Modeling and System Identification – Microexam 3

Prof. Dr. Moritz Diehl, IMTEK, University Freiburg, February 6, 2017, 8:15-9:45

Surname: First Name: Matriculation number:

Subject: Programme: Bachelor ☐ Master ☐ Lehramt ☐ others ☐ Signature:

Please fill in your name above and tick exactly ONE box for the right answer of each question below.

1. Consider the discrete LTI system $y_k = \theta_1 y_{k-1} + \theta_2 \epsilon_k + \theta_3 \epsilon_{k-1}$ with scalar output y and noise ϵ. What of the following abbreviations best describes this system?

(a) ☐ ARX	(b) ☐ FIR	(c) ☐ ARMA	(d) ☐ NARX

2. For which type of system can a global minimum to the estimation problem be guaranteed?

(a) ☐ Output-Error	(b) ☐ Input-Output-Error	(c) ☐ LIP, additive noise	(d) ☐ Equation-Error

3. Which statement concerning the set-up of the Kalman Filter is NOT typically true:

(a) ☐ The larger P, the smaller the innovation step.	(b) ☐ The more trustworthy the model, the smaller P.
(c) ☐ The model is assumed to be linear.	(d) ☐ Process noise effects increase progressively.

4. What does the expression $Q_{N+1}^{-1}\phi(N+1)(y(N+1) - \phi(N+1)^\top \hat{\theta}(N))$ stand for in the context of Recursive Least Squares?

(a) ☐ The innovation update.	(b) ☐ The downweighing of past information.
(c) ☐ The best prior guess.	(d) ☐ The expression is not correct.

5. Consider a linear system defined as $x_i = A_{i-1}x_{i-1} + w_i$ with a linear measurement equation $y_i = C_i x_i + v_i$. If $x_i \in \mathbb{R}^n$ and $y_i \in \mathbb{R}^m$, what is NOT true about the covariance of the process noise W_i and measurement noise V_i of a Kalman Filter?

(a) ☐ W_i and V_i are diagonal matrices.	(b) ☐ W_i has n non-zero singular values.
(c) ☐ $W_i \in \mathbb{R}^{n\times n}$, and $V_i \in \mathbb{R}^{m\times m}$.	(d) ☐ W_i and V_i are positive definite.

6. Given the transfer function $G(j\omega)$, what quantity does the magnitude plot of the Bode diagram show on its y-axis?

(a) ☐ $\|G(j\omega)\|$	(b) ☐ $\lvert G(j\omega)\rvert$	(c) ☐ $\log\lvert G(j\omega)\rvert$	(d) ☐ $\log G(j\omega)^2$

7. For scalar phase shift α, what is the output of the LTI system, described by the transfer function $G(j\omega)$, that is excited with a sinusoidal input $u(t) = U_0 \cdot \sin(\omega \cdot t)$?

(a) ☐ $y(t) = \lvert G(j\omega)\rvert U_0 \cdot \sin(\alpha \cdot t + \omega)$	(b) ☐ $y(t) = \lvert G(j\omega)\rvert U_0 \cdot \sin(\omega \cdot t + \alpha)$
(c) ☐ $y(t) = \frac{U_0}{\lvert G(j\omega)\rvert} \cdot \sin(\omega \cdot t + \alpha)$	(d) ☐ $y(t) = \lvert G(j\omega)\rvert U_0 \cdot \sin(\omega \cdot t) + \alpha$

8. Consider the case in the previous question for an output signal with the magnitude Y_0. Which equation describes the transfer function $G(j\omega)$ for the specific frequency ω?

(a) ☐ $\frac{U_0}{Y_0}e^{j\alpha}$	(b) ☐ $\frac{U_0}{Y_0}e^{j\omega}$	(c) ☐ $\frac{Y_0}{U_0}e^{j\omega}$	(d) ☐ $\frac{Y_0}{U_0}e^{j\alpha}$

9. Regard a periodic signal with base frequency f_{base} that is sampled every $\Delta t = 1s$ (with $1/\Delta t$ a multiple of f_{base}). How many different frequencies are contained in the discretized signal?

