the wake in pictures

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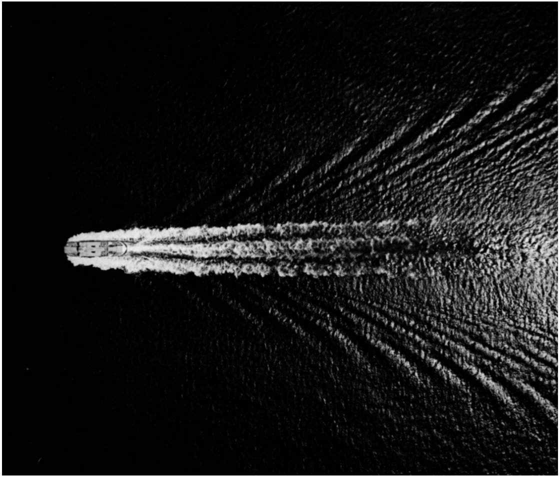
please...

- ask questions!
- wave at me if I speak too quickly/quietly/etc
- sit at the front

what is a wake?

wake:
the disturbed flow
(typically turbulent)
behind an obstacle

here:
ship traveling
right-to-left



Newman 1970, in Van Dyke's *Album of Fluid Motion* (1982, Parabolic, Stanford, CA, USA)

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here:
condensation in Horns Rev, Denmark

Vattenfall 2010, `https://www.flickr.com/photos/vattenfall/4270899001`

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let's zoom in a bit...

Vattenfall 2010, `https://www.flickr.com/photos/vattenfall/4270899001`

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what do we see?
the wake is pretty big,

and some of the turbines are sitting directly in it

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farm perspective:

the flow at 'later' turbines is not fresh and won't contain as much energy

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individual perspective:

wake limits 'extractable' power by up to 60%

and influences local apparent velocity at blades

Vattenfall 2010, https://www.flickr.com/photos/vattenfall/4270899001

what's in here?

- what is the wake doing?
  - → some philosophical implications of lift
  - → an angular momentum detour
  - → some experiments
- how does the wake affect wind turbines
- one conceptual way to model the wake
- where to find more practical information

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some philosophical implications of lift

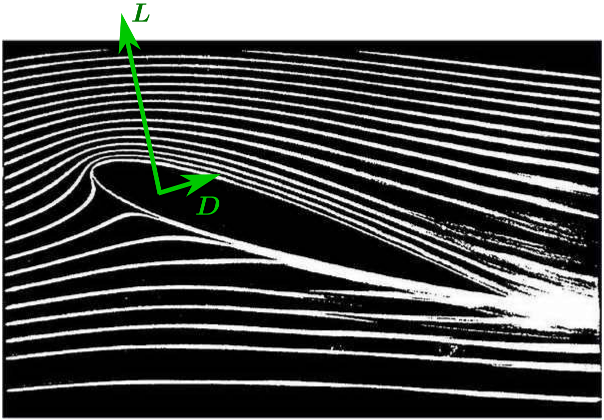
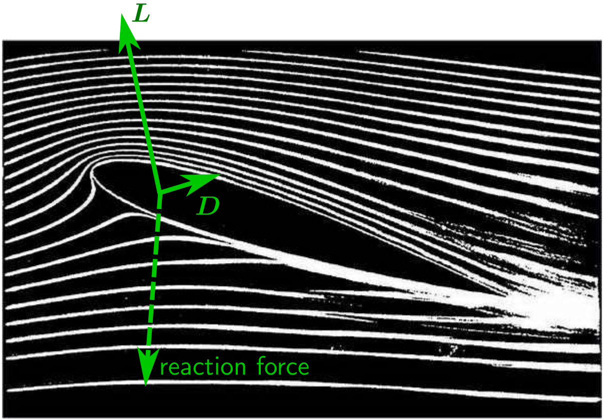


photo source unknown.

some philosophical implications of lift



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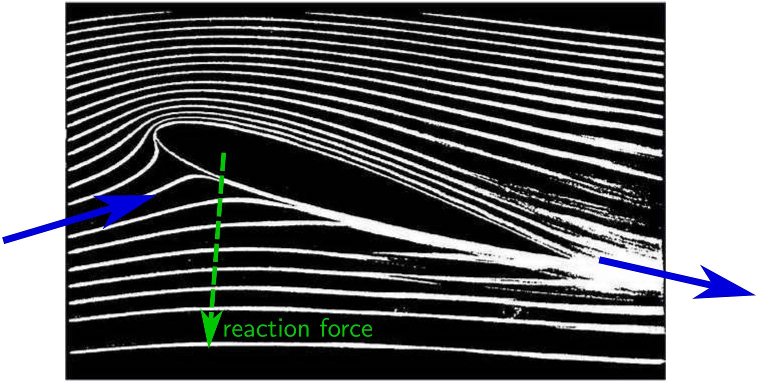


photo source unknown.

