Wind Energy Systems
Albert-Ludwigs-Universität Freiburg – Summer Semester 2018

Exercise Sheet 3: Mechanics for Wind Turbine

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Deadline: midnight before June 26, 2018
`https://goo.gl/forms/0QFRvjHgSHC7FOwH2`

In this exercise sheet we'll explore the role of deflections and vibrations in wind turbine design, focusing on the blades and the tower. To accomplish this exploration, we will play briefly with simple Euler-Bernoulli beam theory, the Rayleigh energy method, and the Campbell diagram.

blade deflection **[*5 pt*]**

1. In this problem, we would like to explore the blade deflection. Let's assume that the blade is approximately straight so that it lays more-or-less in the rotor tip plane, even when deflected.

   We will (only if explicitly stated!) use the same three-bladed demonstration turbine, 'Turbine A' as used by exercise sheet 2. Turbine A is defined by the following parameters: the rotor radius $R = 50$m, and blades of constant chord $c = 5$m and constant profile shape. Turbine A is running in a freestream wind of $u_\infty = 12$m/s with air density $\rho = 1.225$kg/m^3. Remember, that $\mu = r/R$ is our non-dimensional radial location along the blade.

   You're encouraged to use the thrust distribution over the blade from the BEM problem of exercise sheet 2, but the following approximation can be used if necessary:
   $$\mathrm{d}T(\mu) \approx 30\mu^2(4-\mu)q_\infty R\mathrm{d}\mu$$

   (a) For a solid symmetric airfoil, we can[1] approximate the blade's second moment of area as $I_x \approx K_\mathrm{I} c^4 \tau^3$, where K_I approx 0.036 and $\tau = t_\mathrm{max}/c$ is the maximum airfoil thickness to chord length ratio.

      i. Let's assume that the airfoil nondimensional thickness τ =24 percent. What is the maximum thickness t_max of the airfoil? [*0.25 pt*]

      ii. What is the second moment of area I_x in terms of the defined parameters? [*0.25 pt*]

      iii. As mentioned, this approximation assumes a solid airfoil. Do you think a more accurate approximation of I_x would be larger or smaller than this 'solid' approximation? [*0.25 pt*]

   (b) We might make the assumption that the blade behaves as a slender beam. In that case, the downwind-direction blade deflection x could be found with Euler-Bernoulli beam theory from the distributed load q, the Young's modulus E and the cross-section second moment of area I_x:
   $$\frac{\mathrm{d}^2}{\mathrm{d}r^2}\left(EI_x\frac{\mathrm{d}^2 x}{\mathrm{d}r^2}\right) = q$$

      There are some boundary conditions to this integral:
   $$x(0) = 0; \quad x'(0) = 0; \quad x''(R) = 0; \quad x'''(R) = 0$$

      Briefly, what do these boundary conditions mean? [*0.25 pt*]

   (c) What is the relationship between the downwind-direction blade deflection at the tip and the rotor radius R? [*1 pt*]

   (d) For 'Turbine A', what is the ratio between the tip blade deflection and the rotor radius, if the blade is made of the following materials?

      i. carbon-fiber composite ($E \approx 150$GPa), [*0.25 pt*]

      ii. fiberglass aka. glass-reinforced plastic ($E \approx 17$GPa), [*0.25 pt*]

      iii. polystyrene ($E \approx 3$GPa)? [*0.25 pt*]

   (e) What trade-offs might be relevant when selecting blade material? [*0.75 pt*]

   (f) The fact that the blades are rotating will likley lead to a smaller deflection than predicted here. Briefly, why would that be? What is this phenomenon called? [*0.5 pt*]

[1] `https://ocw.mit.edu/courses/aeronautics-and-astronautics/16-01-unified-engineering-i-ii-iii-iv-fall-2005-spring-2006/systems-labs-06/spl10b.pdf`

(g) Qualitatively, what happens to the blade loading under the following conditions?