(a) ☐ $2\Delta t/f_{base}$	(b) ☐ $f_{base}/2\Delta t$	(c) ☐ $2\Delta t f_{base}$	(d) ☐ $1/(2\Delta t f_{base})$

10. At which frequency f [Hz] is the resonance peak of the following transfer function: $G(s) = \frac{a^2}{s^2+2as+a^2}$, with $a \in \mathbb{R}$? …

(a) ☐ $f = \frac{a}{2\pi}$	(b) ☐ $f = \frac{-ja}{2\pi}$	(c) ☐ $f = \frac{ja}{2\pi}$	(d) ☐ $f = \frac{j2\pi}{a}$

Points on page (max. 10) ☐

11. Which slope has the plot of the magnitude of the transfer function $G(s) = \frac{1}{s^2+2s+1}$ at high frequencies?

(a) ☐ 0 dB/decade	(b) ☐ -20 dB/decade	(c) ☐ 20 dB/decade	(d) ☐ -40 dB/decade

12. A continuous function in the time domain $u_\mathrm{c}(t)$ is sampled with a frequency f_s, resulting in a set of discrete values $u = [u(0), \dots, u(N-1)]^\top$. Applying the DFT to u yields $U = [8, 2, 0, 8, 0, 8, 0, 2]^\top$. Reconstruct the function $u_c(t)$ from U. IDFT: $u(n) = \frac{1}{N}\sum_{k=0}^{N-1} U(k)e^{j\frac{2\pi kn}{N}}$, for $n = 0, \dots, N-1$

(a) ☐ $u_\mathrm{c}(t) = 1 + \frac{1}{2}\cos(\frac{\pi f_s t}{8}) + 2\cos(\frac{\pi f_s t}{4})$	(b) ☐ $u_\mathrm{c}(t) = 2 + \cos(\frac{\pi f_s t}{4}) + 2\cos(\frac{3\pi f_s t}{4})$
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13. You want to do multisine excitation to identify the transfer function of an unknown system. How should the multisine be designed, to prevent aliasing errors?

14. What is the name of the theorem that defines the above mentioned condition?

15. You want to do multisine excitation to identify the transfer function of an unknown system. How should the multisine be designed, to prevent leakage errors?

16. What is the frequency resolution of the DFT of a given periodic signal with period T, sampling frequency f_s, and the number of samples per period N?

(a) ☐ $\omega = 2\pi f_s$	(b) ☐ $\omega = \frac{2\pi f_s}{T}$	(c) ☐ $\omega = \frac{2\pi}{N \cdot T}$	(d) ☐ $\omega = \frac{2\pi f_s}{N}$

17. You identify an LTI system with periodic multisine excitations, where each window has length T and the total duration of your experiment is MT with a large integer M. Which procedure should you follow to identify the transfer function $\hat{G}(j\omega_k)$ at a given frequency $\omega_k = \frac{2\pi k}{T}$?

(a) ☐ build the quotients of the M windows, average the quotients and apply the DFT on the quotients	(b) ☐ average the M windows, build the quotient of the average and apply DFT
(c) ☐ compute the DFTs of each window, build the DFT quotients and then average the quotients	(d) ☐ compute the DFTs of each window, then average the DFTs and then build the quotient of the average

18. Which expression describes the Kalman filter innovation step of the covariance P?

(a) ☐ $P_{[k\|k]} = (P_{[k\|k-1]} + C_k^\top V^{-1} C_k)^{-1}$	(b) ☐ $P_{[k\|k]} = (P^{-1}_{[k\|k-1]} + C_k^\top V^{-1} C_k)^{-1}$
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19. What signal-to-noise-ratio (SNR) at a certain frequency f_0 of a measured signal do you want to achieve, in order to be able to estimate the amplitude of this frequency component with an accuracy of 10%?