some philosophical implications of lift

photo source unknown.

some philosophical implications of lift

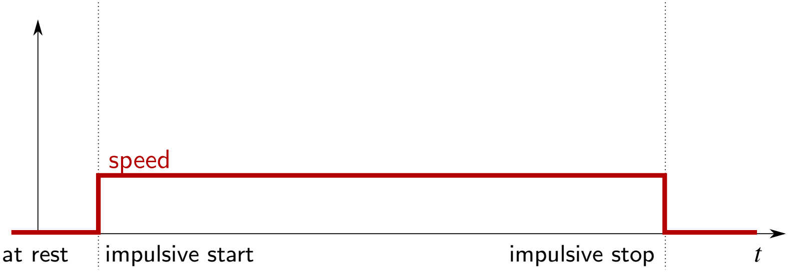
'circulation', not 'rotation'

photo source unknown.

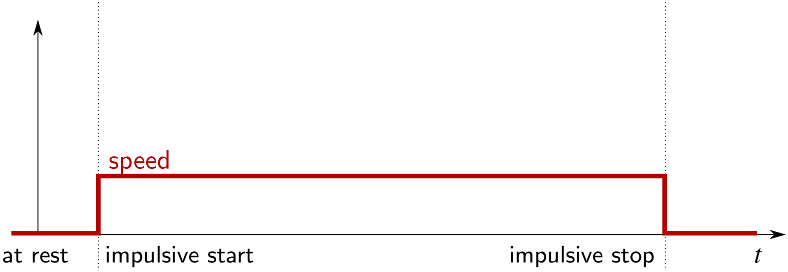
circulation around body $\Leftrightarrow$ lift!

photo source unknown.

wait...



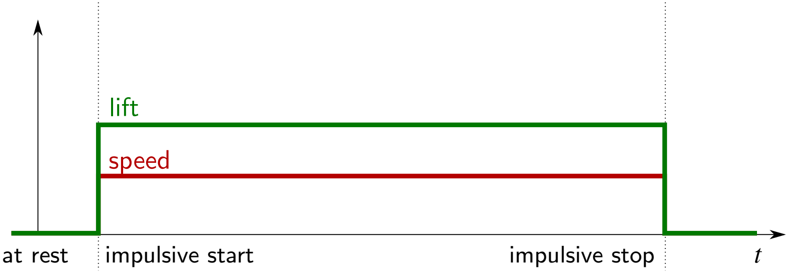
(not to scale!)



for constant lift coefficient, what happens to lift?

(not to scale!)

wait...



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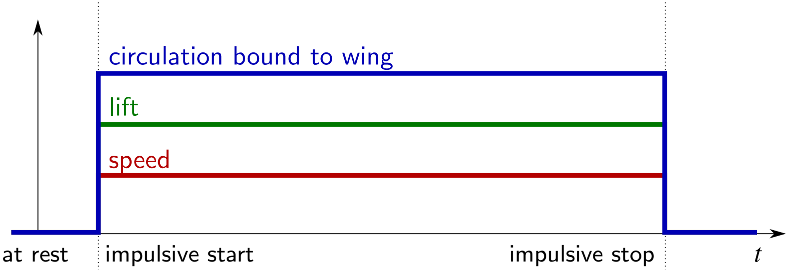

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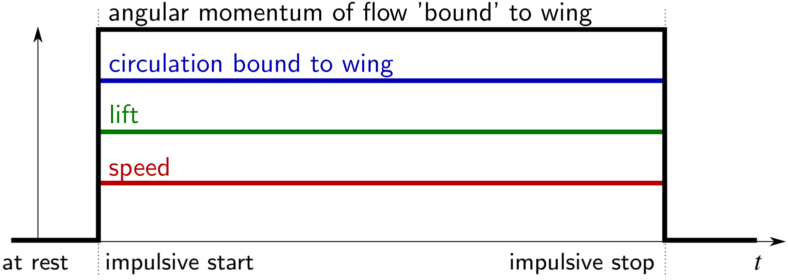

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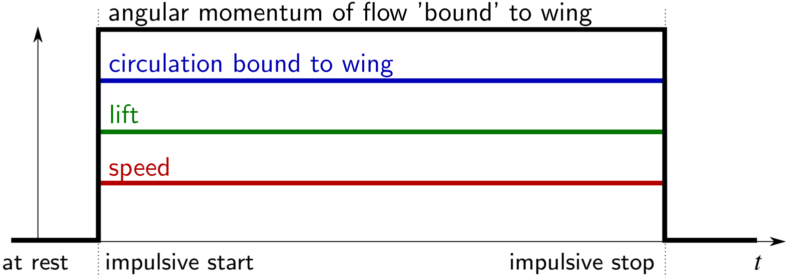
wait... what about conservation of angular momentum?



