p}_L(x) = 0 \end{cases}$$

$$\dot{p}_R\left(x(0)\right) = 0$$

Transition

$$\begin{cases} q^+ = q^- \\ M(q)\left(\dot{q}^+ - \dot{q}^-\right) = \text{Impulse} \\ J_{C_R}(q)\dot{q}^+ = 0 \\ J_{C_L}(q)\dot{q}^+ = 0 \end{cases}$$

Taking into account periodicity constraints

$$t \in [0, t_{TRANS}] \qquad\qquad t \in [t_{TRANS}, t_{END}]$$

$$\begin{matrix} p_R\left(x(0)\right) = 0 \\ \dot{p}_R\left(x(0)\right) = 0 \end{matrix} \quad \begin{cases} \dot{x} = f(x, z) \\ F_{C_L} = 0 \\ \ddot{p}_R(x) = 0 \end{cases} \qquad p_L\left(x(t_{TRANS})\right) = 0 \qquad \begin{cases} \dot{x} = f(x, z) \\ \ddot{p}_R(x) = 0 \\ \ddot{p}_L(x) = 0 \end{cases}$$

Transition

$$\begin{cases} q^+ = q^- \\ M(q)\left(\dot{q}^+ - \dot{q}^-\right) = \text{Impulse} \\ J_{C_R}(q)\dot{q}^+ = 0 \\ J_{C_L}(q)\dot{q}^+ = 0 \end{cases}$$

Taking into account periodicity constraints

$$t \in [0, t_{TRANS}]$$

$$p_R(x(0)) = 0$$
$$\dot{p}_R(x(0)) = 0$$
$$\begin{cases} \dot{x} = f(x,z) \\ F_{C_L} = 0 \\ \ddot{p}_R(x) = 0 \end{cases}$$

$$t \in [t_{TRANS}, t_{END}]$$

$$p_L(x(t_{TRANS})) = 0$$
$$\begin{cases} \dot{x} = f(x,z) \\ \ddot{p}_R(x) = 0 \\ \ddot{p}_L(x) = 0 \end{cases}$$

Transition

$$\begin{cases} q^+ = q^- \\ M(q)\left(\dot{q}^+ - \dot{q}^-\right) = \text{Impulse} \\ J_{C_R}(q)\dot{q}^+ = 0 \\ J_{C_L}(q)\dot{q}^+ = 0 \end{cases}$$

Periodicity constraints implicitly imply

$$p_{L_y}(x(0)) = p_{R_y}(x(t_{END}))$$
$$p_{R_y}(x(0)) = p_{L_y}(x(t_{END}))$$

Taking into account periodicity constraints

$$t \in [0, t_{TRANS}]$$

$$p_R(x(0)) = 0 \qquad \begin{cases} \dot{x} = f(x,z) \\ F_{C_L} = 0 \\ \ddot{p}_R(x) = 0 \end{cases}$$

$$\dot{p}_R(x(0)) = 0$$

$$p_{L_x}(x(t_{TRANS})) = 0$$

$$t \in [t_{TRANS}, t_{END}]$$

$$\begin{cases} \dot{x} = f(x,z) \\ \ddot{p}_R(x) = 0 \\ \ddot{p}_L(x) = 0 \end{cases}$$

Transition

$$\begin{cases} q^+ = q^- \\ M(q)\left(\dot{q}^+ - \dot{q}^-\right) = \text{Impulse} \\ J_{C_R}(q)\dot{q}^+ = 0 \\ J_{C_L}(q)\dot{q}^+ = 0 \end{cases}$$

Periodicity constraints implicitly imply

$$p_{L_y}(x(0)) = p_{R_y}(x(t_{END})) \qquad \dot{p}_L(x(0)) = \dot{p}_R(x(t_{END}))$$

$$p_{R_y}(x(0)) = p_{L_y}(x(t_{END})) \qquad \dot{p}_R(x(0)) = \dot{p}_L(x(t_{END}))$$

