Model Predictive Control of Power Converters:

Control algorithms and numerical optimization methods

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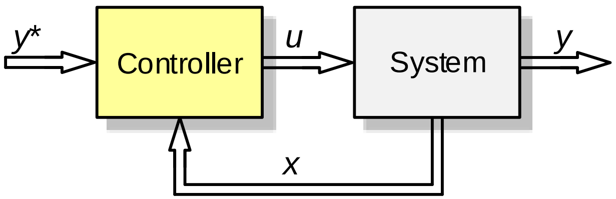
Model Predictive Control of Power Converters
Classification

Direct control



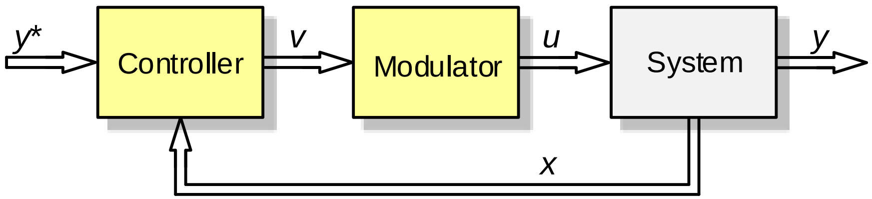
Direct manipulation of switch position:

- Manipulated variable: $\boldsymbol{u} \in \mathbb{Z}^{n_u}$

Control methods:

- Reference tracking (finite control set MPC): enumeration or sphere decoding
- Bounds (MPDxC): branch and bound
- Optimized pulse patterns: QP solver or algebraic manipulation

Indirect control



Indirect manipulation of switch position:

- Manipulated variable: $\boldsymbol{v} \in \mathbb{R}^{n_u}$

Control methods:

- Reference tracking: linearization, QP solver or explicit solution

Model Predictive Control of Power Converters
Outline

Long-horizon direct MPC

- *Integer optimization problem*
- *Sphere decoding*
- *Case study*

Indirect MPC

- *Modular multilevel converter*
- *Controller formulation*
- *Simulation results*

Assessment of control methods for power converters

Conclusions and outlook

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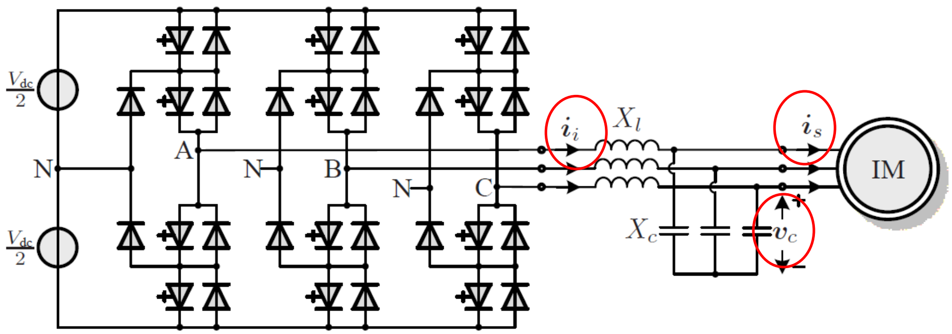
Long-Horizon Direct MPC
Case Study

NPC converter with *LC* filter and ind. machine:



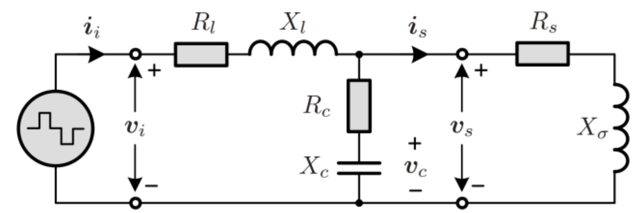
Control objectives:

- Regulate **inverter** currents, **capacitor** voltages and **stator** currents along their references
  $y^* = [(i_i^*)^T \ (v_c^*)^T \ (i_s^*)^T]^T$ with $\alpha\beta$-components
- Minimize the switching frequency

Assessment:

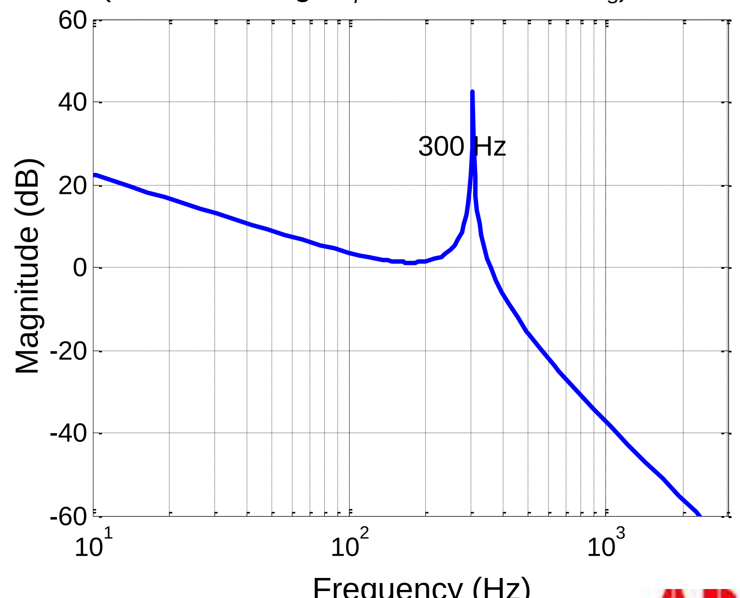
- Two coupled 3rd order systems
- Short horizons lead to **poor performance** (due to the undamped system resonance)
- Long horizons are **mandatory**

System resonance:



Bode magnitude plot:

(inverter voltage v_i to stator current i_s)



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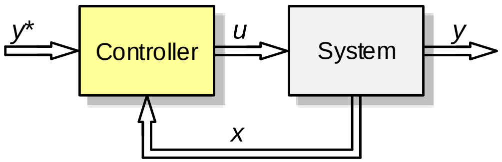
Long-Horizon Direct MPC
Derivation of the Integer Program

- Consider the discrete-time linear time-invariant system with integer inputs

$$\boldsymbol{x}(\ell+1) = \boldsymbol{A}\boldsymbol{x}(\ell) + \boldsymbol{B}\boldsymbol{u}(\ell)$$
$$\boldsymbol{y}(\ell) = \boldsymbol{C}\boldsymbol{x}(\ell)$$

where $\boldsymbol{x} \in \mathbb{R}^{n_x}$, $\boldsymbol{u} \in \mathbb{Z}^{n_u}$, $\boldsymbol{y} \in \mathbb{R}^{n_y}$

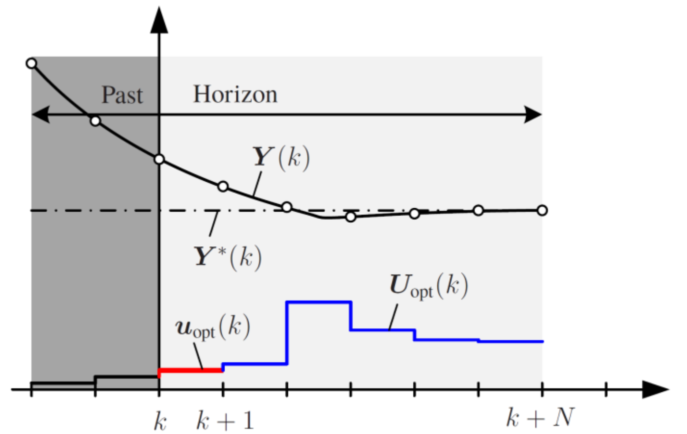


- Consider the quadratic cost function

$$J = \sum_{\ell=k}^{k+N-1} \underbrace{||\boldsymbol{y}^*(\ell+1) - \boldsymbol{y}(\ell+1)||_{\boldsymbol{Q}}^2}_{\substack{\text{Tracking error} \\ \text{(deviation from reference)}}} + \underbrace{\lambda_u ||\Delta \boldsymbol{u}(\ell)||_2^2}_{\substack{\text{Penalty on} \\ \text{control effort}}}$$

$$||\boldsymbol{y}^* - \boldsymbol{y}||_{\boldsymbol{Q}}^2 = (\boldsymbol{y}^* - \boldsymbol{y})^T \boldsymbol{Q} (\boldsymbol{y}^* - \boldsymbol{y})$$

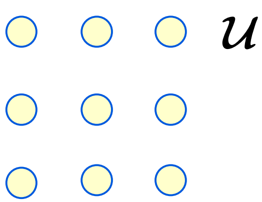
$$\Delta \boldsymbol{u}(\ell) = \boldsymbol{u}(\ell) - \boldsymbol{u}(\ell-1)$$
$$\lambda_u > 0$$



- Consider the input constraints

$$\boldsymbol{u}(\ell) \in \boldsymbol{\mathcal{U}} \quad \forall \ell = k, \dots, k+N-1$$
$$||\Delta \boldsymbol{u}(\ell)||_\infty \leq 1$$

Example of an input set: $\boldsymbol{\mathcal{U}} = \{-1, 0, 1\}^3$



Long-Horizon Direct MPC
Optimization Problem

$$
\begin{aligned}
\underset{\boldsymbol{U}(k)}{\text{minimize}} \quad & \sum_{\ell=k}^{k+N-1} ||\boldsymbol{y}^*(\ell+1) - \boldsymbol{y}(\ell+1)||_{\boldsymbol{Q}}^2 + \lambda_u ||\Delta \boldsymbol{u}(\ell)||_2^2 \\
\text{subject to} \quad & \boldsymbol{x}(\ell+1) = \boldsymbol{A}\boldsymbol{x}(\ell) + \boldsymbol{B}\boldsymbol{u}(\ell) \\
& \boldsymbol{y}(\ell+1) = \boldsymbol{C}\boldsymbol{x}(\ell+1) \\
& \Delta \boldsymbol{u}(\ell) = \boldsymbol{u}(\ell) - \boldsymbol{u}(\ell-1) \\
& \boldsymbol{u}(\ell) \in \boldsymbol{\mathcal{U}} \\
& ||\Delta \boldsymbol{u}(\ell)||_\infty \leq 1, \ \forall \ell = k, \dots, k+N-1
\end{aligned}
$$

With

- The sequence of manipulated variables $\boldsymbol{U}(k) = [\boldsymbol{u}^T(k) \ \boldsymbol{u}^T(k+1) \ \dots \ \boldsymbol{u}^T(k+N-1)]^T$
- The sequence of reference values $\boldsymbol{Y}^*(k) = [\boldsymbol{y}^{*T}(k+1) \ \boldsymbol{y}^{*T}(k+2) \ \dots \ \boldsymbol{y}^{*T}(k+N)]^T$

The optimization problem is a function of

- The “parameters” $\boldsymbol{x}(k), \boldsymbol{u}(k-1), \boldsymbol{Y}^*(k)$
- The optimization variable $\boldsymbol{U}(k)$

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Long-Horizon Direct MPC
Objective Function in Vector Form

- The objective function in vector form:

$$J = J_1 + J_2 = ||\boldsymbol{Y}^*(k) - \boldsymbol{\Gamma}\boldsymbol{x}(k) - \boldsymbol{\Upsilon}\boldsymbol{U}(k)||^2_{\widetilde{\boldsymbol{Q}}} + \lambda_u ||\boldsymbol{S}\boldsymbol{U}(k) - \boldsymbol{E}\boldsymbol{u}(k-1)||^2_2$$

- After some algebraic manipulations:

$$\boxed{J = \boldsymbol{U}^T(k)\boldsymbol{H}\boldsymbol{U}(k) + 2\boldsymbol{\Theta}^T(k)\boldsymbol{U}(k) + \theta(k)}$$

- Where
  - $\boldsymbol{H} = \boldsymbol{\Upsilon}^T\widetilde{\boldsymbol{Q}}\boldsymbol{\Upsilon} + \lambda_u\boldsymbol{S}^T\boldsymbol{S}$

    $\boldsymbol{H} \in \mathbb{R}^{3N\times 3N}$ is the Hessian matrix

    It is a function of $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$ and λ_u

    It holds that $\boldsymbol{H} = \boldsymbol{H}^T$, $\boldsymbol{H} \succ 0$ for $\lambda_u > 0$
  - $\boldsymbol{\Theta}(k) = -\boldsymbol{\Upsilon}^T\widetilde{\boldsymbol{Q}}\big(\boldsymbol{Y}^*(k) - \boldsymbol{\Gamma}\boldsymbol{x}(k)\big) - \lambda_u\boldsymbol{S}^T\boldsymbol{E}\boldsymbol{u}(k-1)$

    $\boldsymbol{\Theta}(k) \in \mathbb{R}^{3N}$ is a time-varying vector

    It is a function of $\boldsymbol{x}(k)$, $\boldsymbol{u}(k-1)$ and $\boldsymbol{Y}^*(k)$
  - $\theta(k) = ||\boldsymbol{Y}^*(k) - \boldsymbol{\Gamma}\boldsymbol{x}(k)||^2_{\widetilde{\boldsymbol{Q}}} + \lambda_u||\boldsymbol{E}\boldsymbol{u}(k-1)||^2_2$

    $\theta(k) \in \mathbb{R}$ is a time-varying scalar

Can we rewrite the problem to solve it more easily?