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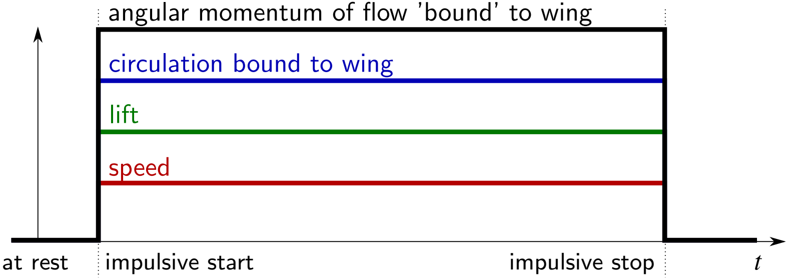

what part of the flow might be acting to conserve angular momentum?

(not to scale!)

wait... what about conservation of angular momentum?

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a hypothesis:
maybe the wake acts to conserve angular momentum!

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a hypothesis:

maybe the wake acts to conserve angular momentum!

hypothetically, how would this work?

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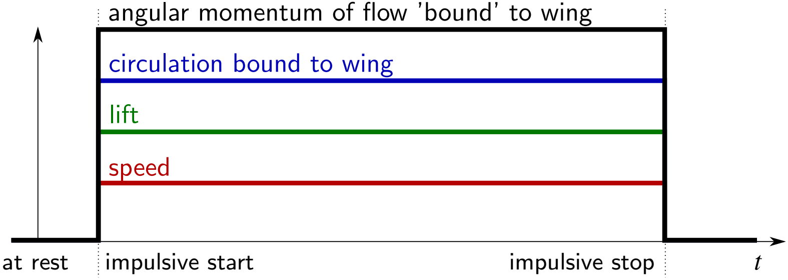

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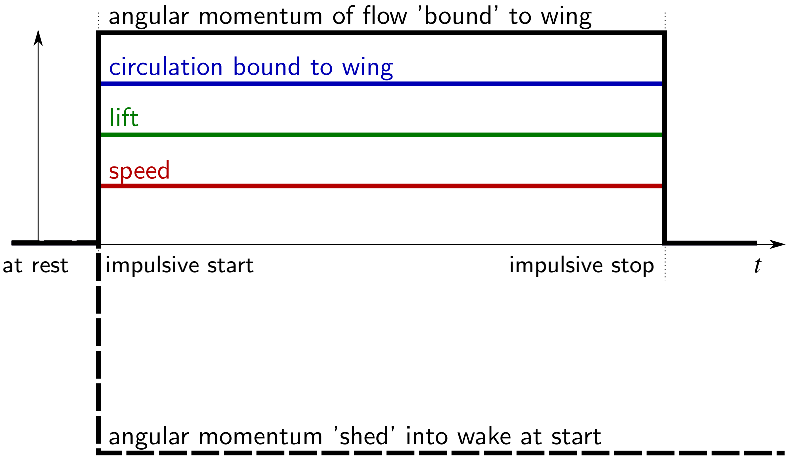

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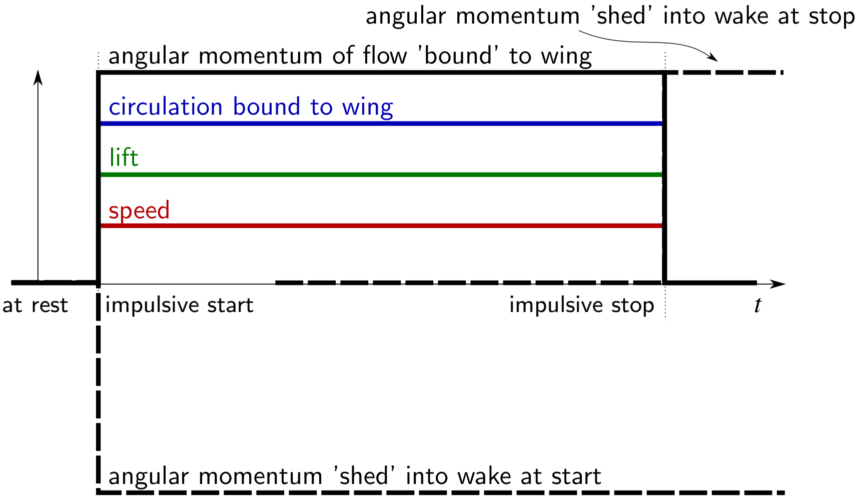