(a) ☐ 1dB	(b) ☐ 10dB	(c) ☐ 20dB	(d) ☐ 40dB

20. Which are the apropriate matrices of the state space model of a system described by the following ODE: $\ddot{a} = c_1\dot{a} + c_2 a + \dot{s}$ $\ddot{s} = c_3\dot{a} + c_4\dot{s} + s$ and the output $y = c_5 s + c_6\dot{a}$, with $c_1, \dots, c_6 \in \mathbb{R}$. Consider the states to be $x = [a, \dot{a}, s, \dot{s}]^\top$.

(a) ☐ $A = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & c_3 & 1 & c_4 \\ 0 & 1 & 0 & 0 \\ c_2 & c_1 & 0 & 1 \end{bmatrix}, C = \begin{bmatrix} c_5 & 0 & 0 & c_6 \end{bmatrix}$	(b) ☐ $A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ c_2 & c_1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & c_3 & 1 & c_4 \end{bmatrix}, C = \begin{bmatrix} c_5 & 0 & 0 & c_6 \end{bmatrix}$
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10. What signal-to-noise-ratio (SNR) at a certain frequency f_0 of a measured signal do you want to achieve, in order to be able to estimate the amplitude of this frequency component with an accuracy of 10%?

(a) ☐ 40dB	(b) ☐ 1dB	(c) ☐ 10dB	(d) ☐ 20dB

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16. What is the name of the theorem that defines the above mentioned condition?

17. Consider the discrete LTI system $y_k = \theta_1 y_{k-1} + \theta_2\epsilon_k + \theta_3\epsilon_{k-1}$ with scalar output y and noise ϵ. What of the following abbreviations best describes this system?

(a) ☐ ARMA	(b) ☐ FIR	(c) ☐ ARX	(d) ☐ NARX

18. A continuous function in the time domain $u_c(t)$ is sampled with a frequency f_s, resulting in a set of discrete values $u = [u(0), \dots, u(N-1)]^\top$. Applying the DFT to u yields $U = [8, 2, 0, 8, 0, 8, 0, 2]^\top$. Reconstruct the function $u_c(t)$ from U. IDFT: $u(n) = \frac{1}{N}\sum_{k=0}^{N-1} U(k)e^{j\frac{2\pi kn}{N}}$, for $n = 0, \dots, N-1$

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19. You identify an LTI system with periodic multisine excitations, where each window has length T and the total duration of your experiment is MT with a large integer M. Which procedure should you follow to identify the transfer function $\hat{G}(j\omega_k)$ at a given frequency $\omega_k = \frac{2\pi k}{T}$?

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17. Which statement concerning the set-up of the Kalman Filter is NOT typically true:

(a) ☐ The model is assumed to be linear.	(b) ☐ The more trustworthy the model, the smaller P.
(c) ☐ The larger P, the smaller the innovation step.	(d) ☐ Process noise effects increase progressively.

18. What does the expression $Q_{N+1}^{-1}\phi(N+1)(y(N+1) - \phi(N+1)^\top\hat{\theta}(N))$ stand for in the context of Recursive Least Squares?

(a) ☐ The expression is not correct.	(b) ☐ The downweighing of past information.
(c) ☐ The best prior guess.	(d) ☐ The innovation update.

19. Which expression describes the Kalman filter innovation step of the covariance P?

(a) ☐ $P_{[k\vert k]} = (P_{[k\vert k-1]} + C_k^\top V^{-1} C_k)^{-1}$	(b) ☐ $P_{[k\vert k]} = P_{[k\vert k-1]}^{-1} + C_k^\top V^{-1} C_k$
(c) ☐ $P_{[k\vert k-1]} = (P_{[k-1\vert k-1]}^{-1} + C_k^\top V^{-1} C_k)^{-1}$	(d) ☐ $P_{[k\vert k]} = (P_{[k\vert k-1]}^{-1} + C_k^\top V^{-1} C_k)^{-1}$

20. Which are the apropriate matrices of the state space model of a system described by the following ODE: $\ddot{a} = c_1\dot{a} + c_2 a + \dot{s}$ $\ddot{s} = c_3\dot{a} + c_4\dot{s} + s$ and the output $y = c_5 s + c_6\dot{a}$, with $c_1, \dots, c_6 \in \mathbb{R}$. Consider the states to be $x = [a, \dot{a}, s, \dot{s}]^\top$.