Long-Horizon Direct MPC
Objective Function in Vector Form

- Starting with

$$J = \boldsymbol{U}^T(k)\boldsymbol{H}\boldsymbol{U}(k) + 2\boldsymbol{\Theta}^T(k)\boldsymbol{U}(k) + \theta(k)$$

- we "complete the squares"

$$J = \big(\boldsymbol{U}(k) + \boldsymbol{H}^{-1}\boldsymbol{\Theta}(k)\big)^T \boldsymbol{H}\big(\boldsymbol{U}(k) + \boldsymbol{H}^{-1}\boldsymbol{\Theta}(k)\big) - \underbrace{\boldsymbol{\Theta}^T(k)\boldsymbol{H}^{-1}\boldsymbol{\Theta}(k) + \theta(k)}_{\text{Independent of } \boldsymbol{U}(k)}$$

- and write the obj. function in the quadratic form

$$J = \big(\boldsymbol{U}(k) - \boldsymbol{U}_{\text{unc}}(k)\big)^T \boldsymbol{H}\big(\boldsymbol{U}(k) - \boldsymbol{U}_{\text{unc}}(k)\big)$$

- with the unconstrained solution $\boldsymbol{U}_{\text{unc}}(k) = -\boldsymbol{H}^{-1}\boldsymbol{\Theta}(k)$

  Real-valued vector
  A function of ***x****(k)*, ***u****(k-1) and* ***Y*****(k)*

- *The Hessian* ***H*** is symmetric and positive definite for $\lambda_u > 0$

  => *A unique invertible and lower triangular matrix* ***V*** *exists which satisfies* $\boldsymbol{V}^T\boldsymbol{V} = \boldsymbol{H}$

- This leads to optimization problem

$$\boxed{J = ||\boldsymbol{V}\boldsymbol{U}(k) - \boldsymbol{V}\boldsymbol{U}_{\text{unc}}(k)||_2^2}$$

Derivation of the Integer Program
Reformulated Optimization Problem

$$\boldsymbol{U}_{\text{opt}}(k) = \arg \min_{\boldsymbol{U}(k)} ||\boldsymbol{V}\boldsymbol{U}(k) - \boldsymbol{V}\boldsymbol{U}_{\text{unc}}(k)||_2^2$$

$$\text{subject to } \boldsymbol{U}(k) \in \boldsymbol{\mathcal{U}}^N$$

$$||\Delta \boldsymbol{u}(\ell)||_\infty \leq 1, \; \forall \ell = k, \ldots, k+N-1$$

Switching sequence (optimizer)

$$\boldsymbol{U}(k) = [\boldsymbol{u}^T(k) \ldots \boldsymbol{u}^T(k+N-1)]^T$$

Integer-valued vector

$$\mathcal{U}^N = \{-1, 0, 1\}^{3N}$$

Unconstrained solution

$$\boldsymbol{U}_{\text{unc}}(k) = -\boldsymbol{H}^{-1}\boldsymbol{\Theta}(k)$$

Real-valued vector
*Function of **x**(k), **u**(k-1) and **Y***(k)*

The optimization problem amounts to finding the integer vector ***U****(k)* that minimizes the Euclidian distance to ***U***$_{\text{unc}}$(*k*) in the space spanned by ***V***

V is the **generator matrix** (with $\boldsymbol{V}^T\boldsymbol{V} = \boldsymbol{H}$)

Geyer and Quevedo: "Performance of multistep finite control set model predictive control for power electronics", IEEE Trans. on Power Electronics, Mar. 2015

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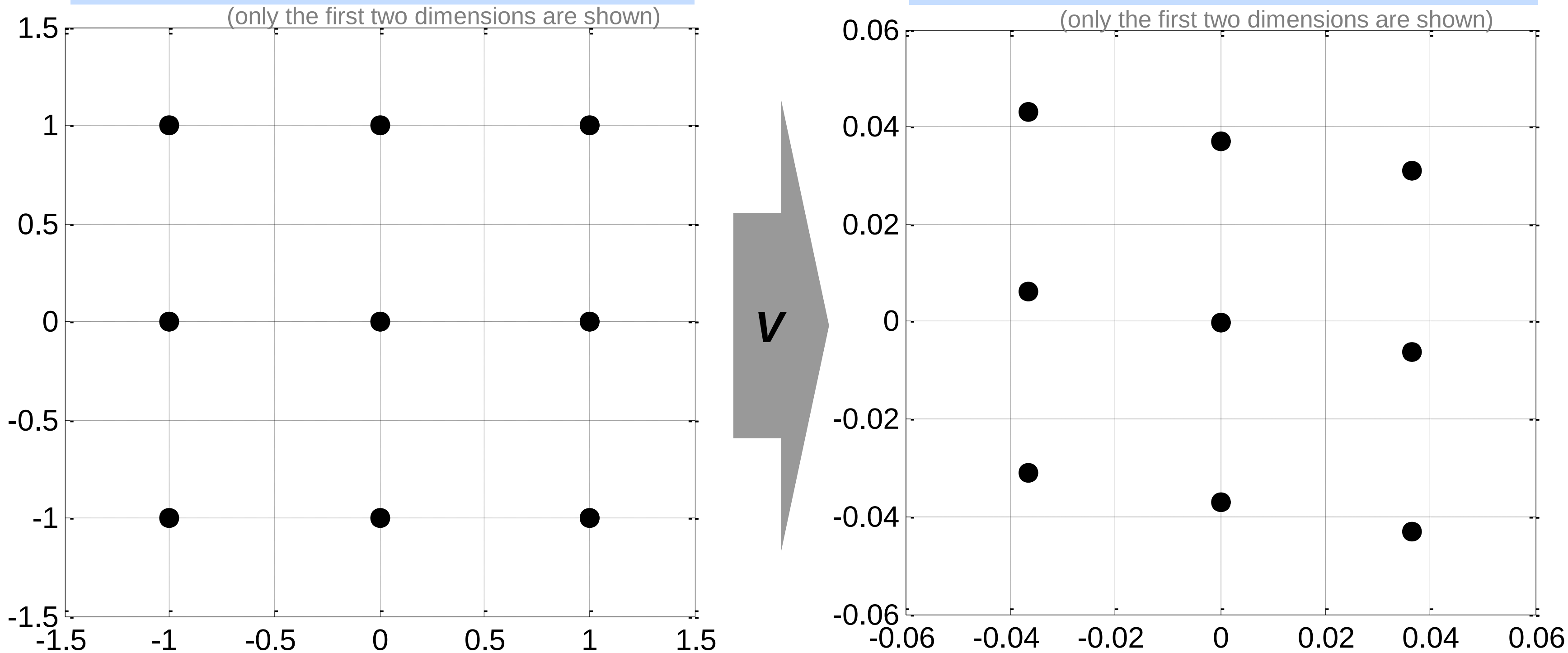
Conclusions and outlook

Solving the Integer Program

Example: Optimization Problem with $\mathcal{U} \in \{-1, 0, 1\}^3$

$\mathcal{U}$ in the orthogonal coordinate system

$\mathcal{U}$ in the transformed coordinate system



"Generator" matrix (for T_s=25µs, λ_u=10^{-3}):

$$\boldsymbol{V} = \begin{bmatrix} 36.45 & 0 & 0 \\ -6.068 & 36.95 & 0 \\ -5.265 & -5.265 & 37.32 \end{bmatrix} \cdot 10^{-3} \quad \text{with} \quad \boldsymbol{V}^T\boldsymbol{V} = \boldsymbol{H}$$

almost diagonal

Solving the Integer Program
Example: Optimization Problem with $\mathcal{U} \in \{-1, 0, 1\}^3$

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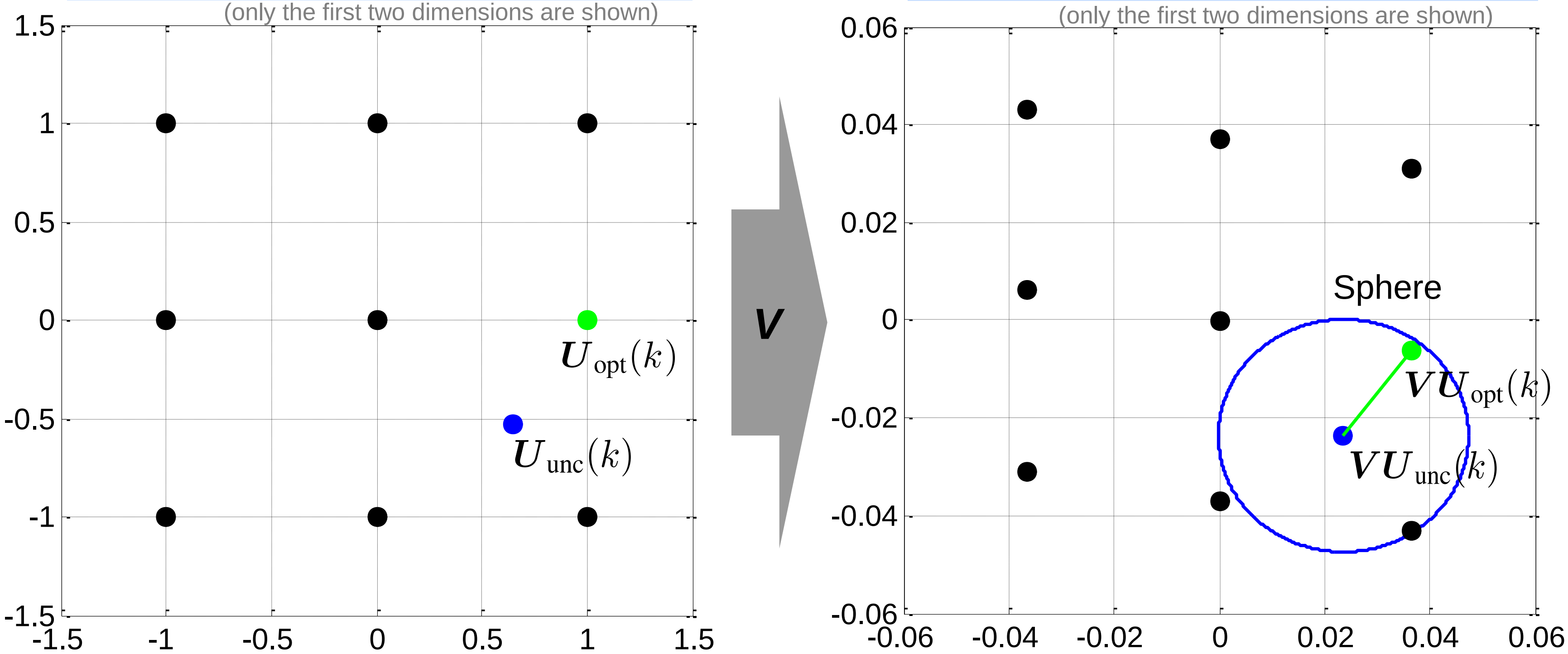


In the **transformed** coordinate system:

- The optimal solution has the **minimal distance** to the unconstrained solution
- The **optimal** solution lies within a **sphere** (centered at the **unconstrained solution**)

Modified **sphere decoder** => solve the integer optimization problem

Solving the Integer Program
Sphere Decoder

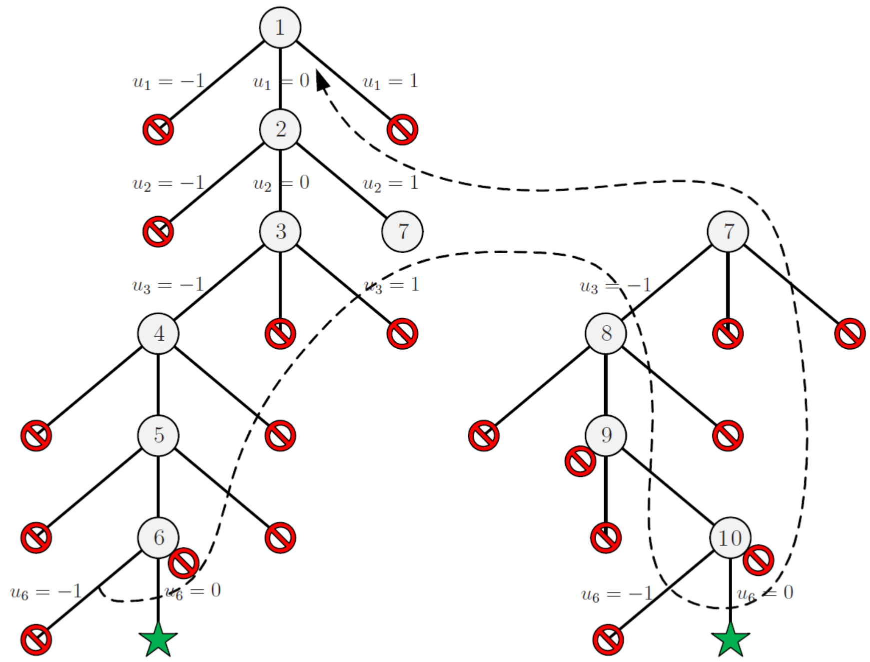
Branch and bound algorithm

- Branching over the set of single-phase switch positions $\mathcal{U} = \{-1, 0, 1\}$ that meet the switching constraint $||\Delta \boldsymbol{u}(\ell)||_\infty \leq 1$
- Bounding: consider solutions only within the sphere of radius $\rho(k)$:

  $$||\boldsymbol{V}\boldsymbol{U}(k) - \boldsymbol{V}\boldsymbol{U}_{\text{unc}}(k)||_2 \leq \rho(k)$$

  If the radius is exceeded => certificate has been found that the branch is suboptimal
- The sphere is tightened whenever a better solution is found

Example: search tree for $\boldsymbol{\mathcal{U}}^N = \{-1, 0, 1\}^6$



Fincke and Pohst: "Improved methods for calculating vectors of short length in a lattice, including a complexity analysis," Math. Comput., Apr. 1985

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Solving the Integer Program
Sphere Decoder

Branch and bound algorithm

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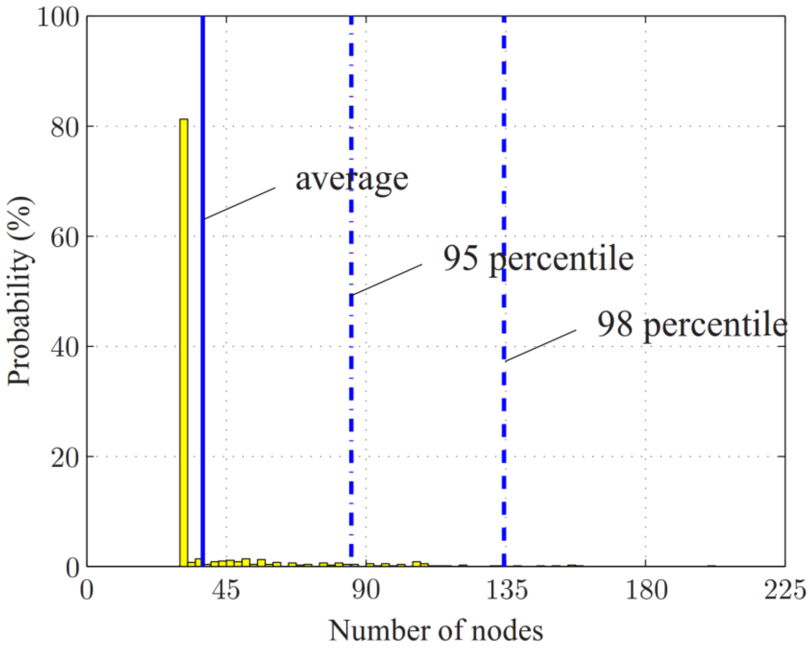
Number of nodes explored for $\boldsymbol{\mathcal{U}}^N = \{-1, 0, 1\}^{30}$



The optimal solution is found in 80% of the cases by exploring only **one** switching sequence => **tight sphere** / strong bounding

Fincke and Pohst: “Improved methods for calculating vectors of short length in a lattice, including a complexity analysis,” Math. Comput., Apr. 1985

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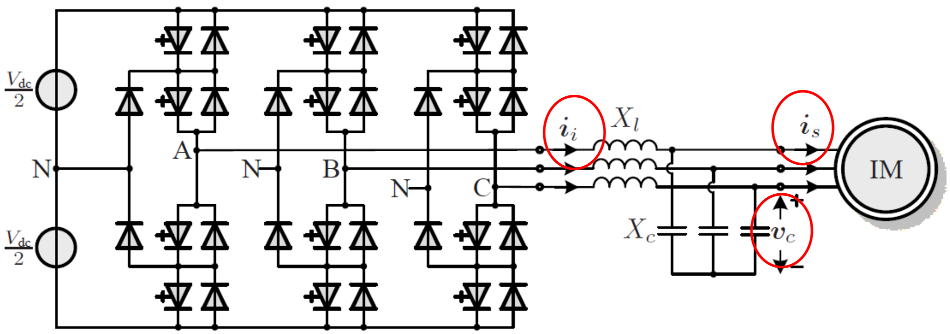
Drive System with *LC* Filter
Case Study