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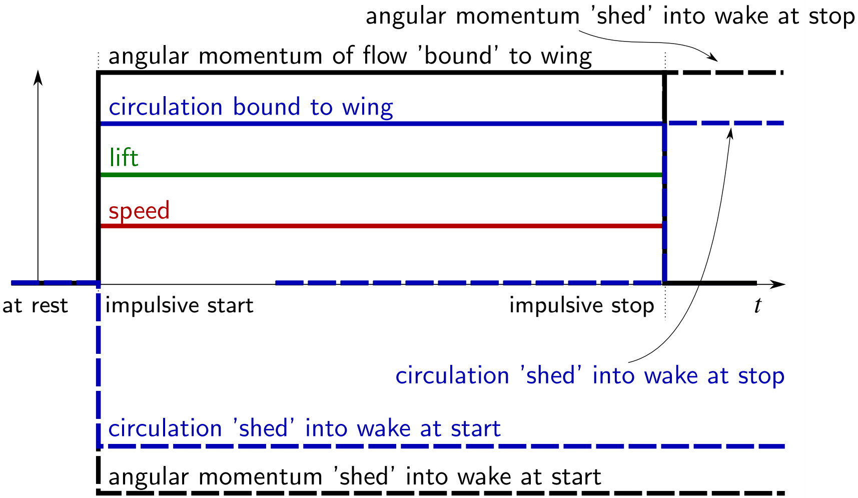

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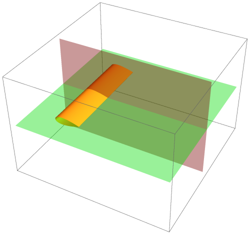
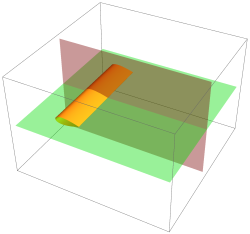
a plan of experiments



2D experiment #1: impulsive start

is 'anything' shed into wake?



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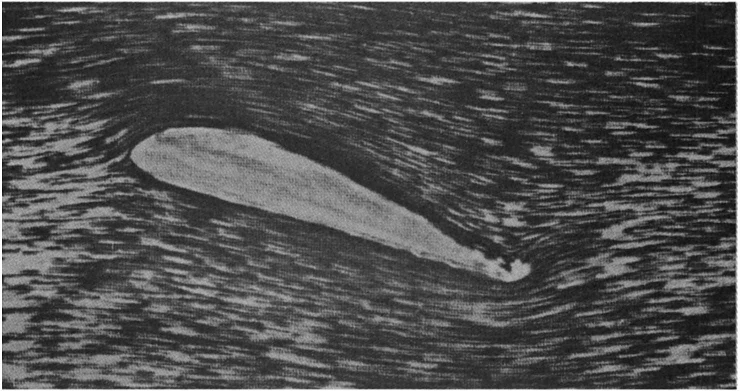

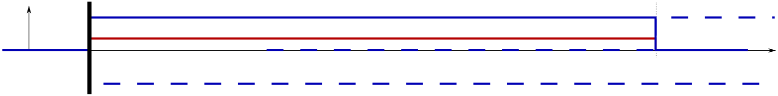

Tietjens 1934, in Tietjens and Prandtl's *Fundamentals of Hydro- and Aeromechanics* (1957, tr. Rosenhead, Dover, New York, NY, USA)

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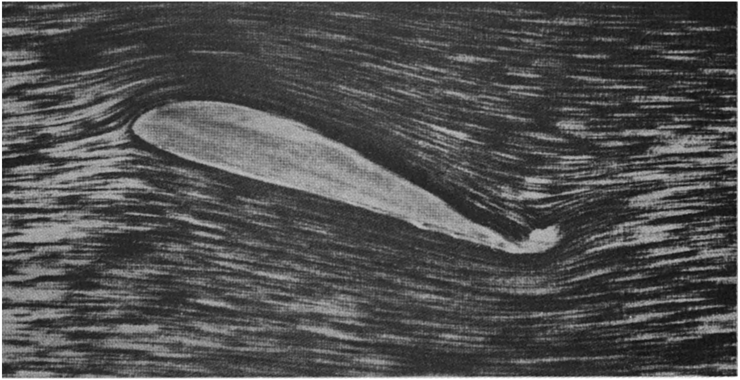

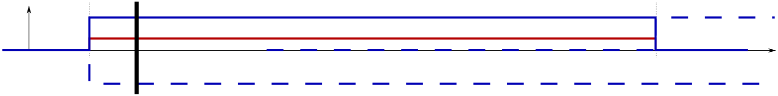

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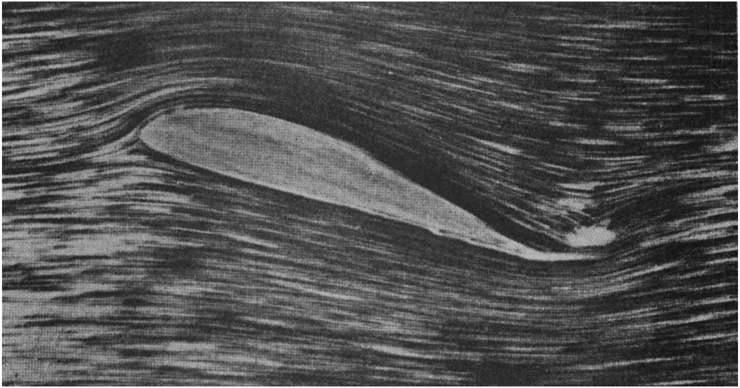