i. yawed flow [*0.25 pt*]

ii. shaft tilt [*0.25 pt*]

iii. wind shear [*0.25 pt*]

iv. tower shadow [*0.25 pt*]

preliminary tower design **[*10 pt*]**

2. We would like to make a preliminary design of a wind turbine tower. This tower should support an un-yawed and un-tilted three-bladed wind turbine ('Turbine B'), with the following dimensions:

Table 1: wind turbine dimensions and properties for Turbine B

property	symbol	value
tower height	L	84 m
nacelle + hub mass	m_{nac}	143 tonnes
rotor radius	R	12 m
design tip speed ratio	λ_{rated}	5
cut-in wind speed	$u_{\text{cut-in}}$	3 m/s
rated wind speed	u_{rated}	12 m/s
cut-out wind speed	$u_{\text{cut-out}}$	25 m/s

Some other information that you might find useful is as follows:

Table 2: other potentially useful information

property	symbol	value
density of A36 structural steel	ρ_{steel}	$7.8 \cdot 10^3$ kg/m^3
Young's modulus of A36 structural steel	E_{steel}	200 GPa
yield stress of A36 structural steel	U_{steel}	250 MPa
air density	ρ_{air}	1.225 kg/m^3
surface roughness length for low crops w. occasional obstacles	z_0	0.1 m
meterological mast height	z_{Ref}	10 m
approx. drag coefficient for cylinder	C_{D}	1
typical wind turbine structural safety factor	f_{safety}	1.35

(a) **rotor thrust** **[*1 pt*]**

i. What is the design angular velocity Ω_{rated} of the wind turbine? [*0.25 pt*]

ii. Suppose that the angular velocity $\Omega(u_\infty)$ of the wind turbine is piecewise linear with the free-stream velocity at hub height u_∞. That is: $\Omega(u \leq u_{\text{cut-in}}) = 0\text{rad/s}$, $\Omega(u_{\text{rated}}) = \Omega_{\text{rated}}$, $\Omega(u_{\text{cut-out}}) = \Omega_{\text{rated}}$, and $\Omega(u > u_{\text{cut-out}}) = 0\text{rad/s}$.

Considering a logarithmic wind profile, where a met. mast of height z_{Ref} measures a reference wind speed of u_{Ref}, above a landscape of low crops with occasional larger obstacles, What is the angular velocity $\Omega(u_{\text{Ref}})$ as a function of the reference wind speed? [*0.25 pt*]

iii. You happen to learn that the thrust coefficient C_{T} of this wind turbine can roughly be approximated with the following function:

$$C_{\text{T}}(\lambda) \approx \frac{0.8}{\pi} \left(\arctan \left(0.5\lambda - 2 \right) \right) + 0.3$$

.

What is the relationship between C_{T} and u_{Ref}? [*0.25 pt*]

iv. What is the magnitude of the thrust force F on the rotor as a function of u_{Ref}? [*0.25 pt*]

(b) **tower bending stress** **[*3 pt*]**

Let's consider the tower as a simple cantilevered beam, where an aerodynamic drag force is acting continuously along the tower length, and the rotor thrust is acting at the top of the tower.

Let's assume that the tower is a thin walled tube with a constant cross-section along its length. This constant cross-section is an annulus, with an outer radius of r and a thickness τ.

i. Make a contour plot of the total mass of steel in the tower, based on $r \in [1\text{m}, 6\text{m}]$ and $\tau \in [0\text{m}, 0.15\text{m}]$. *[0.25 pt]*

ii. You happen to know that the second moment of area of a filled circlular area with radius a is $\pi a^4/4$. What is the second moment of area I_x of the tower cross-section? *[0.25 pt]*

iii. What is the distance d between the beam's neutral axis and the outer radius? *[0.25 pt]*

iv. Considering the logarithmic wind profile, what is the bending moment of the tower at the ground due only to the drag along the tower length M_D? *[0.25 pt]*

v. What is the bending moment of the tower at the ground due only to the thrust on the rotor M_T? *[0.25 pt]*

vi. What is the total bending moment of the tower at the ground M? *[0.25 pt]*

vii. What is the maximum stress σ_max due to bending on the tower? *[0.25 pt]*

viii. Considering the safety factor f_safety, please devise a ratio ϕ which indicates whether the tower can safely support the maximum bending stress. Let's define $\phi < 1$ as safe, and $\phi > 1$ as unsafe. *[0.25 pt]*

ix. Over all reference velocities u_Ref that the wind turbine is likley to experience in its lifetime, when will the bending stress criteria be the strictest? *[0.25 pt]*

x. For the following proposed tower outer diameters r, what thickness τ would you propose? Please motivate your choices. Also, please round thicknesses to the nearest 5mm.