(a) ☐ $A = \begin{bmatrix} c_2 & c_1 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & c_3 & 1 & c_4 \end{bmatrix}, C = \begin{bmatrix} 0 & c_6 & c_5 & 0 \end{bmatrix}$	(b) ☐ $A = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & c_3 & 1 & c_4 \\ 0 & 1 & 0 & 0 \\ c_2 & c_1 & 0 & 1 \end{bmatrix}, C = \begin{bmatrix} c_5 & 0 & 0 & c_6 \end{bmatrix}$
(c) ☐ $A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ c_2 & c_1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & c_3 & 1 & c_4 \end{bmatrix}, C = \begin{bmatrix} c_5 & 0 & 0 & c_6 \end{bmatrix}$	(d) ☐ $A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ c_2 & c_1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & c_3 & 1 & c_4 \end{bmatrix}, C = \begin{bmatrix} 0 & c_6 & c_5 & 0 \end{bmatrix}$

Points on page (max. 9)	

Modeling and System Identification – Microexam 3

Prof. Dr. Moritz Diehl, IMTEK, University Freiburg, February 6, 2017, 8:15-9:45

Surname: First Name: Matriculation number:

Subject: Programme: Bachelor ☐ Master ☐ Lehramt ☐ others ☐ Signature:

Please fill in your name above and tick exactly ONE box for the right answer of each question below.

1. Which expression describes the Kalman filter innovation step of the covariance P?

(a) ☐ $P_{[k\|k]} = (P_{[k\|k-1]} + C_k^\top V^{-1} C_k)^{-1}$	(b) ☐ $P_{[k\|k]} = (P^{-1}_{[k\|k-1]} + C_k^\top V^{-1} C_k)^{-1}$
(c) ☐ $P_{[k\|k-1]} = (P^{-1}_{[k-1\|k-1]} + C_k^\top V^{-1} C_k)^{-1}$	(d) ☐ $P_{[k\|k]} = P^{-1}_{[k\|k-1]} + C_k^\top V^{-1} C_k$

2. What signal-to-noise-ratio (SNR) at a certain frequency f_0 of a measured signal do you want to achieve, in order to be able to estimate the amplitude of this frequency component with an accuracy of 10%?

(a) ☐ 20dB	(b) ☐ 1dB	(c) ☐ 40dB	(d) ☐ 10dB

3. Which are the apropriate matrices of the state space model of a system described by the following ODE: $\ddot{a} = c_1\dot{a} + c_2 a + \dot{s}$ $\ddot{s} = c_3\dot{a} + c_4\dot{s} + s$ and the output $y = c_5 s + c_6 \dot{a}$, with $c_1, \ldots, c_6 \in \mathbb{R}$. Consider the states to be $x = [a, \dot{a}, s, \dot{s}]^\top$.

(a) ☐ $A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ c_2 & c_1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & c_3 & 1 & c_4 \end{bmatrix}, C = \begin{bmatrix} 0 & c_6 & c_5 & 0 \end{bmatrix}$	(b) ☐ $A = \begin{bmatrix} c_2 & c_1 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & c_3 & 1 & c_4 \end{bmatrix}, C = \begin{bmatrix} 0 & c_6 & c_5 & 0 \end{bmatrix}$
(c) ☐ $A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ c_2 & c_1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & c_3 & 1 & c_4 \end{bmatrix}, C = \begin{bmatrix} c_5 & 0 & 0 & c_6 \end{bmatrix}$	(d) ☐ $A = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & c_3 & 1 & c_4 \\ 0 & 1 & 0 & 0 \\ c_2 & c_1 & 0 & 1 \end{bmatrix}, C = \begin{bmatrix} c_5 & 0 & 0 & c_6 \end{bmatrix}$

4. For which type of system can a global minimum to the estimation problem be guaranteed?

(a) ☐ LIP, additive noise	(b) ☐ Output-Error	(c) ☐ Equation-Error	(d) ☐ Input-Output-Error

5. Which statement concerning the set-up of the Kalman Filter is NOT typically true:

(a) ☐ Process noise effects increase progressively.	(b) ☐ The more trustworthy the model, the smaller P.
(c) ☐ The larger P, the smaller the innovation step.	(d) ☐ The model is assumed to be linear.