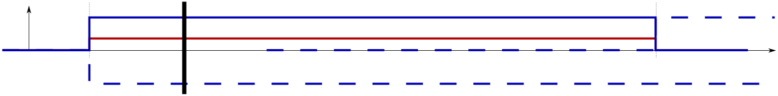

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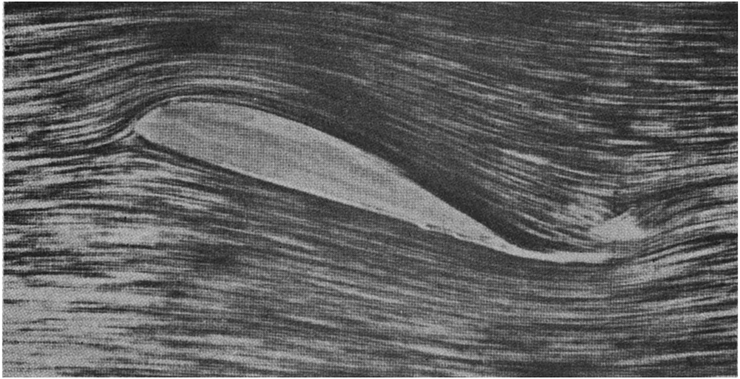

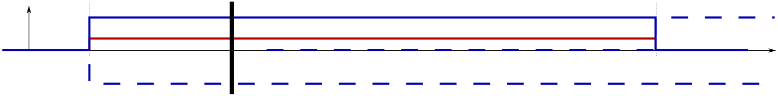

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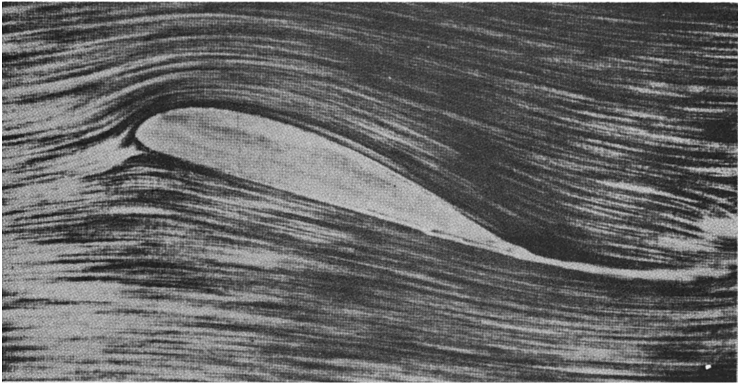

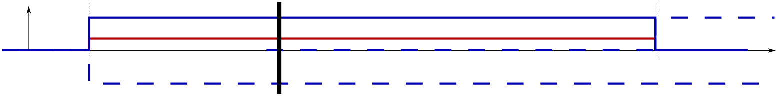

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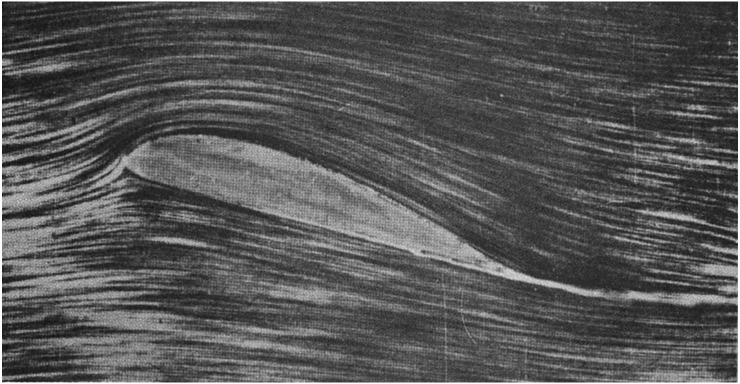

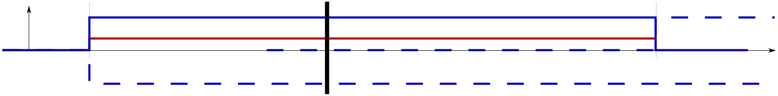

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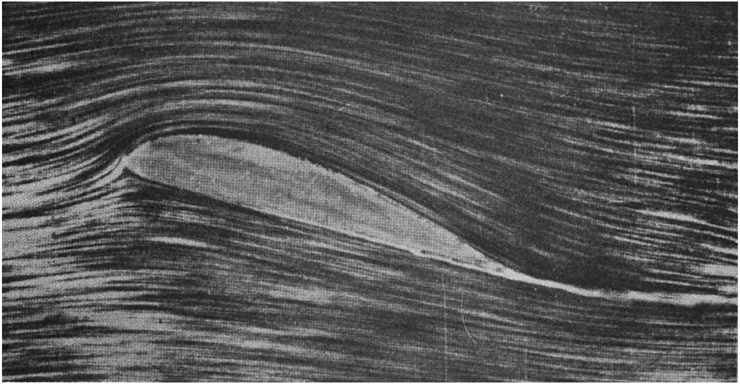