   A. $r = 5.5$ m *[0.25 pt]*

   B. $r = 3.0$ m *[0.25 pt]*

   C. $r = 1.5$ m *[0.25 pt]*

(c) **tower natural frequency** ***[3 pt]***

Let's use Rayleigh's energy method to estimate the natural frequency of the tower. In this method, we assume that the strain energy from bending perfectly trades off with the kinetic energy of the tower's displacement x. We will again approximate the tower as a cantilevered beam.

Let's assume that the tower's displacement is sinusoidal in time:

$$x(t) = x_0 \sin(\omega t)$$

and that the tower remains approximately straight during its displacement.

Futher, we know that the strain energy from bending can be found as:

$$V = \frac{1}{2}kx^2, \text{ where } k = 3\frac{E_\text{steel} I_x}{L^3}.$$

i. What is $\dot{x}(t)$? *[0.25 pt]*

ii. What is the kinetic energy due to the nacelle displacement T_nac? *[0.25 pt]*

iii. What is the kinetic energy due to the displacement of the tower T_t? (*Hint: the tower is not massless...*) (*Hint: also, you might assume that the deflection of the tower is roughly proportional to the distance to the fixed point.*) *[0.5 pt]*

iv. What is the total kinetic energy T of the swaying cantilevered beam? *[0.25 pt]*

v. What equation can you formulate, that would implicitly define the vibration frequency ω? *[0.5 pt]*

vi. Please find ω. *[0.25 pt]*

vii. What is the natural frequency f_nat of the cantilevered tower? *[0.25 pt]*

viii. What is the natural frequency of each of the three potential tower designs (defined by r and τ) that you determined in (2(b)x)? (*Hint: If you do not have a solution to (2(b)x), you can use the following combinations of* (r, τ)*:* $(1.5m, 0.05m)$, $(3.0m, 0.02m)$, $(5.5m, 0.01m)$.)

   A. $r = 5.5$ m *[0.25 pt]*

   B. $r = 3.0$ m *[0.25 pt]*

   C. $r = 1.5$ m *[0.25 pt]*

(d) **Campbell diagram** ***[3 pt]***

i. With what frequency (1P, 2P, 3P, ...) would you expect the tower to experience the following effects? What is this frequency (in Hertz), as a function of the wind turbine's rotor speed (in RPM)?

   A. 'rotor-rotation' effects, such as having unequally dirty blades? *[0.25 pt]*

B. 'blade-passing' effects, such as tower shadow? [*0.25 pt*]

ii. Make a plot of frequency [Hz] vs rotor speed [RPM] that we will call the Campbell plot. Add the 'rotor-rotation' and 'blade-passing' frequencies into the Campbell plot. Include a 15 percent safety margin to either side of each curve. [*0.25 pt*]

iii. What is the design rotor speed (in RPM) of the wind turbine? [*0.25 pt*]

iv. Please show the design rotor speed in the Campbell plot. [*0.25 pt*]

v. Please add the tower natural frequencies corresponding to your three possible tower designs (one for each of the outer diameters 5.5m, 3.0m and 1.5m) into the Campbell plot. [*0.25 pt*]

vi. Which of the three investigated tower designs (outer diameters 5.5m, 3.0m, 1.5m) can be classified as the following? Please explain briefly.

A. soft-soft [*0.25 pt*]

B. soft-stiff [*0.25 pt*]

C. stiff-stiff [*0.25 pt*]

vii. Suggest some considerations you might have when chosing between your three proposed tower designs? [*1 pt*]