6. What does the expression $Q_{N+1}^{-1}\phi(N+1)(y(N+1) - \phi(N+1)^\top \hat{\theta}(N))$ stand for in the context of Recursive Least Squares?

(a) ☐ The best prior guess.	(b) ☐ The downweighing of past information.
(c) ☐ The expression is not correct.	(d) ☐ The innovation update.

7. Consider the discrete LTI system $y_k = \theta_1 y_{k-1} + \theta_2 \epsilon_k + \theta_3 \epsilon_{k-1}$ with scalar output y and noise ϵ. What of the following abbreviations best describes this system?

(a) ☐ NARX	(b) ☐ ARX	(c) ☐ FIR	(d) ☐ ARMA

8. Regard a periodic signal with base frequency f_{base} that is sampled every $\Delta t = 1s$ (with $1/\Delta t$ a multiple of f_{base}). How many different frequencies are contained in the discretized signal?

(a) ☐ $2\Delta t/f_{base}$	(b) ☐ $2\Delta t f_{base}$	(c) ☐ $1/(2\Delta t f_{base})$	(d) ☐ $f_{base}/2\Delta t$

9. What is the frequency resolution of the DFT of a given periodic signal with period T, sampling frequency f_s, and the number of samples per period N?

(a) ☐ $\omega = 2\pi f_s$	(b) ☐ $\omega = \frac{2\pi f_s}{N}$	(c) ☐ $\omega = \frac{2\pi f_s}{T}$	(d) ☐ $\omega = \frac{2\pi}{N \cdot T}$

Points on page (max. 9) ☐

10. You identify an LTI system with periodic multisine excitations, where each window has length T and the total duration of your experiment is MT with a large integer M. Which procedure should you follow to identify the transfer function $\hat{G}(j\omega_k)$ at a given frequency $\omega_k = \frac{2\pi k}{T}$?

(a) ☐ build the quotients of the M windows, average the quotients and apply the DFT on the quotients	(b) ☐ compute the DFTs of each window, build the DFT quotients and then average the quotients
(c) ☐ average the M windows, build the quotient of the average and apply DFT	(d) ☐ compute the DFTs of each window, then average the DFTs and then build the quotient of the average

11. You want to do multisine excitation to identify the transfer function of an unknown system. How should the multisine be designed, to prevent leakage errors?

12. For scalar phase shift α, what is the output of the LTI system, described by the transfer function $G(j\omega)$, that is excited with a sinusoidal input $u(t) = U_0 \cdot \sin(\omega \cdot t)$?

(a) ☐ $y(t) = \lvert G(j\omega)\rvert U_0 \cdot \sin(\alpha \cdot t + \omega)$	(b) ☐ $y(t) = \lvert G(j\omega)\rvert U_0 \cdot \sin(\omega \cdot t + \alpha)$
(c) ☐ $y(t) = \lvert G(j\omega)\rvert U_0 \cdot \sin(\omega \cdot t) + \alpha$	(d) ☐ $y(t) = \frac{U_0}{\lvert G(j\omega)\rvert} \cdot \sin(\omega \cdot t + \alpha)$

13. Consider the case in the previous question for an output signal with the magnitude Y_0. Which equation describes the transfer function $G(j\omega)$ for the specific frequency ω?