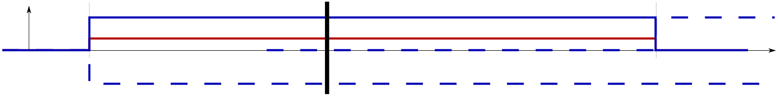

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a plan of experiments

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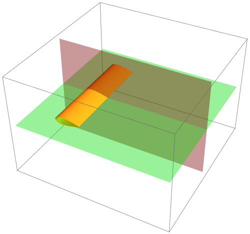
2D experiment: start and stop
is circulation shed into wake?



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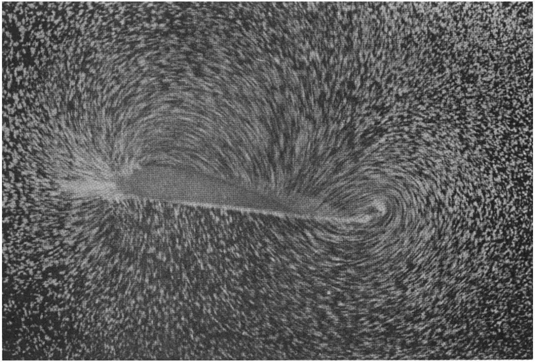

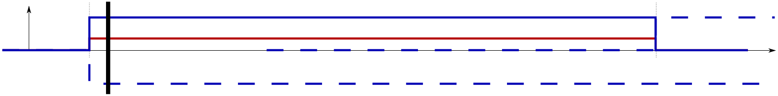

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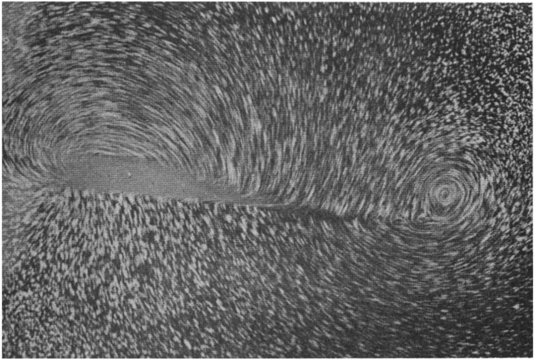

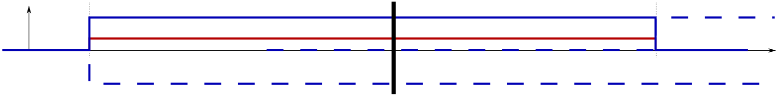

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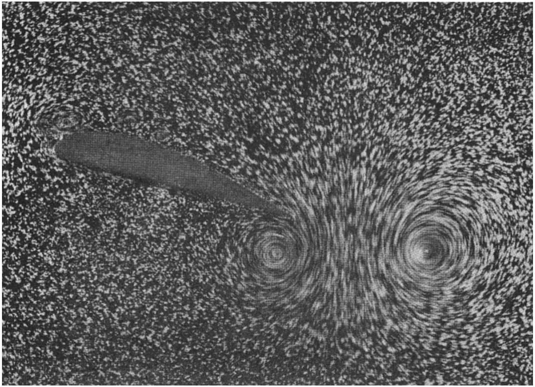

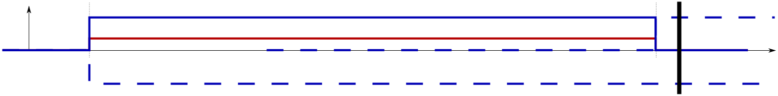

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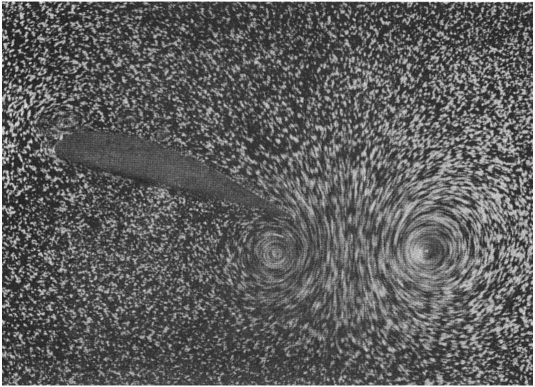

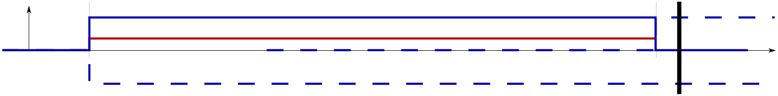

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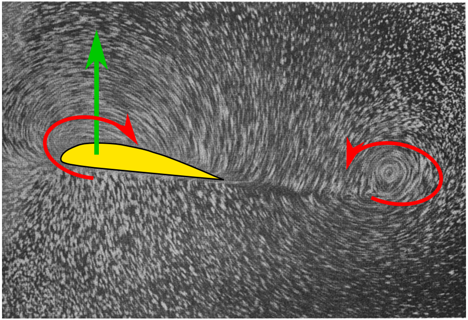
from infinite to finite wings...