NPC converter with *LC* filter and ind. machine:



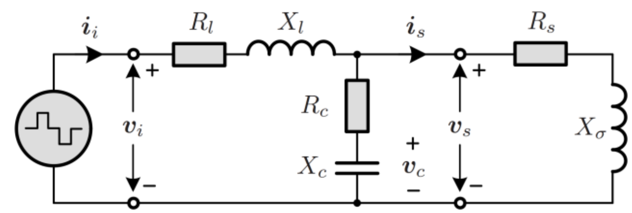
Control objectives:

- Regulate **inverter** currents, **capacitor** voltages and **stator** currents along their references
  $\boldsymbol{y}^* = [(\boldsymbol{i}_i^*)^T \ (\boldsymbol{v}_c^*)^T \ (\boldsymbol{i}_s^*)^T]^T$ with $\alpha\beta$-components
- Minimize the switching frequency

Assessment:

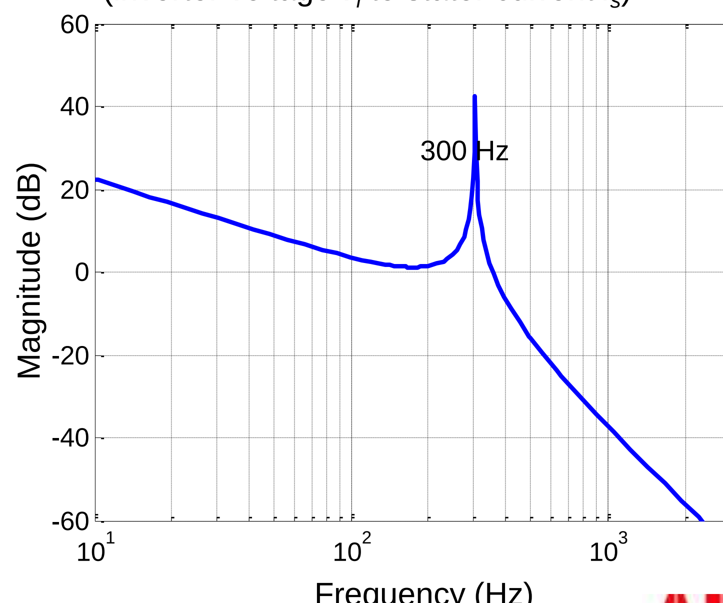
- Two coupled 3rd order systems
- Short horizons lead to **poor performance** (due to the undamped system resonance)
- Long horizons are **mandatory**

System resonance:



Bode magnitude plot:

(inverter voltage v_i to stator current i_s)



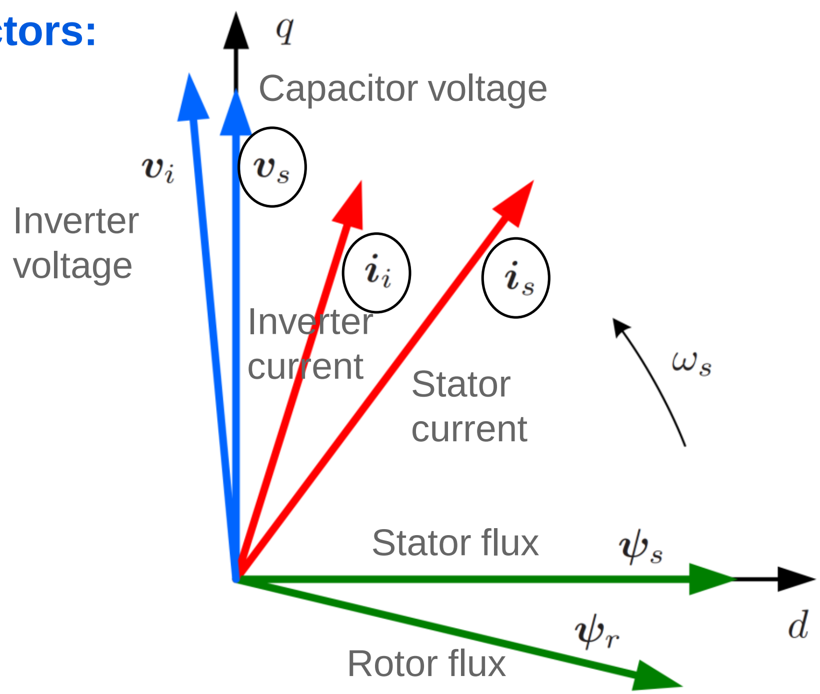
Drive System with *LC* Filter
Control Problem Formulation

Performance index: $$J = \sum_{\ell=k}^{k+N-1} \underbrace{||\boldsymbol{y}^*(\ell+1) - \boldsymbol{y}(\ell+1)||_{\boldsymbol{Q}}^2}_{\text{Tracking error (deviation from reference)}} + \underbrace{\lambda_u ||\boldsymbol{\Delta u}(\ell)||_2^2}_{\text{Penalty on switching effort}}$$

$$\boldsymbol{y} = [\boldsymbol{i}_i^T \ \boldsymbol{v}_c^T \ \boldsymbol{i}_s^T]^T$$

$$\boldsymbol{\Delta u}(\ell) = \boldsymbol{u}(\ell) - \boldsymbol{u}(\ell-1)$$

Output reference vectors:



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$$\boldsymbol{y} = [\boldsymbol{i}_i^T \ \boldsymbol{v}_c^T \ \boldsymbol{i}_s^T]^T \qquad \boldsymbol{\Delta u}(\ell) = \boldsymbol{u}(\ell) - \boldsymbol{u}(\ell-1)$$

Model:

$$\boldsymbol{x}(\ell+1) = \boldsymbol{A}\boldsymbol{x}(\ell) + \boldsymbol{B}\boldsymbol{u}(\ell)$$

$$\boldsymbol{y}(\ell) = \boldsymbol{C}\boldsymbol{x}(\ell)$$

with $\boldsymbol{x} = [\boldsymbol{i}_i^T \ \boldsymbol{v}_c^T \ \boldsymbol{i}_s^T \ \boldsymbol{\psi}_r^T]^T \in \mathbb{R}^8$

$\boldsymbol{u} \in \mathbb{Z}^3$

Input constraints:

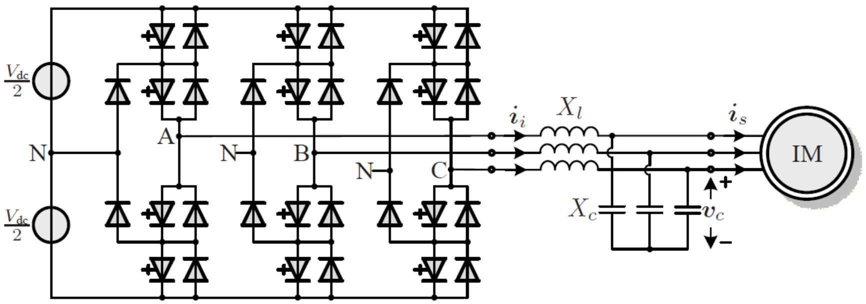
$$\boldsymbol{u}(\ell) \in \{-1, 0, 1\}^3$$

$$||\boldsymbol{\Delta u}(\ell)||_\infty \le 1$$

The outputs are linear in the initial state vector ***x***(*k*) and the sequence of manipulated variables (the switching sequence)
$$\boldsymbol{U}(k) = [\boldsymbol{u}^T(k) \ \boldsymbol{u}^T(k+1) \ldots \boldsymbol{u}^T(k+N-1)]^T$$

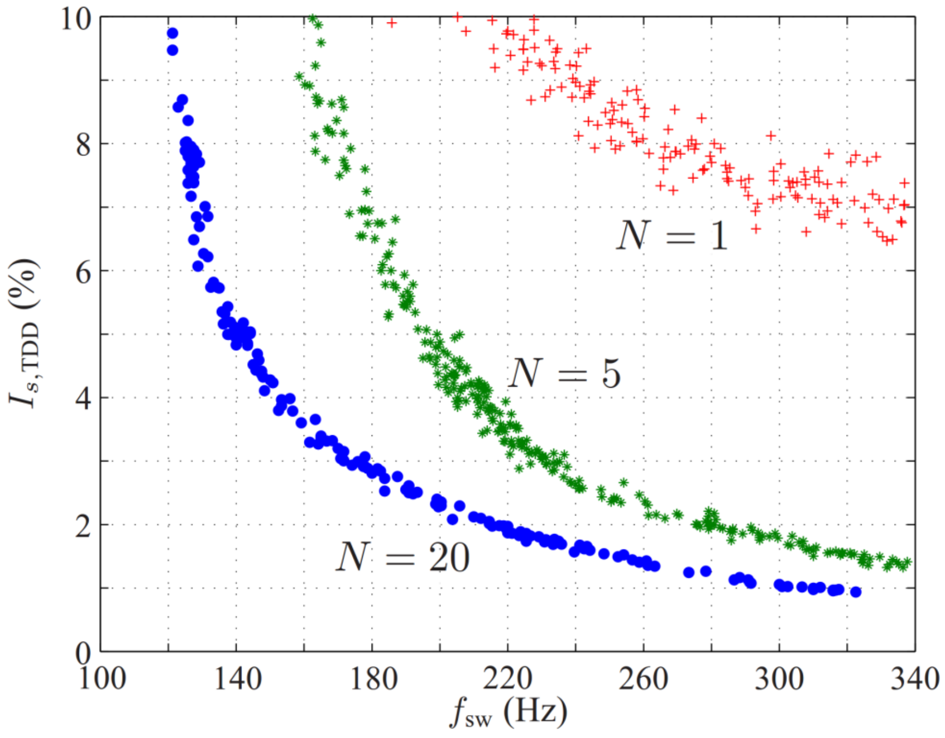
Drive System with *LC* Filter
System Parameters and Current TDD

System parameters



- MV induction **machine**: 3.3kV, 2MVA, 50Hz, X_σ = 0.25pu
- **Filter**: X_l = 0.117pu, X_c = 0.336pu, f_{res} = 304Hz
- Sampling **interval**: T_s = 125μs

Current TDD vs switching frequency

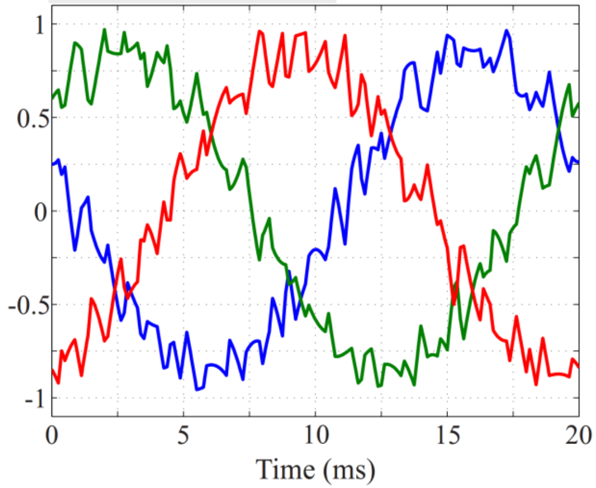


Long prediction horizons enable operation at switching frequencies f_{sw} below 50% of the resonance frequency f_{res}

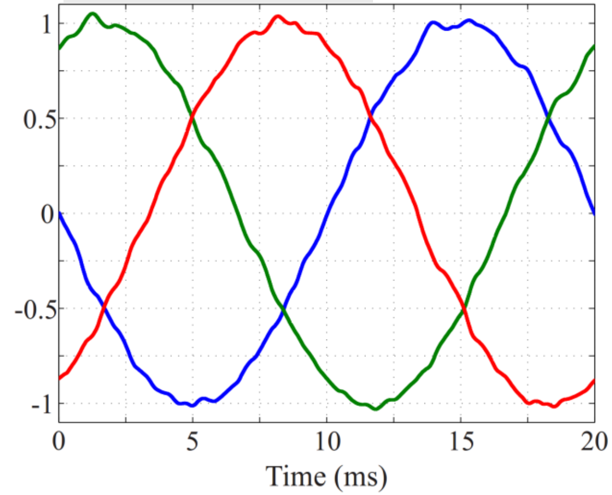
Drive System with *LC* Filter
Steady-State Operation

- Penalty matrix on output var.s $\boldsymbol{Q} = \text{diag}(1, 1, 5, 5, 150, 150)$
- Penalty on switching $\lambda_u = 0.28$
- Sampling interval $T_s = 125\ \mu\text{s}$
- Prediction horizon $N = 15$