(a) ☐ $\frac{Y_0}{U_0} e^{j\omega}$	(b) ☐ $\frac{U_0}{Y_0} e^{j\alpha}$	(c) ☐ $\frac{U_0}{Y_0} e^{j\omega}$	(d) ☐ $\frac{Y_0}{U_0} e^{j\alpha}$

14. A continuous function in the time domain $u_c(t)$ is sampled with a frequency f_s, resulting in a set of discrete values $u = [u(0), \ldots, u(N-1)]^\top$. Applying the DFT to u yields $U = [8, 2, 0, 8, 0, 8, 0, 2]^\top$. Reconstruct the function $u_c(t)$ from U. IDFT: $u(n) = \frac{1}{N}\sum_{k=0}^{N-1} U(k) e^{j\frac{2\pi k n}{N}}$, for $n = 0, \ldots, N-1$

(a) ☐ $u_c(t) = 2 + \cos(\frac{\pi f_s t}{4}) + 2\cos(\pi f_s t)$	(b) ☐ $u_c(t) = 1 + \frac{1}{2}\cos(\frac{\pi f_s t}{4}) + 2\cos(\frac{3\pi f_s t}{4})$
(c) ☐ $u_c(t) = 1 + \frac{1}{2}\cos(\frac{\pi f_s t}{8}) + 2\cos(\frac{\pi f_s t}{4})$	(d) ☐ $u_c(t) = 2 + \cos(\frac{\pi f_s t}{4}) + 2\cos(\frac{3\pi f_s t}{4})$

15. At which frequency f [Hz] is the resonance peak of the following transfer function: $G(s) = \frac{a^2}{s^2 + 2as + a^2}$, with $a \in \mathbb{R}$? ...

(a) ☐ $f = \frac{ja}{2\pi}$	(b) ☐ $f = \frac{-ja}{2\pi}$	(c) ☐ $f = \frac{a}{2\pi}$	(d) ☐ $f = \frac{j2\pi}{a}$

16. You want to do multisine excitation to identify the transfer function of an unknown system. How should the multisine be designed, to prevent aliasing errors?

17. What is the name of the theorem that defines the above mentioned condition?

18. Consider a linear system defined as $x_i = A_{i-1}x_{i-1} + w_i$ with a linear measurement equation $y_i = C_i x_i + v_i$. If $x_i \in \mathbb{R}^n$ and $y_i \in \mathbb{R}^m$, what is NOT true about the covariance of the process noise W_i and measurement noise V_i of a Kalman Filter?

(a) ☐ $W_i \in \mathbb{R}^{n \times n}$, and $V_i \in \mathbb{R}^{m \times m}$.	(b) ☐ W_i and V_i are positive definite.
(c) ☐ W_i has n non-zero singular values.	(d) ☐ W_i and V_i are diagonal matrices.

19. Given the transfer function $G(j\omega)$, what quantity does the magnitude plot of the Bode diagram show on its y-axis?

(a) ☐ $\lVert G(j\omega)\rVert$	(b) ☐ $\log G(j\omega)^2$	(c) ☐ $\log\lvert G(j\omega)\rvert$	(d) ☐ $\lvert G(j\omega)\rvert$

20. Which slope has the plot of the magnitude of the transfer function $G(s) = \frac{1}{s^2 + 2s + 1}$ at high frequencies?

(a) ☐ -40 dB/decade	(b) ☐ 20 dB/decade	(c) ☐ 0 dB/decade	(d) ☐ -20 dB/decade

Modeling and System Identification – Microexam 3

Prof. Dr. Moritz Diehl, IMTEK, University Freiburg, February 6, 2017, 8:15-9:45

Surname: First Name: Matriculation number:

Subject: Programme: Bachelor ☐ Master ☐ Lehramt ☐ others ☐ Signature:

Please fill in your name above and tick exactly ONE box for the right answer of each question below.