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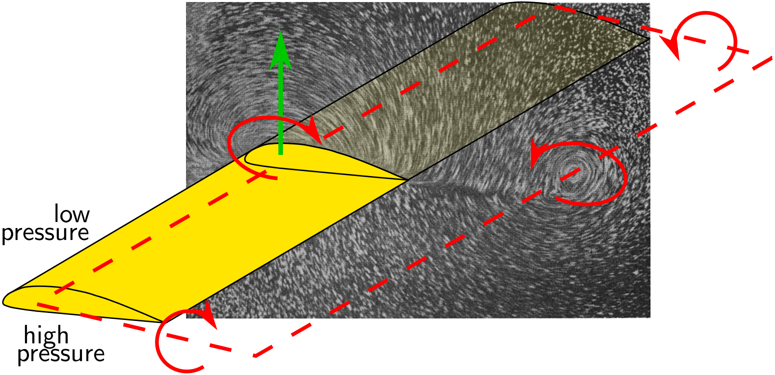

air 'escapes' around the wing tips

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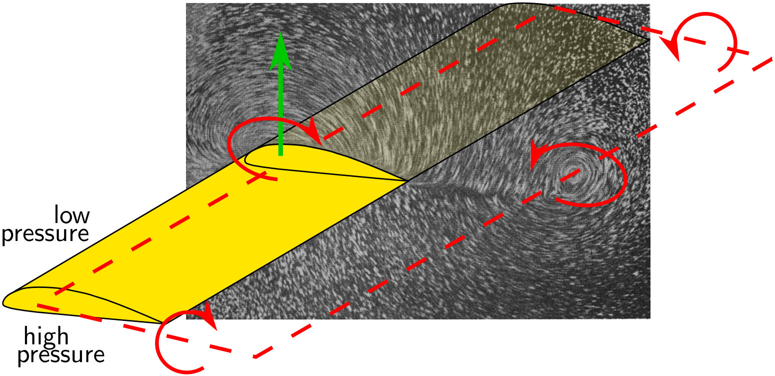

air 'escapes' around the wing tips

is there circulating motion trailing behind wing tips?

a plan of experiments

2D experiment: impulsive start
is 'anything' shed into wake?

2D experiment: start and stop
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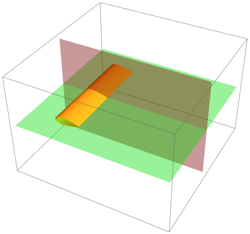
3D experiment: steady, level flight
does circulation trail behind wingtips?



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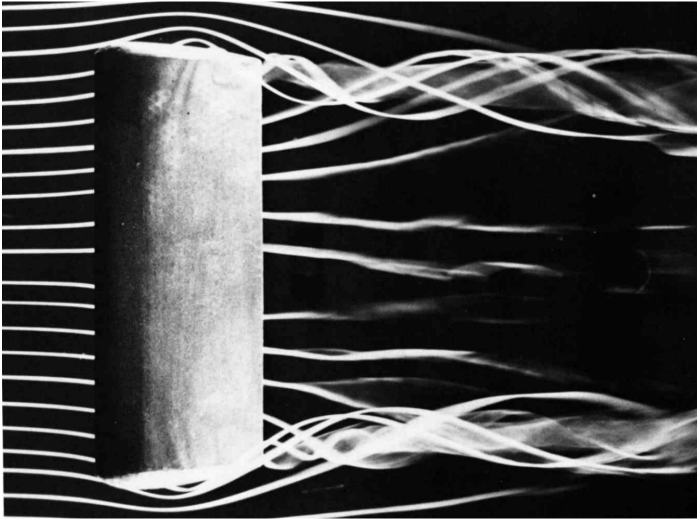