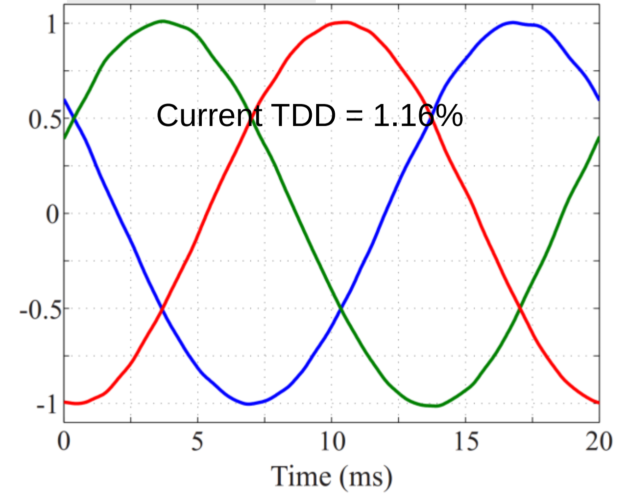
Inverter currents



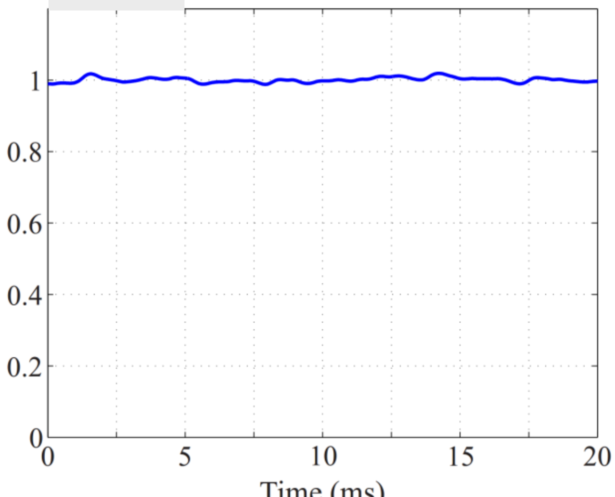
Capacitor voltages



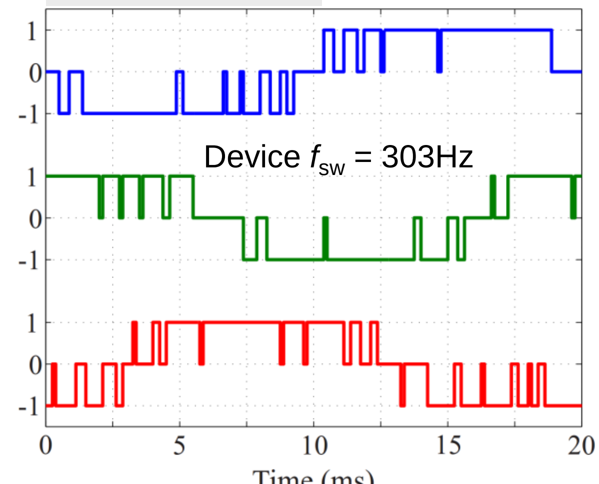
Stator currents



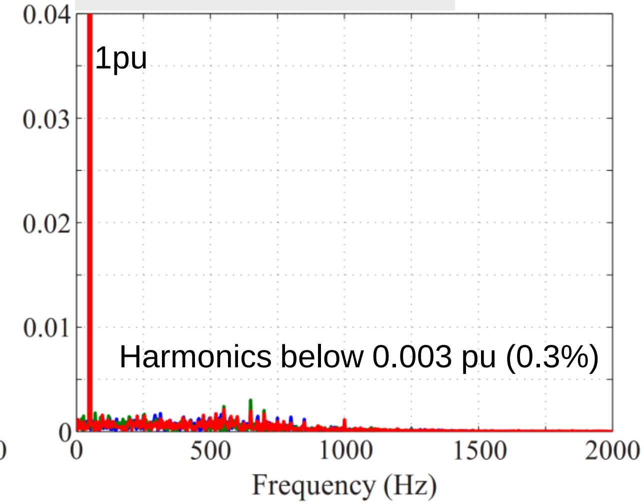
Torque



Switch positions



Stator current spectrum

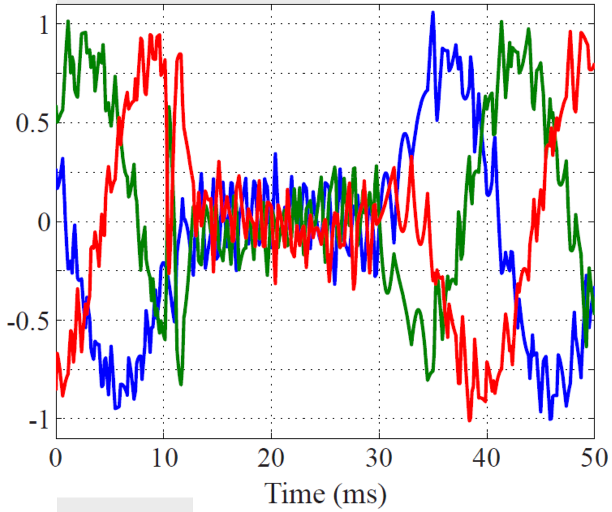


Geyer, Karamanakos and Kennel: “On the benefit of long-horizon direct model predictive control for drives with LC filters”, ECCE, USA, Sep. 2014

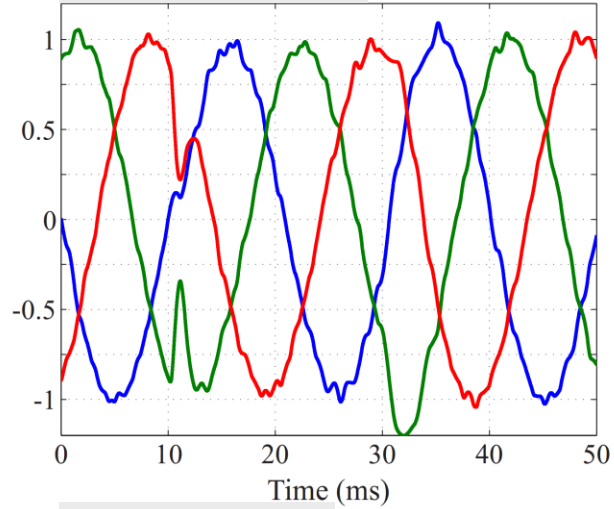
Drive System with *LC* Filter
Torque Steps

- Penalty matrix on output var.s $Q = \mathrm{diag}(1, 1, 5, 5, 150, 150)$
- Penalty on switching $\lambda_u = 0.28$
- Sampling interval $T_s = 125\ \mu\mathrm{s}$
- Prediction horizon $N = 15$

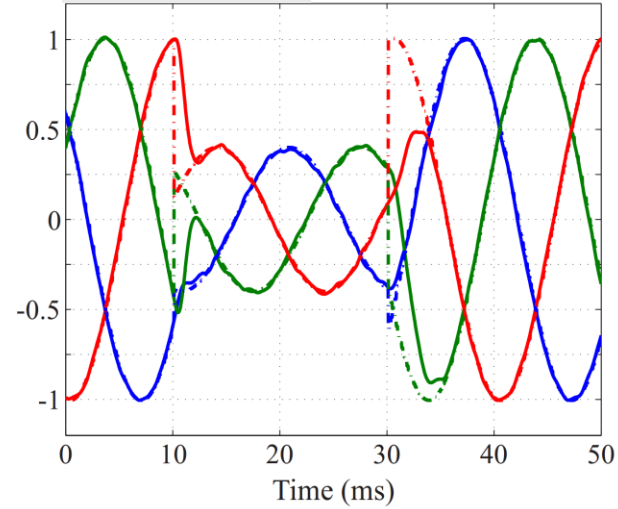
Inverter currents



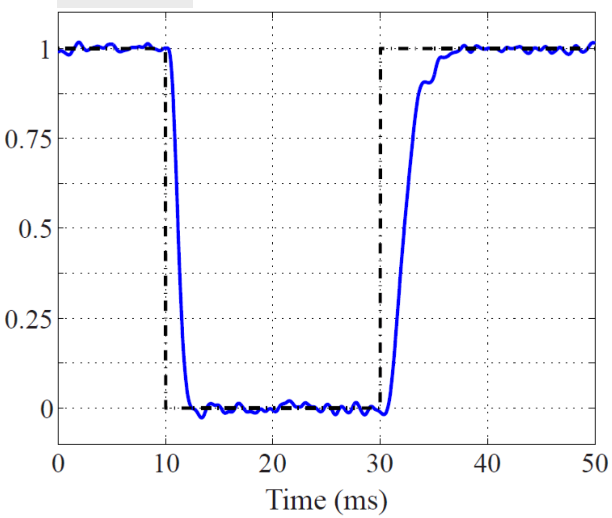
Capacitor voltages



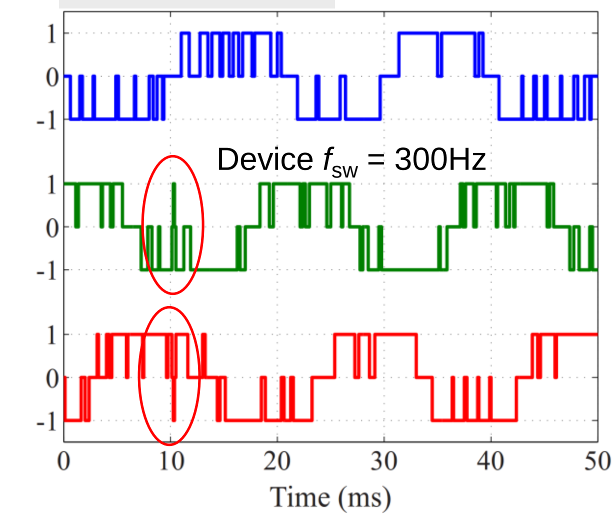
Stator currents



Torque



Switch positions



During transient, reduce penalty from $\mathrm{diag}(1, 1, 5, 5, 150, 150)$ to $\mathrm{diag}(1, 1, 5, 5, 15, 15)$

$\Rightarrow$ Almost no overshoot

$\Rightarrow$ Settling times of < 3ms

$\Rightarrow$ Constant switching frequency

$\Rightarrow$ Inversion of voltage (subject to switching constraint)

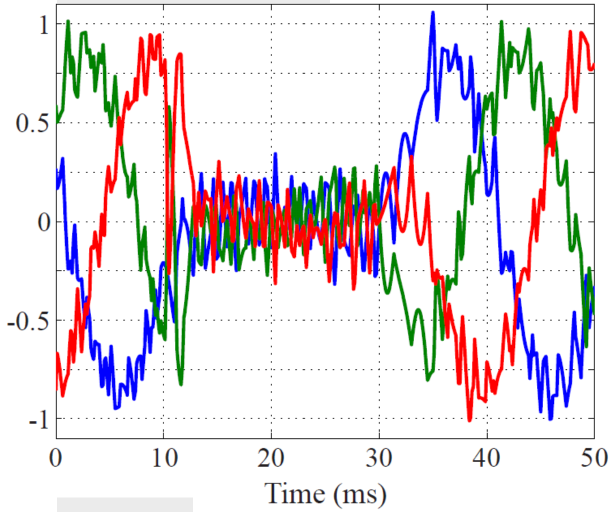
Geyer, Karamanakos and Kennel: “On the benefit of long-horizon direct model predictive control for drives with LC filters”, ECCE, USA, Sep. 2014

ABB

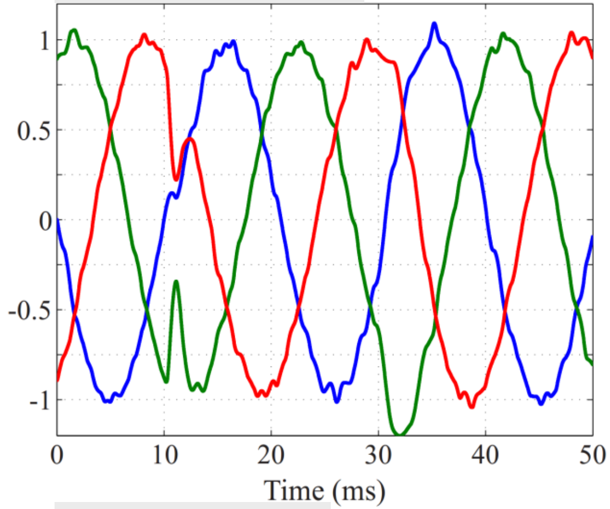
Drive System with *LC* Filter
Torque Steps

- Penalty matrix on output var.s $Q = \mathrm{diag}(1, 1, 5, 5, 150, 150)$
- Penalty on switching $\lambda_u = 0.28$
- Sampling interval $T_s = 125\ \mu\mathrm{s}$
- Prediction horizon $N = 15$

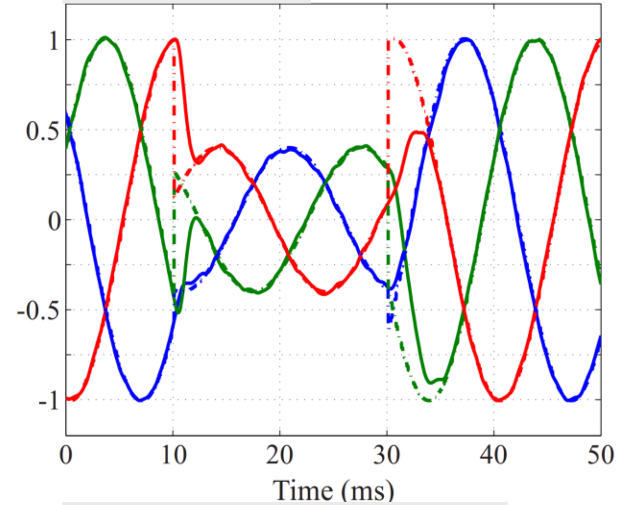
Inverter currents



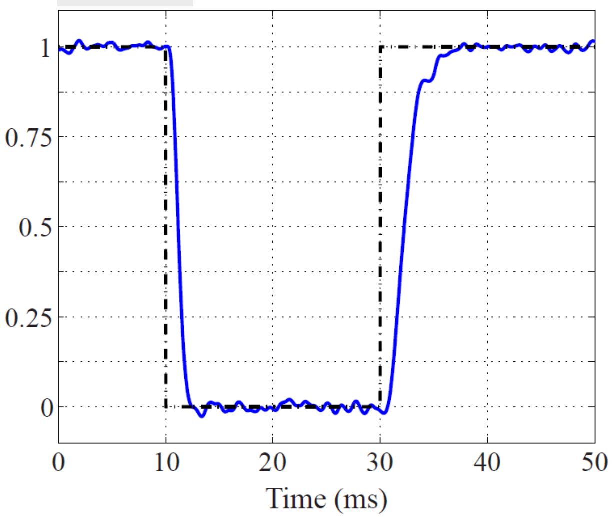
Capacitor voltages



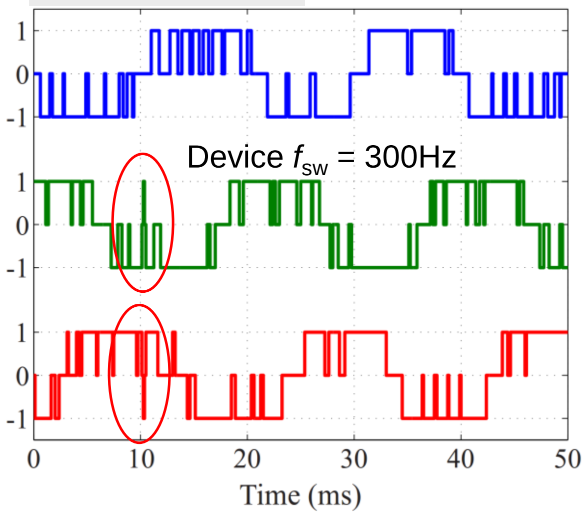
Stator currents



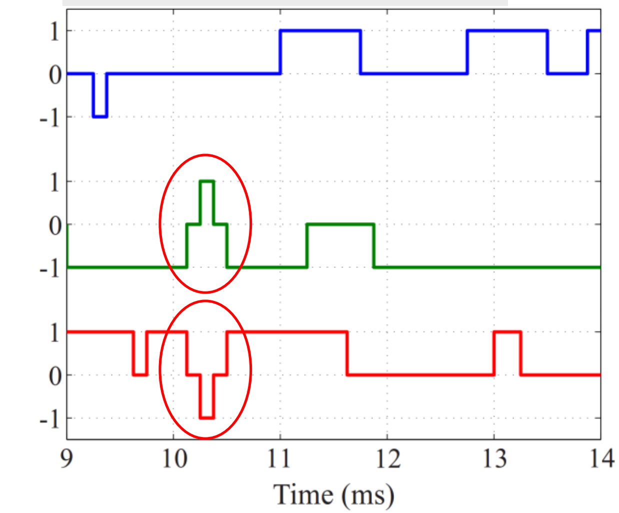
Torque



Switch positions



Switch positions (zoomed in)



Geyer, Karamanakos and Kennel: “On the benefit of long-horizon direct model predictive control for drives with LC filters”, ECCE, USA, Sep. 2014