Taking into account periodicity constraints

$$t \in [0, t_{TRANS}]$$

$$p_R\left(x(0)\right) = 0$$
$$\dot{p}_R\left(x(0)\right) = 0$$

$$\begin{cases} \dot{x} = f(x,z) \\ F_{C_L} = 0 \\ \ddot{p}_R(x) = 0 \end{cases}$$

$$p_{L_x}\left(x(t_{TRANS})\right) = 0$$

$$t \in [t_{TRANS}, t_{END}]$$

$$\begin{cases} \dot{x} = f(x,z) \\ \ddot{p}_R(x) = 0 \\ \ddot{p}_L(x) = 0 \end{cases}$$

Transition

$$\begin{cases} q^+ = q^- \\ M(q)\left(\dot{q}^+ - \dot{q}^-\right) = \text{Impulse} \\ J_{C_R}(q)\dot{q}^+ = 0 \\ J_{C_L}(q)\dot{q}^+ = 0 \end{cases}$$

Periodicity constraints implicitly imply

$$p_{L_y}\left(x(0)\right) = p_{R_y}\left(x(t_{END})\right) \qquad \dot{p}_L\left(x(0)\right) = \dot{p}_R\left(x(t_{END})\right)$$
$$p_{R_y}\left(x(0)\right) = p_{L_y}\left(x(t_{END})\right) \qquad \dot{p}_R\left(x(0)\right) = \dot{p}_L\left(x(t_{END})\right)$$

Taking into account periodicity constraints

$$t \in [0, t_{TRANS}] \qquad\qquad t \in [t_{TRANS}, t_{END}]$$

$$p_R\left(x(0)\right) = 0 \qquad \begin{cases} \dot{x} = f(x,z) \\ F_{C_L} = 0 \\ \ddot{p}_R(x) = 0 \end{cases} \qquad p_{L_x}\left(x(t_{TRANS})\right) = 0 \qquad \begin{cases} \dot{x} = f(x,z) \\ \ddot{p}_R(x) = 0 \\ \ddot{p}_L(x) = 0 \end{cases}$$

$$\dot{p}_R\left(x(0)\right) = 0$$

Transition

$$\begin{cases} q^+ = q^- \\ M(q)\left(\dot{q}^+ - \dot{q}^-\right) = \text{Impulse} \\ J_{C_R}(q)\dot{q}^+ = 0 \end{cases}$$

Periodicity constraints implicitly imply

$$p_{L_y}\left(x(0)\right) = p_{R_y}\left(x(t_{END})\right) \qquad \dot{p}_L\left(x(0)\right) = \dot{p}_R\left(x(t_{END})\right)$$

$$p_{R_y}\left(x(0)\right) = p_{L_y}\left(x(t_{END})\right) \qquad \dot{p}_R\left(x(0)\right) = \dot{p}_L\left(x(t_{END})\right)$$

What do we optimize for?

- Minimize the positive work:

$$\sum_{k=0}^{N} \sum_{i \in \mathscr{A}} \max\left(\tau_{i,k} \dot{q}_{i,k}, 0\right) h_k$$

Slack variables to deal with non-smooth objective:

$$\sum_{k=0}^{N} \sum_{i \in \mathscr{A}} s_{i,k} h_k \qquad \begin{cases} s_{i,k} \geq 0 \\ s_{i,k} \geq \tau_{i,k} \dot{q}_{i,k} \end{cases}$$

- Minimize control variations:

$$\sum_{k=0}^{N-1} \sum_{i \in \mathscr{A}} \left(\tau_{i,k+1} - \tau_{i,k}\right)^2$$

What do we optimize for?

- Minimize the positive work:

$$\sum_{k=0}^{N} \sum_{i \in \mathscr{A}} \max\left(\tau_{i,k} \dot{q}_{i,k}, 0\right) h_k$$

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- Minimize control variations:

$$\sum_{k=0}^{N-1} \sum_{i \in \mathscr{A}} \left(\tau_{i,k+1} - \tau_{i,k}\right)^2$$

Horizontal walk

Climb

Steep climb

Thank you!