1. A continuous function in the time domain $u_\mathrm{c}(t)$ is sampled with a frequency f_s, resulting in a set of discrete values $u = [u(0), \dots, u(N-1)]^\top$. Applying the DFT to u yields $U = [8, 2, 0, 8, 0, 8, 0, 2]^\top$. Reconstruct the function $u_c(t)$ from U. IDFT: $u(n) = \frac{1}{N}\sum_{k=0}^{N-1} U(k) e^{j\frac{2\pi kn}{N}}$, for $n = 0, \dots, N-1$

(a) ☐ $u_\mathrm{c}(t) = 2 + \cos(\frac{\pi f_s t}{4}) + 2\cos(\frac{3\pi f_s t}{4})$	(b) ☐ $u_\mathrm{c}(t) = 1 + \frac{1}{2}\cos(\frac{\pi f_s t}{4}) + 2\cos(\frac{3\pi f_s t}{4})$
(c) ☐ $u_\mathrm{c}(t) = 2 + \cos(\frac{\pi f_s t}{4}) + 2\cos(\pi f_s t)$	(d) ☐ $u_\mathrm{c}(t) = 1 + \frac{1}{2}\cos(\frac{\pi f_s t}{8}) + 2\cos(\frac{\pi f_s t}{4})$

2. You want to do multisine excitation to identify the transfer function of an unknown system. How should the multisine be designed, to prevent aliasing errors?

3. What is the name of the theorem that defines the above mentioned condition?

4. For scalar phase shift α, what is the output of the LTI system, described by the transfer function $G(j\omega)$, that is excited with a sinusoidal input $u(t) = U_0 \cdot \sin(\omega \cdot t)$?

(a) ☐ $y(t) = \lvert G(j\omega)\rvert U_0 \cdot \sin(\omega \cdot t + \alpha)$	(b) ☐ $y(t) = \lvert G(j\omega)\rvert U_0 \cdot \sin(\omega \cdot t) + \alpha$
(c) ☐ $y(t) = \lvert G(j\omega)\rvert U_0 \cdot \sin(\alpha \cdot t + \omega)$	(d) ☐ $y(t) = \frac{U_0}{\lvert G(j\omega)\rvert} \cdot \sin(\omega \cdot t + \alpha)$

5. Consider the case in the previous question for an output signal with the magnitude Y_0. Which equation describes the transfer function $G(j\omega)$ for the specific frequency ω?

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6. Which expression describes the Kalman filter innovation step of the covariance P?

(a) ☐ $P_{[k\vert k]} = (P_{[k\vert k-1]}^{-1} + C_k^\top V^{-1} C_k)^{-1}$	(b) ☐ $P_{[k\vert k]} = P_{[k\vert k-1]}^{-1} + C_k^\top V^{-1} C_k$
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7. What signal-to-noise-ratio (SNR) at a certain frequency f_0 of a measured signal do you want to achieve, in order to be able to estimate the amplitude of this frequency component with an accuracy of 10%?

(a) ☐ 20dB	(b) ☐ 40dB	(c) ☐ 10dB	(d) ☐ 1dB

8. Which are the apropriate matrices of the state space model of a system described by the following ODE: $\ddot{a} = c_1\dot{a} + c_2 a + \dot{s}$ $\ddot{s} = c_3\dot{a} + c_4\dot{s} + s$ and the output $y = c_5 s + c_6\dot{a}$, with $c_1, \dots, c_6 \in \mathbb{R}$. Consider the states to be $x = [a, \dot{a}, s, \dot{s}]^\top$.

(a) ☐ $A = \begin{bmatrix} c_2 & c_1 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & c_3 & 1 & c_4 \end{bmatrix}, C = \begin{bmatrix} 0 & c_6 & c_5 & 0 \end{bmatrix}$	(b) ☐ $A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ c_2 & c_1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & c_3 & 1 & c_4 \end{bmatrix}, C = \begin{bmatrix} c_5 & 0 & 0 & c_6 \end{bmatrix}$
(c) ☐ $A = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & c_3 & 1 & c_4 \\ 0 & 1 & 0 & 0 \\ c_2 & c_1 & 0 & 1 \end{bmatrix}, C = \begin{bmatrix} c_5 & 0 & 0 & c_6 \end{bmatrix}$	(d) ☐ $A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ c_2 & c_1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & c_3 & 1 & c_4 \end{bmatrix}, C = \begin{bmatrix} 0 & c_6 & c_5 & 0 \end{bmatrix}$

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Points on page (max. 8) ☐

10. What is the frequency resolution of the DFT of a given periodic signal with period T, sampling frequency f_s, and the number of samples per period N?