Drive System with *LC* Filter
Assessment

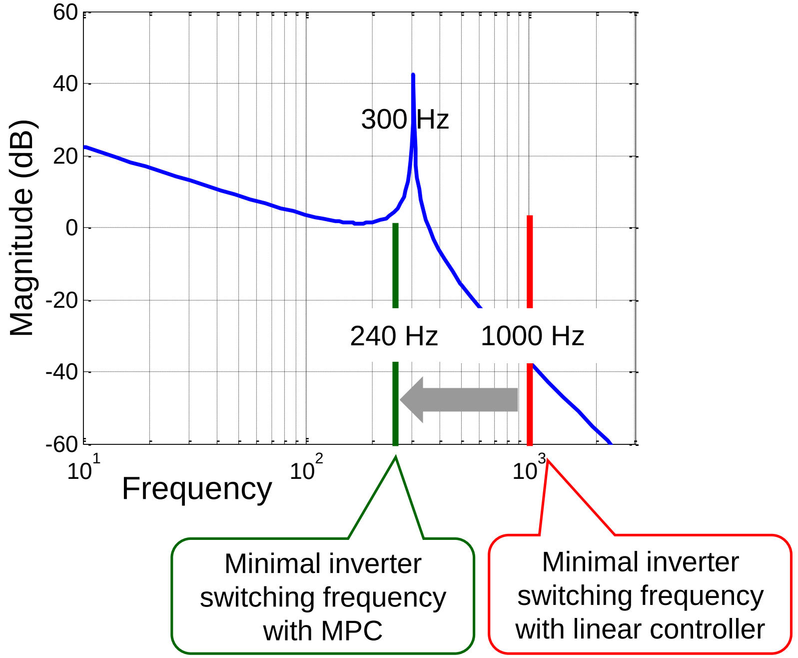
Advantages

- **Sphere decoding** exploits the problem structure *=> low computational burden*
- Part of the problem is solved **offline** *=> generator matrix*
- **Simple** controller design with one loop *=> active damping loop not required*

Performance

- Long horizons reduce the **current distortions** by an order of magnitude
- Can operate at **switching frequencies** below the resonance frequency => f_{sw}=120 *vs* f_{res}=304 Hz
- MIMO approach *=> excellent transient response* (<3ms)

Bode magnitude plot:
(inverter voltage v_i to stator current i_s)



The power electronics **community** focuses almost exclusively on the horizon **one** case
=> sphere decoder enables **long** horizons

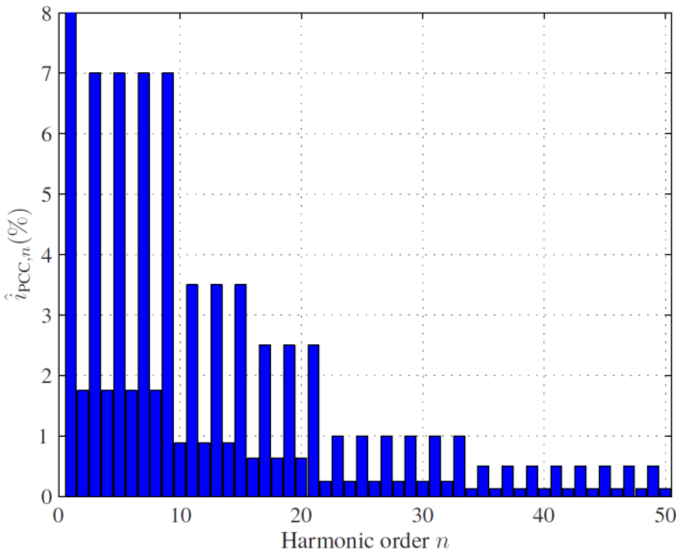
Drive System with *LC* Filter
Assessment

Limitations

- The **Hessian** matrix must be time invariant
- Sphere decoding is restricted to **linear** systems with **integer** inputs
- **Even-order** harmonics limit the applicability to grid-side converters

Further reduction of the computation time

- Preprocessing
  => well-conditioned generator matrix
- Allow for suboptimal solutions
  => impose upper bound on the solution time
- Project the unconstrained solution onto the convex hull
  => tight sphere during transients



Other extensions

- FPGA implementation
- Terminal weight
- State constraints (e.g. on currents)
- Voronoi diagrams
- Shaping of the harmonic spectrum

Model Predictive Control of Power Converters
Outline

Long-horizon direct MPC

- *Integer optimization problem*
- *Sphere decoding*
- *Case study*

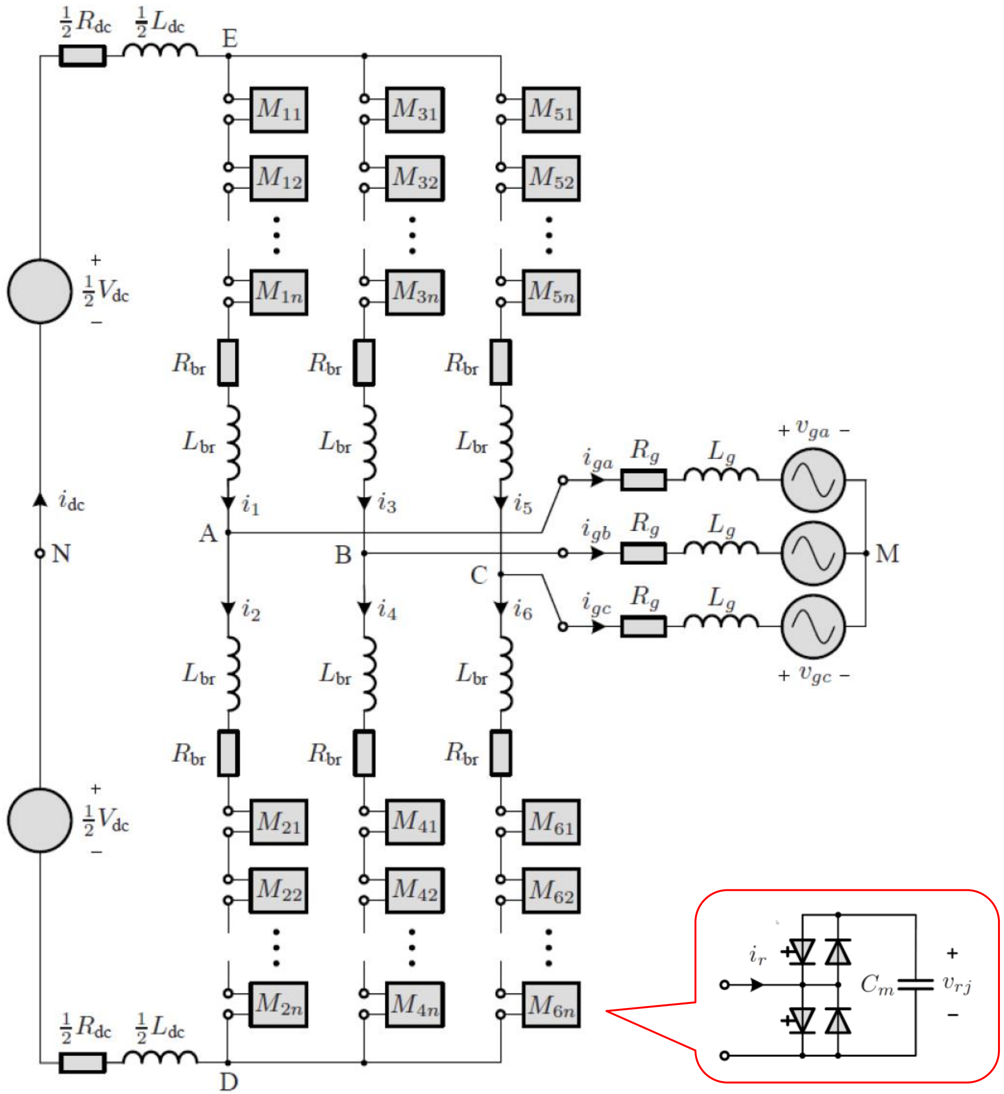
Indirect MPC

- *Modular multilevel converter*
- *Controller formulation*
- *Simulation results*

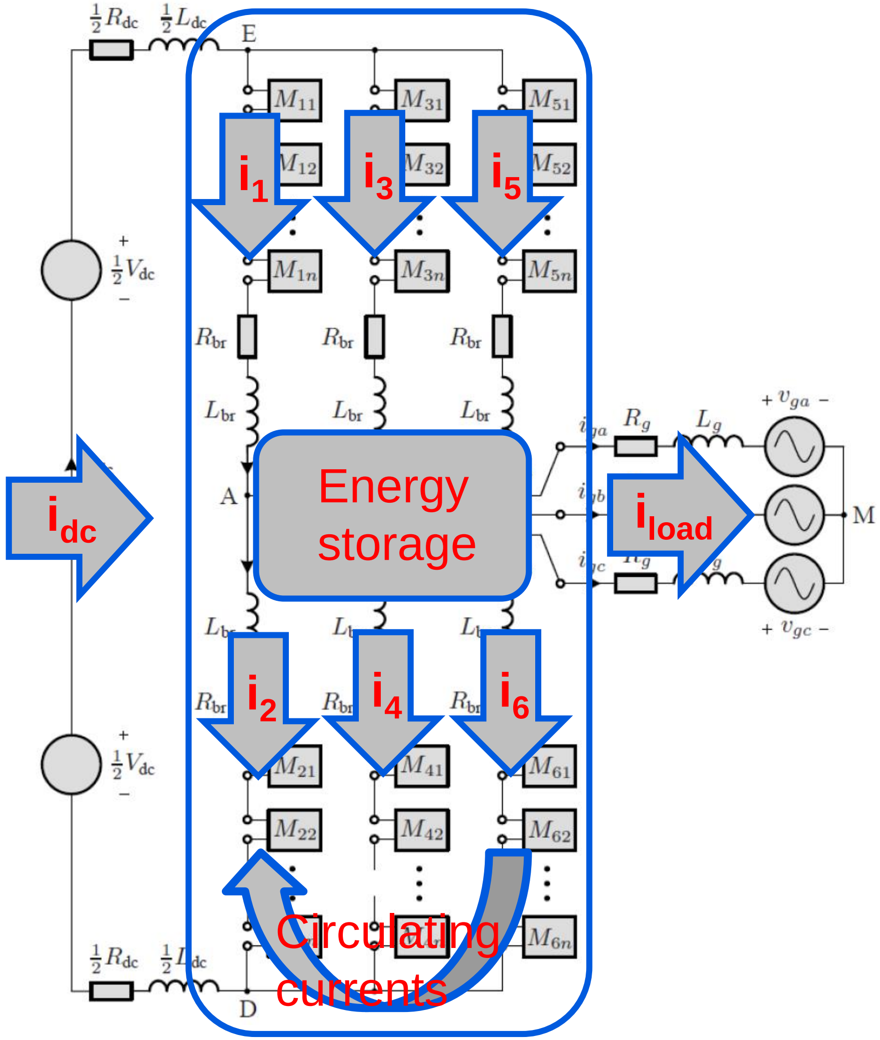
Assessment of control methods for power converters

Conclusions and outlook

Modular Multilevel Converter
Topology



Modular Multilevel Converter
Control Problem



- **MMC:**
  - **Dc-link** current i_{dc}
  - Internal currents: **branch** currents and **circulating** currents
  - Storage: **energy** per branch
- **Output:** load current i_{load}
- **Actuator:** number of modules inserted per branch

Model Predictive Control of Power Converters
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Conclusions and outlook

MPC of Modular Multilevel Converters
State-Space Model

=> Linearized **continuous-time** model:

$$\frac{d\boldsymbol{x}(t)}{dt} = \boldsymbol{A}_c(t_0)\boldsymbol{x}(t) + \boldsymbol{B}_c(t_0)\boldsymbol{u}(t) + \boldsymbol{f}_c(t_0)$$
$$\boldsymbol{y}(t) = \boldsymbol{C}_c\boldsymbol{x}(t)$$

with state vector $\boldsymbol{x} = \left[i_1 \dots i_4 \ i_{\text{dc}} \ v_1^{\Sigma} \dots v_6^{\Sigma} \ v_{g\alpha} \ v_{g\beta}\right]^T$

input vector $\boldsymbol{u} = [\Delta n_1 \dots \Delta n_6]^T$ Change in the insertion indices

output vector $\boldsymbol{y} = [i_\alpha \ i_\beta \ v_1^{\Sigma} \dots v_6^{\Sigma}]^T$

=> **Discrete-time** model:

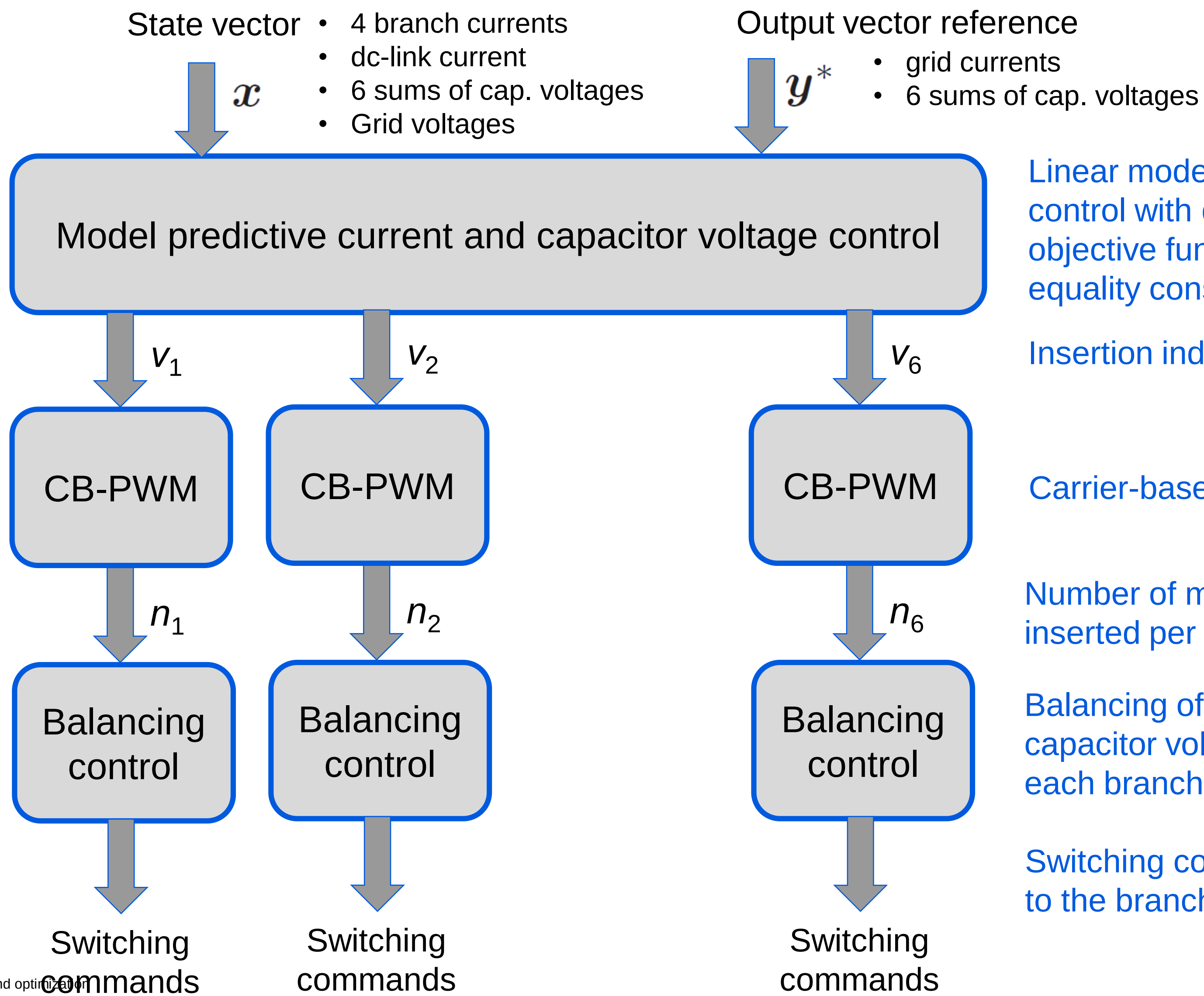
$$\boldsymbol{x}(k+1) = \boldsymbol{A}_d(t_0)\boldsymbol{x}(k) + \boldsymbol{B}_d(t_0)\boldsymbol{u}(k) + \boldsymbol{f}_d(t_0)$$

