Exercises for Lecture Course on Modeling and System Identification (MSI)
Albert-Ludwigs-Universität Freiburg – Winter Term 2017

Exercise 10: Frequency Domain Identification

(to be returned on Jan 30th, 2017, 8:15 in SR 00-010/014, or before in building 102, 1st floor, 'Anbau')

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In this exercise you will identify an unknown system using a nonparametric identification method. More precisely, you will use multisine excitation to determine the Bode diagram characteristic of the unknown system.
Create a MATLAB script called `main.m` with your code, possibly calling other functions/scripts. From running this script, all the necessary results and plots should be clearly visible. Compress all the files necessary to run your code in a `.zip` file and send it to `msi.syscop@gmail.com`. Please state your name and the names of your team members in the e-mail.

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1. **Multisine Excitation**

   Given an unknown test system, estimate the Bode diagram characteristic of this system by analyzing the output to a multisine input. Please download the provided function `test_system.p` from the course webpage. This MATLAB function accepts an $N \times 1$ input vector u [V] and a sampling time dt [s], and returns an $N \times 1$ output vector as follows: `y = test_system(u,dt)`. The system has a built-in safety: the amplitude of the input signal should be between -10V and $+10$V.

   (a) Generate a periodic signal with a single frequency $f_1 = 1\,\text{Hz}$, e.g. $u = \cos(2\pi f_1 t)\,\text{V}, t \in [0, t_f]$. Apply this signal to the test system and plot both the input and the output w.r.t. time. What is the amplification (in dB) and the phase shift (in degrees) with respect to the input? (1 point)

   (b) Describe how you chose the sampling time Δt in the previous question, and state the corresponding theorem. Which error is prevented by choosing the sampling frequency according to this theorem? (1 point)

   (c) A "multisine" signal allows us to estimate amplification and phase shift for a bunch of different frequencies at the same time. Generate a few periods of the following multisine:

   $$m(t) = A \sum_{k=0}^{N-1} \cos(2\pi k f_{\text{base}} t)\,\text{V}, \quad t \in [0, t_f],$$

   for some amplitude A and base frequency $f_{\text{base}} = f_{\text{max}}/N$. (1 point)

   (d) Apply the multisine to the system. Plot the Bode diagram for this system in the frequency range $[0, 20]$ Hz. Please give an interpretation of the Bode diagram (i.e. which frequencies are attenuated or phase shifted) (1 point)

   (e) What A did you choose in the previous question as to not go over the safety bounds of the system? What might be the reason that you have to choose such an A? (1 point)

   (f) What does the crest factor describe? Compute the crest factor of the previous multisine. How can it be improved? (1 point)

(g) Now, use random phases for the multisine signal. Such a signal is designed elegantly in the frequency domain. Generate a multisine signal in the frequency domain and compute the time domain signal using Inverse Fourier Transformation (`ifft`). Use uniform random phases inside the interval $[0, 2\pi]\mathrm{rad}$. *Hint: A real-valued time domain signal is a complex conjugate symmetric signal in the frequency domain using the Fourier transform.* (1 point)

(h) Apply the multisine you designed in the previous question to the test system and estimate the Bode diagram again. Compare with the Bode diagram from question 3. Compute the crest factor again. What are the advantages of using random phases? (2 points)

(i) In which case does it make sense to use a Frequency Sweep instead of multisine excitation to identify a system? Why? (1 point)

2. Consider a vector of real numbers $u(0), \dots, u(N-1)$. We recall that the DFT is defined by:

$$U(m) := \sum_{k=0}^{N-1} u(k) e^{-j\frac{2\pi mk}{N}}.$$

Compute and plot the real parts of the DFT signal U corresponding to the following input signals u with length $N = 8$: (1 point)

(a) $u_1 = [3, 2, 0, 2, 3, 2, 0, 2]^\top$:

(b) $u_2 = [4, 0, 0, 0, 0, 0, 0, 0]^\top$:

(c) Assume now that we sample new measurements u_3 at $f_s = 100\,\mathrm{Hz}$ and we take $N = 1000$ samples. What is the frequency resolution of the DFT signal, i.e. what is the smallest nonzero frequency on the frequency axis? (1 point)

This sheet gives in total 12 points