(a) ☐ $\omega = \frac{2\pi f_s}{T}$	(b) ☐ $\omega = 2\pi f_s$	(c) ☐ $\omega = \frac{2\pi}{N \cdot T}$	(d) ☐ $\omega = \frac{2\pi f_s}{N}$

11. Consider the discrete LTI system $y_k = \theta_1 y_{k-1} + \theta_2 \epsilon_k + \theta_3 \epsilon_{k-1}$ with scalar output y and noise ϵ. What of the following abbreviations best describes this system?

(a) ☐ ARMA	(b) ☐ ARX	(c) ☐ NARX	(d) ☐ FIR

12. Consider a linear system defined as $x_i = A_{i-1}x_{i-1} + w_i$ with a linear measurement equation $y_i = C_i x_i + v_i$. If $x_i \in \mathbb{R}^n$ and $y_i \in \mathbb{R}^m$, what is NOT true about the covariance of the process noise W_i and measurement noise V_i of a Kalman Filter?

(a) ☐ W_i has n non-zero singular values.	(b) ☐ W_i and V_i are positive definite.
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13. Given the transfer function $G(j\omega)$, what quantity does the magnitude plot of the Bode diagram show on its y-axis?

(a) ☐ $\log\lvert G(j\omega)\rvert$	(b) ☐ $\lVert G(j\omega)\rVert$	(c) ☐ $\log G(j\omega)^2$	(d) ☐ $\lvert G(j\omega)\rvert$

14. Regard a periodic signal with base frequency f_{base} that is sampled every $\Delta t = 1s$ (with $1/\Delta t$ a multiple of f_{base}). How many different frequencies are contained in the discretized signal?

(a) ☐ $f_{base}/2\Delta t$	(b) ☐ $1/(2\Delta t f_{base})$	(c) ☐ $2\Delta t/f_{base}$	(d) ☐ $2\Delta t f_{base}$

15. For which type of system can a global minimum to the estimation problem be guaranteed?

(a) ☐ Input-Output-Error	(b) ☐ Output-Error	(c) ☐ Equation-Error	(d) ☐ LIP, additive noise

16. Which statement concerning the set-up of the Kalman Filter is NOT typically true:

(a) ☐ Process noise effects increase progressively.	(b) ☐ The more trustworthy the model, the smaller P.
(c) ☐ The model is assumed to be linear.	(d) ☐ The larger P, the smaller the innovation step.

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(a) ☐ The best prior guess.	(b) ☐ The expression is not correct.
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18. At which frequency f [Hz] is the resonance peak of the following transfer function: $G(s) = \frac{a^2}{s^2+2as+a^2}$, with $a \in \mathbb{R}$? ...

(a) ☐ $f = \frac{j2\pi}{a}$	(b) ☐ $f = \frac{a}{2\pi}$	(c) ☐ $f = \frac{ja}{2\pi}$	(d) ☐ $f = \frac{-ja}{2\pi}$

19. You identify an LTI system with periodic multisine excitations, where each window has length T and the total duration of your experiment is MT with a large integer M. Which procedure should you follow to identify the transfer function $\hat{G}(j\omega_k)$ at a given frequency $\omega_k = \frac{2\pi k}{T}$?

(a) ☐ average the M windows, build the quotient of the average and apply DFT	(b) ☐ compute the DFTs of each window, build the DFT quotients and then average the quotients
(c) ☐ compute the DFTs of each window, then average the DFTs and then build the quotient of the average	(d) ☐ build the quotients of the M windows, average the quotients and apply the DFT on the quotients

20. Which slope has the plot of the magnitude of the transfer function $G(s) = \frac{1}{s^2+2s+1}$ at high frequencies?

(a) ☐ -20 dB/decade	(b) ☐ -40 dB/decade	(c) ☐ 0 dB/decade	(d) ☐ 20 dB/decade