Time-varying system matrices

Time-varying offset vector

ABB

MPC of Modular Multilevel Converters
Control Method



Linear model predictive control with quadratic objective function and equality constraints => QP

Insertion indices [0, 1]

Carrier-based PWM

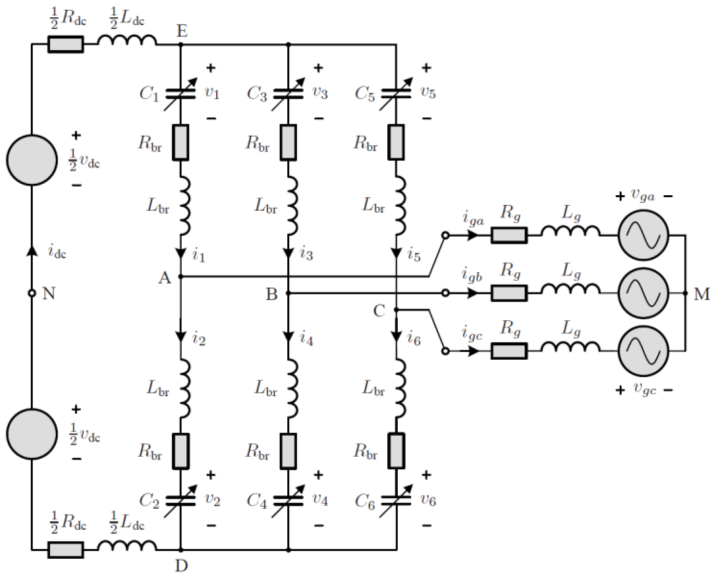
Number of modules to be inserted per branch

Balancing of the module capacitor voltages **within** each branch

Switching commands sent to the branch modules

MPC of Modular Multilevel Converters
MPC Formulation

System model:



Linearized discrete-time MMC model:

$$\boldsymbol{x}(k+1) = \boldsymbol{A}(t_0)\boldsymbol{x}(k) + \boldsymbol{B}(t_0)\boldsymbol{u}(k) + \boldsymbol{b}(t_0)$$

$$\boldsymbol{y}(k) = \boldsymbol{C}\boldsymbol{x}(k)$$

- Insertion indices

- Sums of capacitor voltages
- Branch currents
- Dc-link current
- Grid voltage

- Grid currents
- Sums of capacitor voltages
- Branch currents

Cost function:

$$J_1 = \sum_{\ell=k}^{k+N_p-1} \big(\boldsymbol{y}^*(\ell) - \boldsymbol{y}(\ell)\big)^T \boldsymbol{Q} \big(\boldsymbol{y}^*(\ell) - \boldsymbol{y}(\ell)\big)$$

Output reference tracking

$$J_2 = \sum_{\ell=k}^{k+N_p-1} \big(\Delta \boldsymbol{u}(\ell)\big)^T \boldsymbol{R}\, \Delta \boldsymbol{u}(\ell)$$

Changes in insertion indices

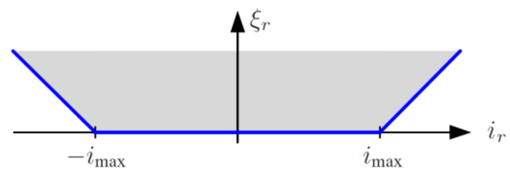
$$J_3 = \sum_{\ell=k}^{k+N_p-1} \lambda_\xi \|\boldsymbol{\xi}(\ell)\|_1 + \lambda_\zeta \|\boldsymbol{\zeta}(\ell)\|_1$$

Soft constraints

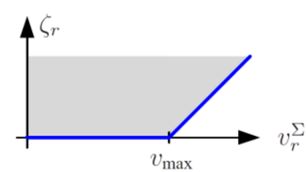
Constraints:

$$0 \le \boldsymbol{u}(\ell) \le 1$$

Hard constraints on insertion indices



Soft constraints on branch currents and dc-link current



Soft constraints on sums of capacitor voltages

min J_1+J_2+J_3 subj. to model and constraints => QP

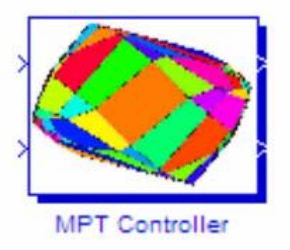
Model predictive current control of modular multilevel converters, ECCE, USA, Sep. 2014

MPC of Modular Multilevel Converters
Solving the QP

- Prediction horizon N_p=6
- Manipulated variables: 6 N_p
- Slack variables: 12 N_p

Dimension of QP: 108

- Problem formulation: MPT Toolbox 3.0
- QP solution: Gurobi



Gurobi Optimization

ABB

Model Predictive Control of Power Converters
Outline

Long-horizon direct MPC

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Indirect MPC

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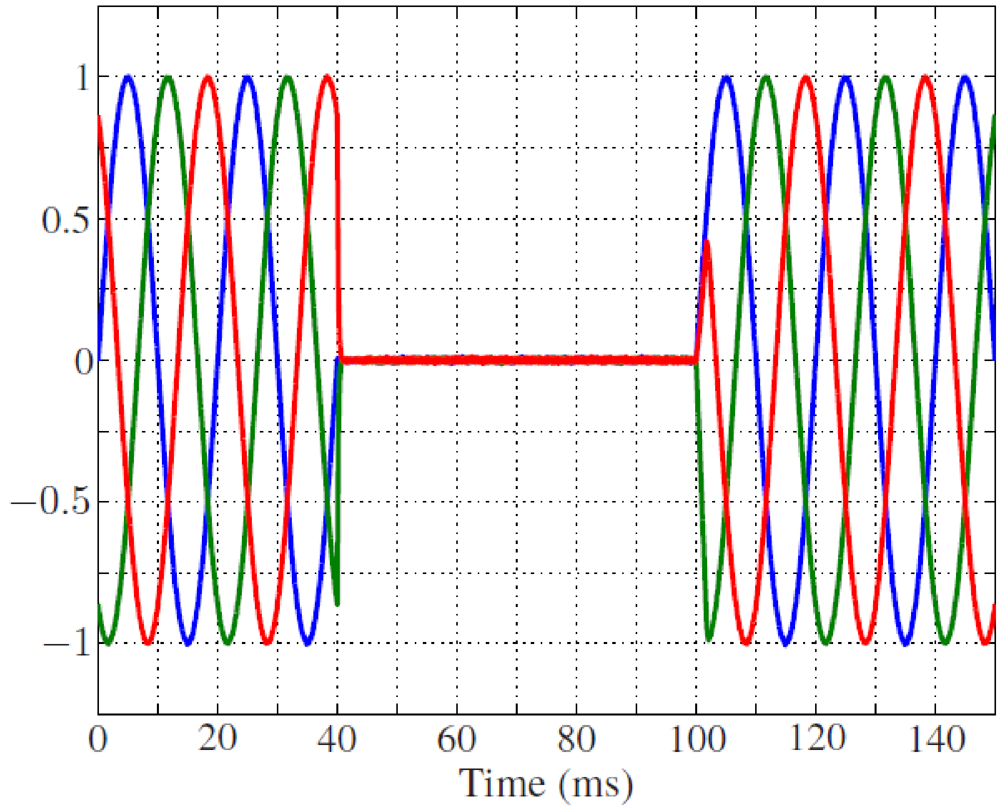
- *Simulation results*

Assessment of control methods for power converters

Conclusions and outlook

MPC of Modular Multilevel Converter
Grid Currents

Active power steps from 1 to 0 and back to 1pu

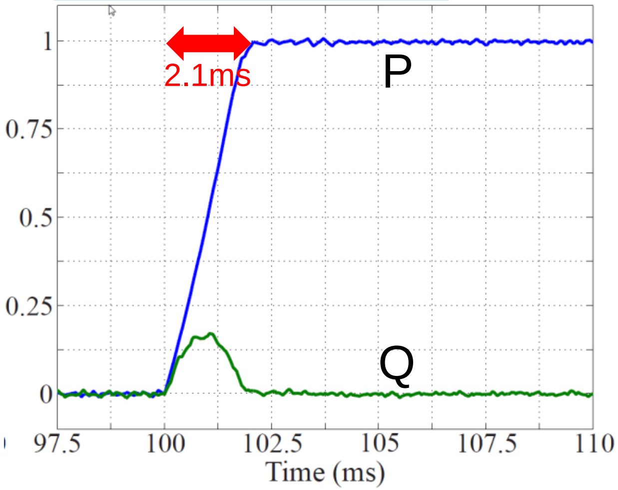


(a) Three-phase grid current i_g

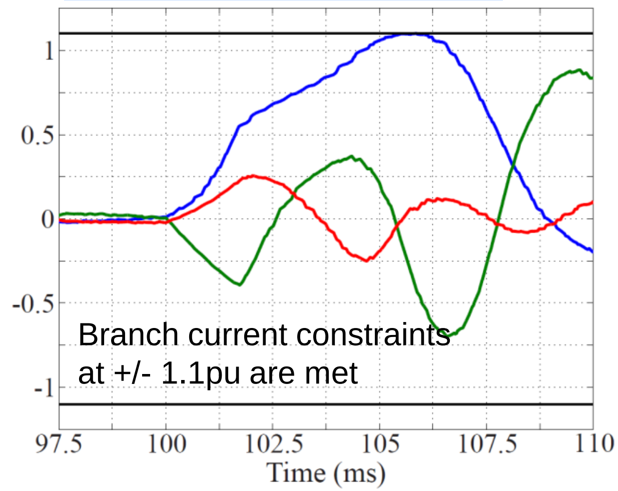
MPC of Modular Multilevel Converters

Power Step from P=0 to 1pu

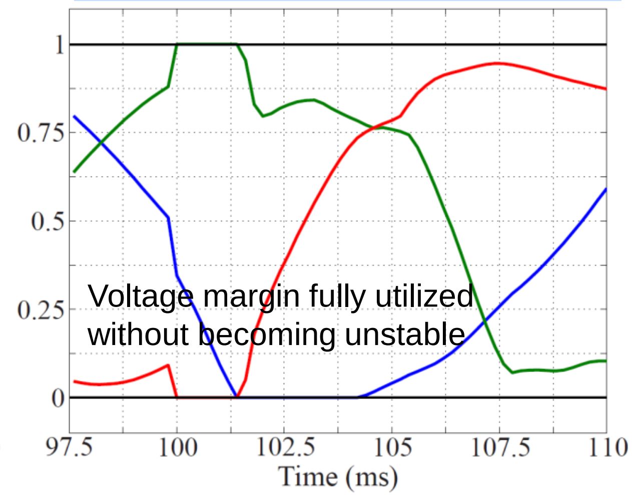
Real and reactive power:



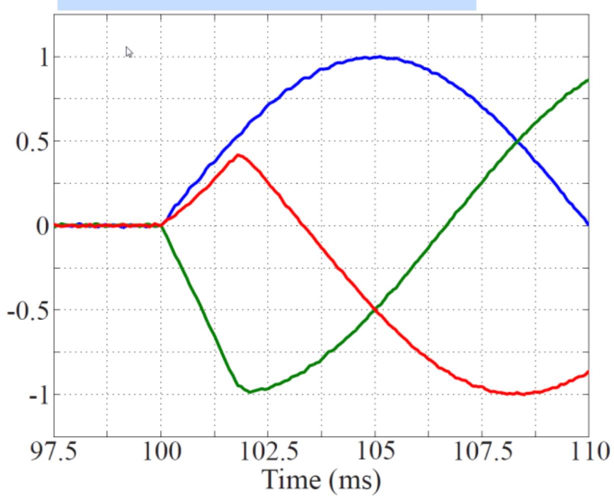
Currents in upper branches:



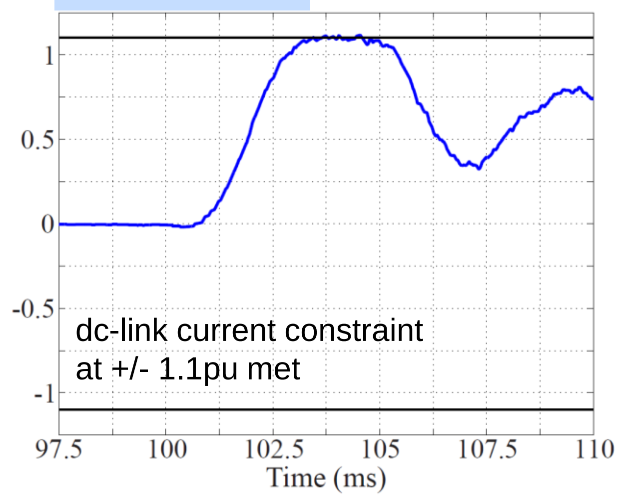
Insertion indices in upper branches:



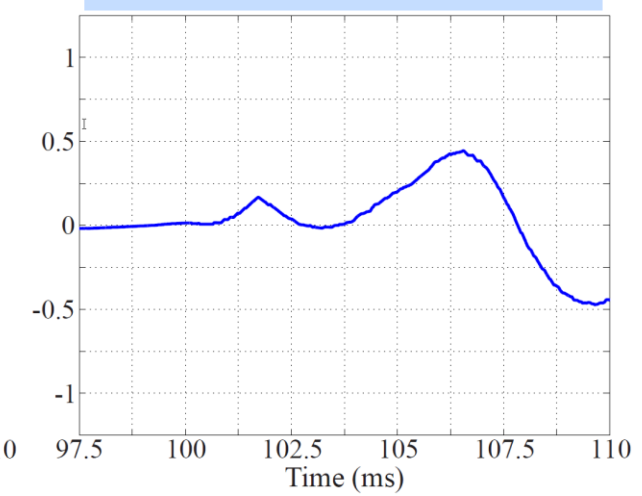
Three-phase grid currents:



Dc-link current:



Circulating current in phase leg *a*:



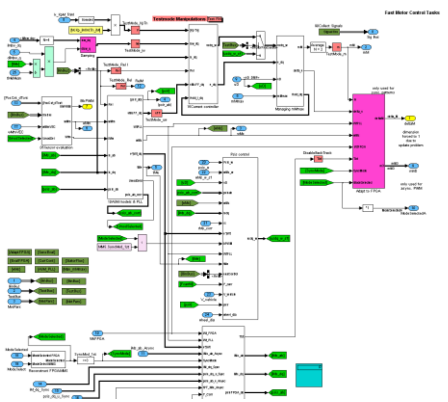
G. Darivianakis, T. Geyer and W. v. d. Merwe: "Model predictive current control of modular multilevel converters", ECCE, Sep. 2014

