

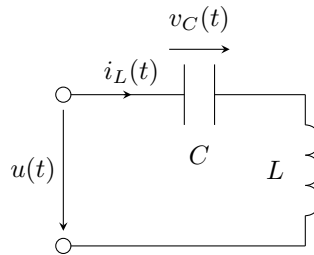
Exercise Sheet 2

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Problem 4: LQR and prefilter

1. The aim of this task is to design an LQR controller for a given CL-oscillator circuit to **track** a voltage reference w at the capacitor. The state vector is defined as $\mathbf{x} := [v_C \quad i_L]^T$ and the system is assumed to be controllable.



- (a) Draw the block-diagram of an $(\mathbf{A}, \mathbf{B}, \mathbf{C})$ -system and use the provided template files to simulate the system without applying any control ($u = 0$). The initial state-vector is $\mathbf{x}_0 := [10 \quad 0]^T$ and the simulation time is 0.2 s. What can you say about the eigenvalues?
- (b) Define weighting matrices \mathbf{Q} and \mathbf{R} such that the performance-index reads as

$$J = \frac{1}{2} \int_0^\infty \left(\frac{v_C^2(t)}{[V]^2} + 0.1 \frac{i_L^2(t)}{[I]^2} + \frac{u^2(t)}{[V]^2} \right) dt \quad (1)$$

and calculate the feedback-gain \mathbf{K} using the MATLAB function `lqr(A, B, Q, R, [])`.

- (c) Draw the closed-loop block diagram, write down matrix \mathbf{A}_{cl} and simulate the system in MATLAB.
- (d) Draw the closed-loop block diagram including input w and simulate the system with input $w = 100$. What do you observe for the steady-state value of the capacitor voltage? How can you make the voltage v_C follow the input w ?
(Remark: we denote the input w to distinguish from actual control u)
- (e) Implement the prefilter \mathbf{N} and simulate the system that can track a referenc w at the capacitor voltage v_C .
- (f) What do you observe regarding the control u when you tune the controller to be more aggressive (\mathbf{R} small, e.g. $\frac{0.01}{[V]^2}$)?