MPC of Modular Multilevel Converter
Concluding Remarks

- MMC is MIMO control problem
- Soft and hard constraints can be imposed in MPC

  => allows for aggressive controller tuning

  => very fast response during transients
- Receding horizon policy => robustness



$$\min_{U(k)} \sum_{\ell=k}^{k+N-1} x^T(\ell) Q x(\ell) + u^T(\ell) R u(\ell)$$

$$\text{subj. to} \quad x(\ell+1) = A x(\ell) + B u(\ell)$$

$$F x(\ell) \leq g$$

But:
- Optimization problem is time varying
- T_s = 1/5000 = 200us is little time to solve the QP

MPC of Modular Multilevel Converter
Concluding Remarks

- MPC scheme is applicable to **any MMC setup** (circuit parameters, phase configuration and number of modules)
- MPC outperforms most of the existing control approaches for the MMC, particularly **during transients**
- Operation of the converter within **safe operating limits** is ensured under all circumstances
- **Overshoots** in the capacitor voltages and branch currents are avoided
- Very low **current THD** of about 0.5%
- Low device **switching frequency** of less than 400Hz

Model Predictive Control of Power Converters
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Assessment of control methods for power converters

Conclusions and outlook

Assessment of the Control Methods
Field / Voltage Oriented Control with SVM

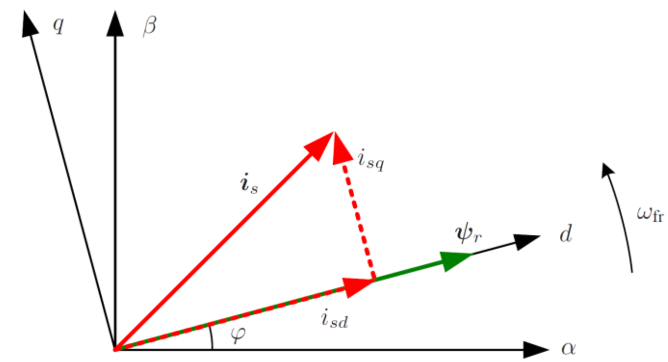
Advantages:

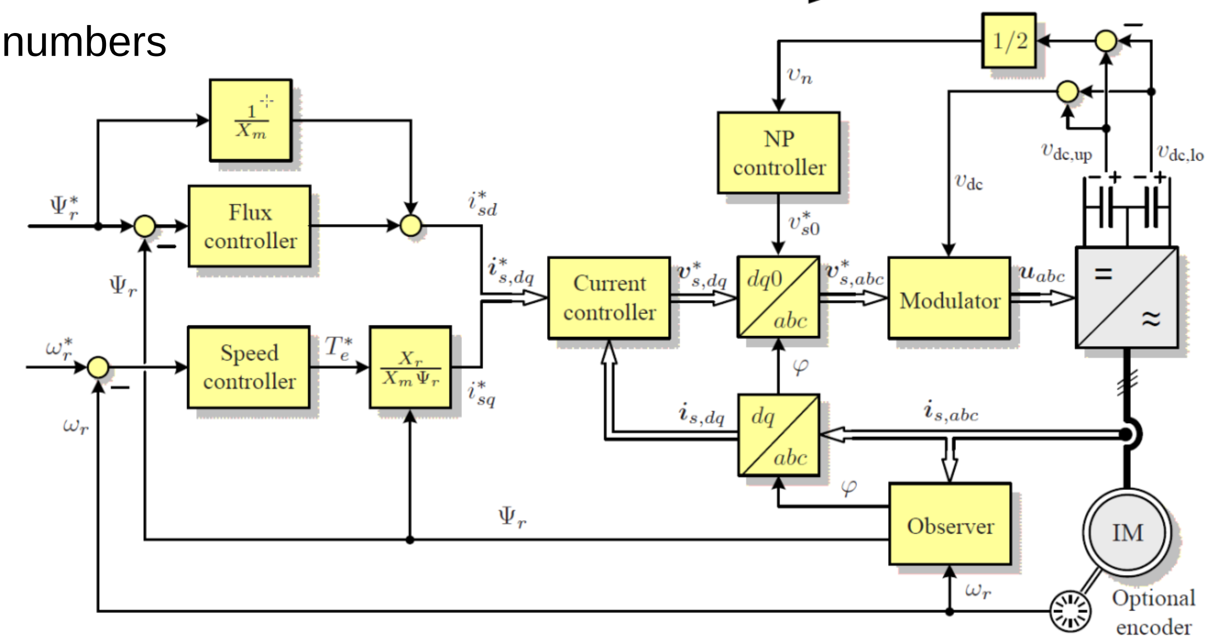
- Very well understood and widely used
- Discrete and deterministic harmonic spectrum

Disadvantages:

- Works poorly at very low pulse numbers

Rotor field oriented control:





Assessment of the Control Methods
Direct Torque / Power Control

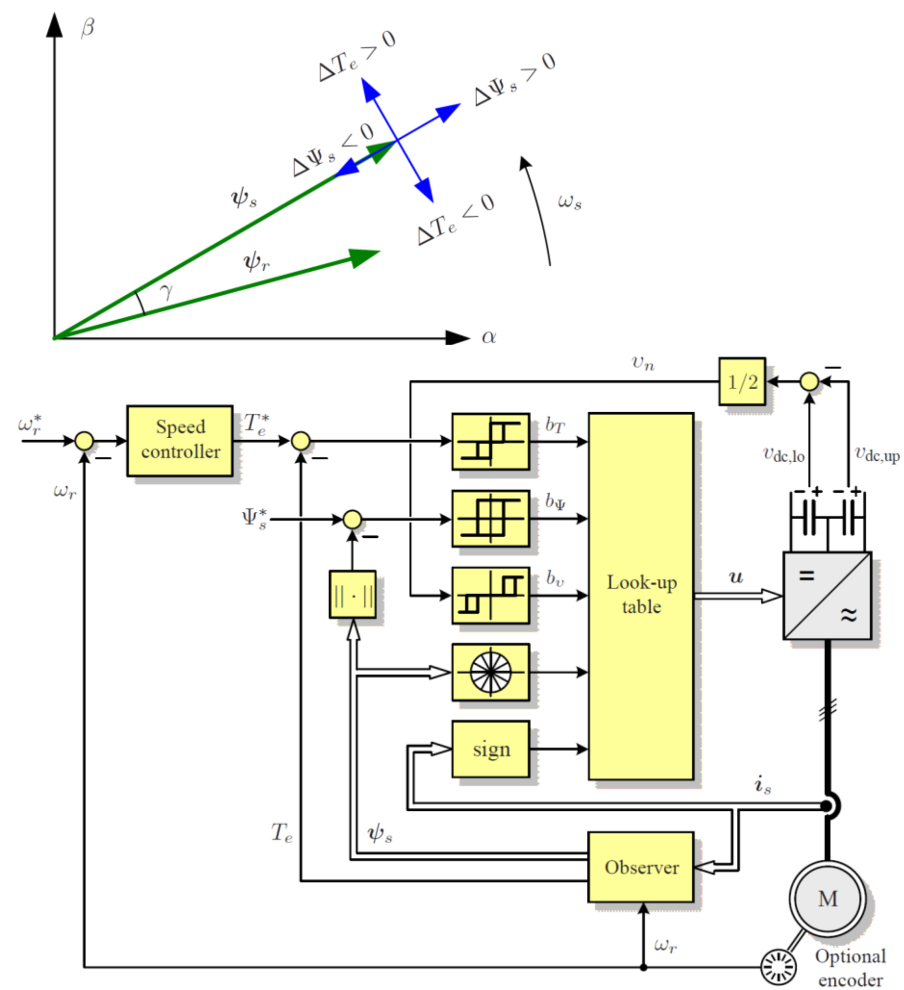
Advantages:

- Very robust
- Very fast dynamic response
- Few system parameters

Disadvantages:

- Significant harmonic distortions
- Non-deterministic harmonic spectrum
- Works poorly at very low pulse numbers
- Requires high sampling frequency
- (Deadlocks)

Direct torque control:



Assessment of the Control Methods
Direct MPC with Bounds (MPDxC)

Advantages:

- Very fast dynamic response
- Robust
- Simple tuning (for MPDCC and MPDPC)

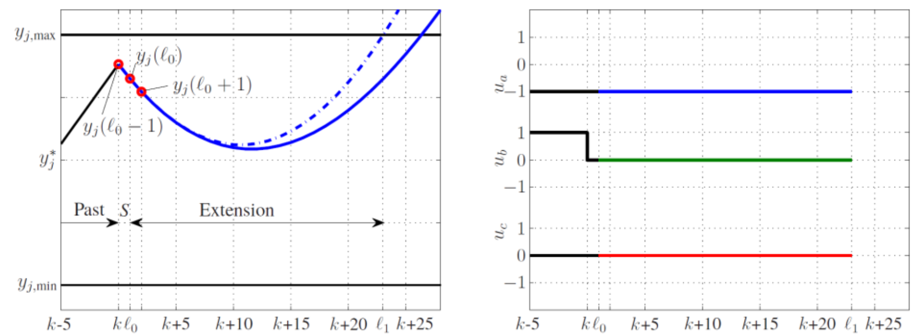
Disadvantages:

- Non-deterministic harmonic spectrum
- Requires high sampling frequency
- Conceptually difficult
- Deadlocks

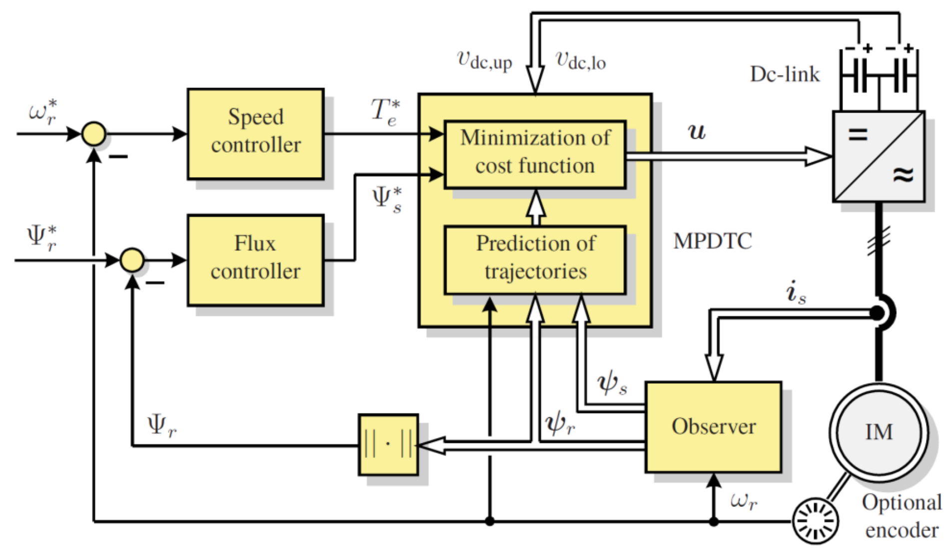
Comment:

- Branch and bound enables the use of long prediction horizons

Model predictive direct torque control:



(a) Predicted output trajectory
(b) Predicted switching sequence



Assessment of the Control Methods
Direct MPC with Reference Tracking

Advantages:

- Conceptually simple
- Very fast dynamic response
- Suitable for higher-order systems

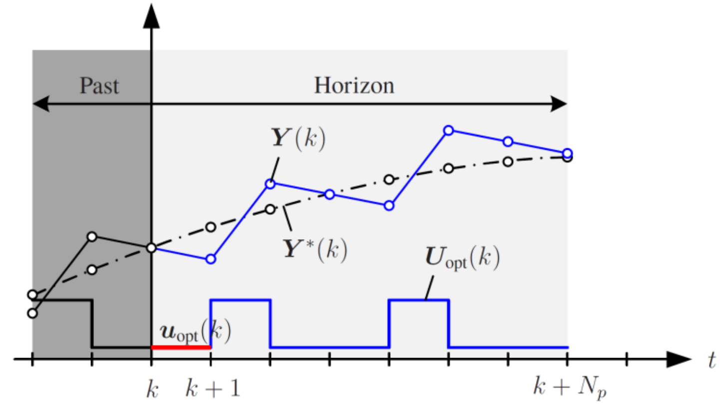
Disadvantages:

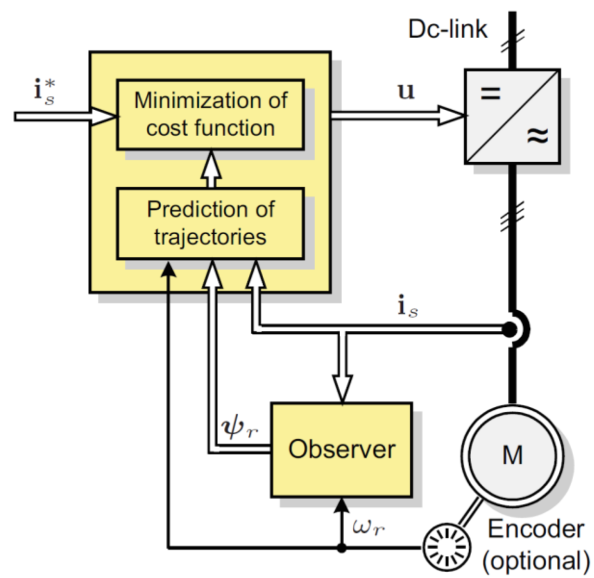
- Non-deterministic harmonic spectrum
- Requires high sampling frequency
- Tuning difficult

Comment:

- Sphere decoding enables the use of long prediction horizons

Current control:





Assessment of the Control Methods
MPC Based on Optimized Pulse Patterns (MP^{3}C)

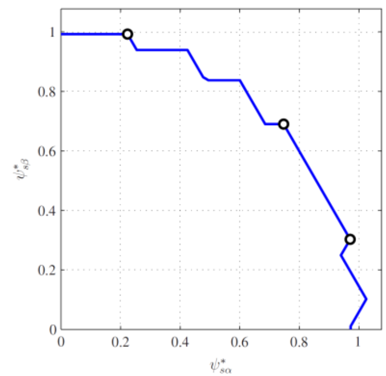
Advantages:

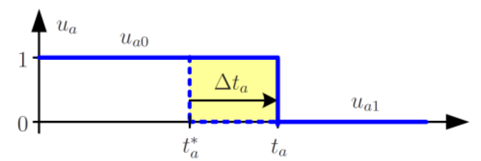
- Low harmonic distortions per switching effort
- Harmonic spectrum is discrete, deterministic and can be shaped
- Fast dynamic response (with pulse insertion)

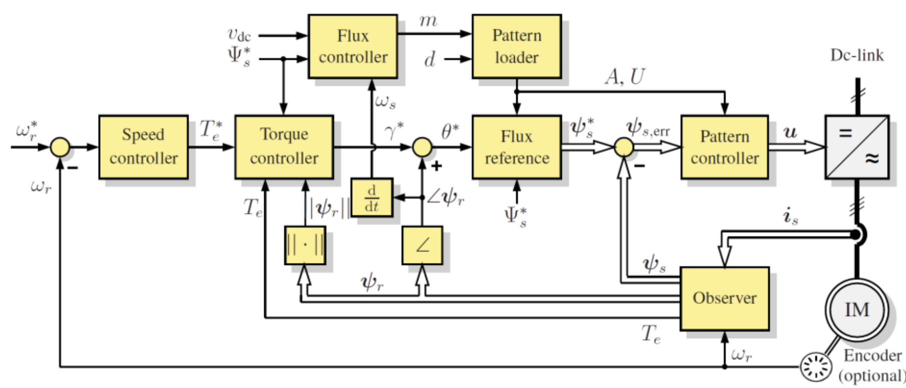
Disadvantages:

- Inflexible (OPPs are precomputed): non-uniform voltage steps, unbalanced load, additional control objectives (such as control of NP potential)
- Conceptually difficult
- Computation of OPPs is time consuming for multilevel converter and high pulse numbers
- Switching frequency is integer multiple of the fundamental frequency

Model predictive pulse pattern control:







Assessment of the Control Methods
Indirect MPC

Advantages:

- Well established / studied MPC framework
- Discrete and deterministic harmonic spectrum

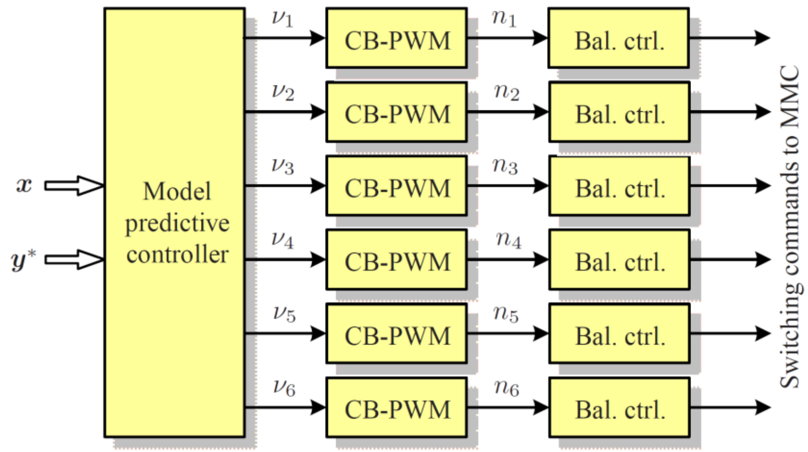
Disadvantages:

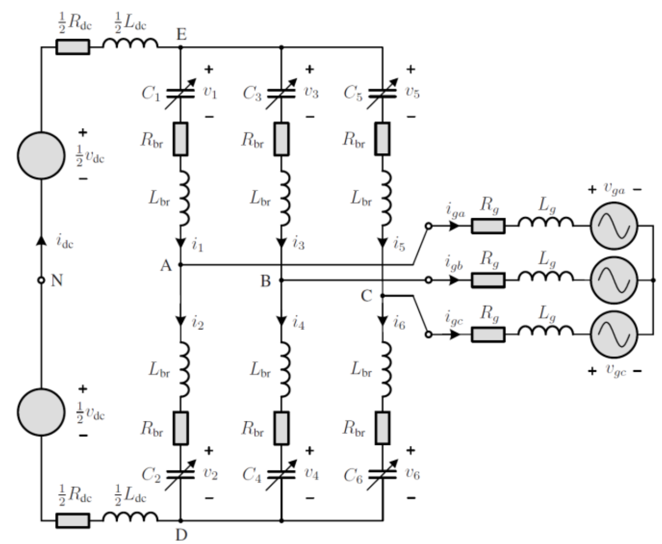
- Solving the QP in real time is challenging

Comments:

- Largely unexplored
- Suitable for “complex” systems and relatively high pulse numbers

Indirect MPC for MMC:





Model Predictive Control of Power Converters
Outline

Long-horizon direct MPC

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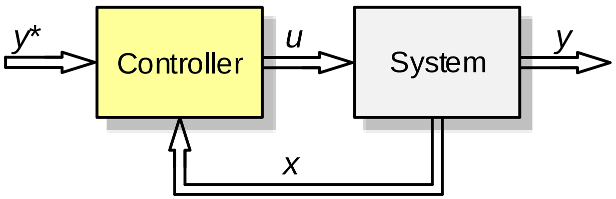
Assessment of control methods for power converters

Conclusions and outlook

Model Predictive Control of Power Converters
Classification

Direct control



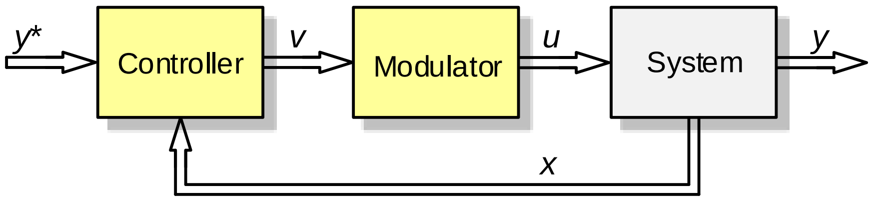
Direct manipulation of switch position:

- Manipulated variable: $\boldsymbol{u} \in \mathbb{Z}^{n_u}$

Control methods:

- Reference tracking (finite control set MPC): enumeration or sphere decoding
- Bounds (MPDxC): branch and bound
- Optimized pulse patterns: QP solver or algebraic manipulation

Indirect control



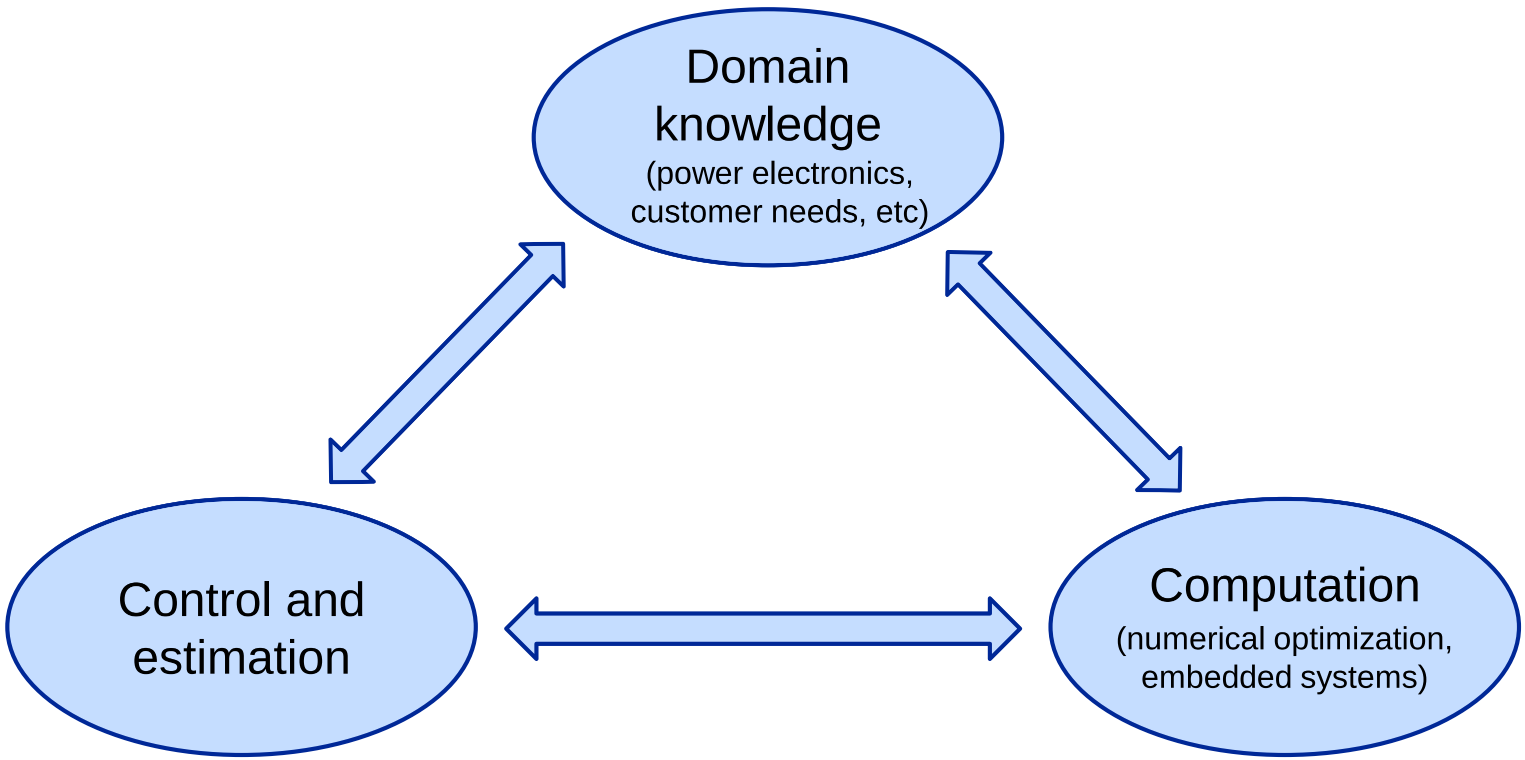
Indirect manipulation of switch position:

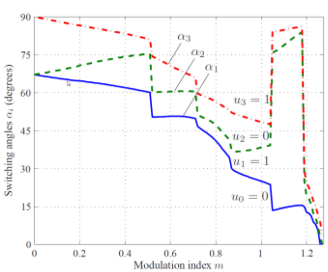
- Manipulated variable: $\boldsymbol{v} \in \mathbb{R}^{n_u}$

Control methods:

- Reference tracking: linearization, QP solver or explicit solution

Model Predictive Control of Power Converters
Outlook



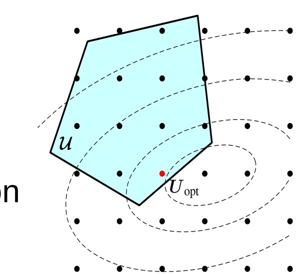


Challenges:

- Control of **optimized pulse patterns**
- Combination of constrained (time-domain) and **frequency-domain** control

Challenges:

- **Integer** optimization
- Numerical optimization on **embedded hardware**



Model Predictive Control of Power Converters
Vision

Develop new **control** methods that

- **fully utilize** the hardware **capability** and/or
- **reduce** the hardware **requirement**

of power electronic systems

Acknowledgements

ABB Corporate Research:
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- Tobias Thurnherr

ETH Zurich, Switzerland:
- Manfred Morari

Stellenbosch University, South Africa:
- Toit Mouton

Tampere University, Finland:
- Petros Karamanakos

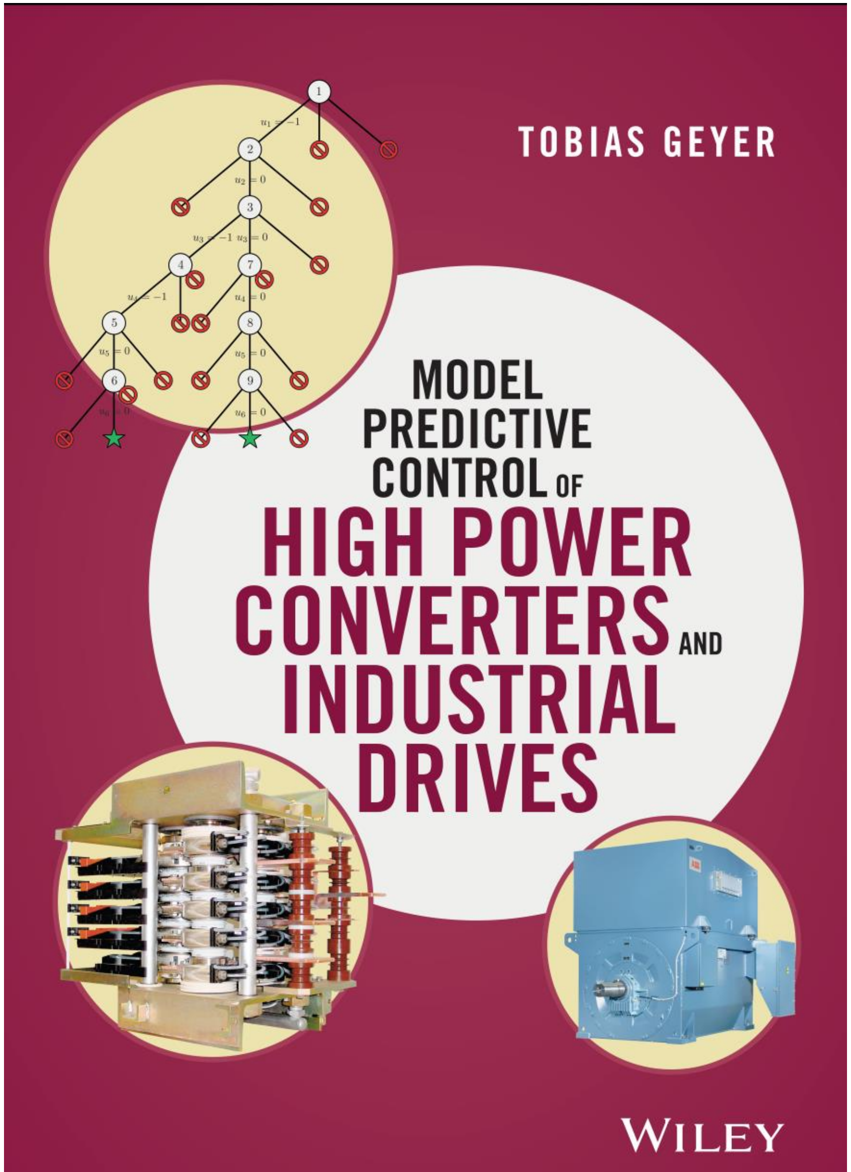
University of Paderborn, Germany:
- Daniel E. Quevedo

Univ. of Auckland, New Zealand:
- Udaya Madawala

TU Munich, Germany:
- Ralph Kennel

Univ. of Technology Sydney, Australia:
- Ricardo P. Aguilera

MPC of High Power Electronics and Industrial Drives



Five main parts:

- Introduction: MPC, machines, semiconductors, topologies, MV inverters, requirements, CB-PWM, OPPs, field oriented control, direct torque control
- Direct MPC with reference tracking (FCS-MPC): predictive current control, predictive torque control, integer quadratic programming formulation, sphere decoding, performance evaluation for NPC inverter drive system without and with LC filter
- Direct MPC with bounds: model predictive direct torque control, extension methods, performance evaluation for 3L and 5L inverter drive systems, state-feedback control law, deadlocks, branch and bound methods, model predictive direct current control, model predictive direct power control
- MPC based on PWM: model predictive pulse pattern control, pulse insertion, performance evaluation for NPC inverter drive system, experimental results for 5L inverter drive system, MPC of an MMC using CB-PWM
- Summary and conclusions: performance comparison of direct MPC schemes, assessment, summary and discussion, outlook

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