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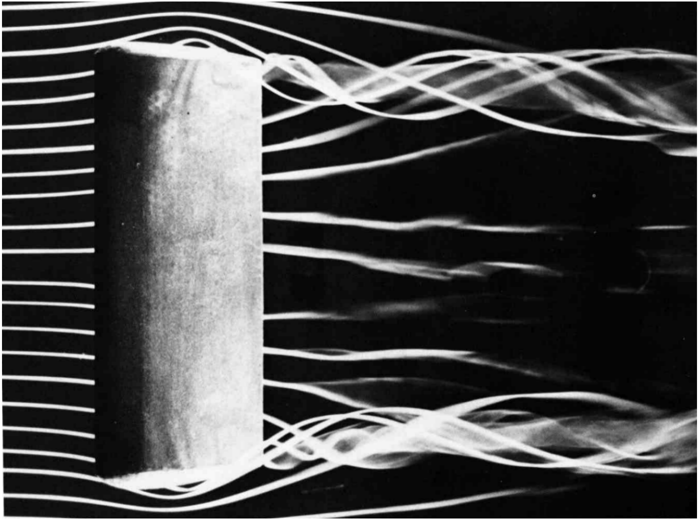

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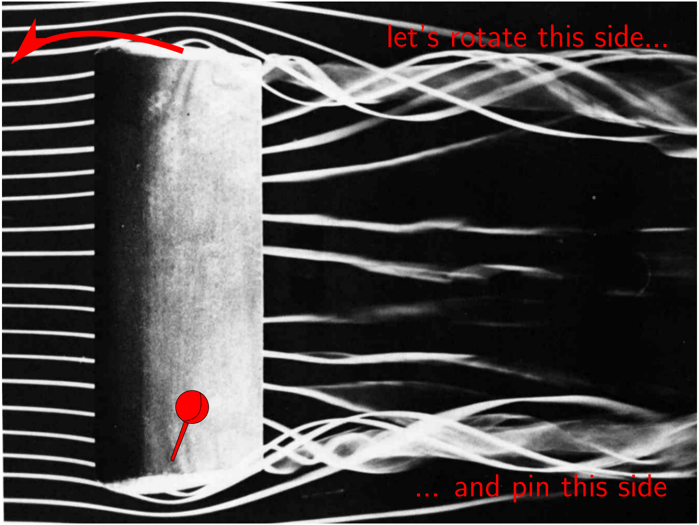
from steady level flight to rotors...



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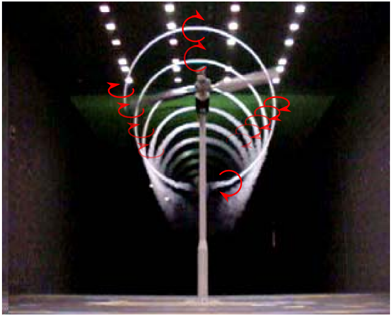

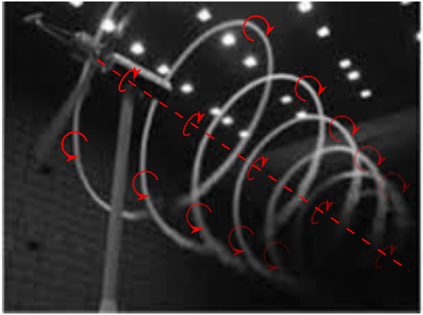

is there circulating motion trailing along the helical path behind the rotating wingtips?

left: in Hand, et al. *Unsteady Aerodynamics Experiment Phase VI: Wind tunnel...*. (2001, NREL/TP-500-29955, Golden, CO, USA.)
right: source unknown.

a plan of experiments

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3D experiment: steady turbine rotor
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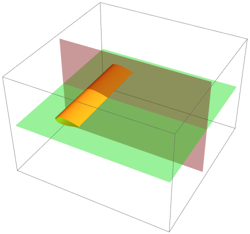

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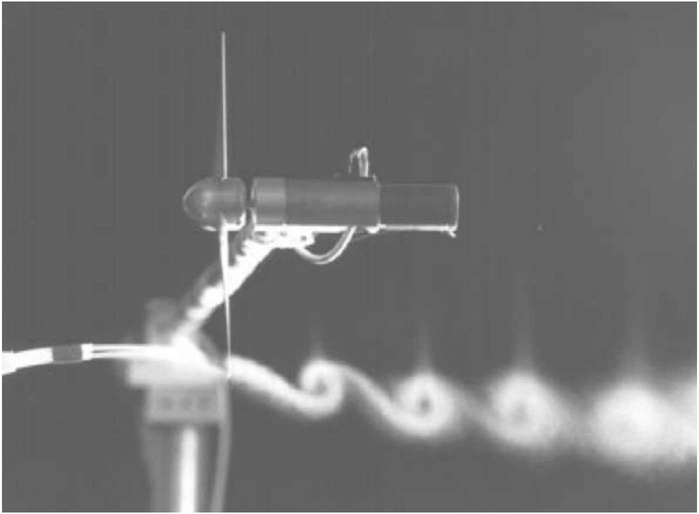

in Vermeer, 'A review of wind turbine wake research...' in *ASME Wind Energy Symp. Tech. Papers* (2001, ASME, New York, NY, USA)

3D experiment: steady turbine rotor
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in Alfredsson and Dahlberg *A preliminary wind tunnel study of windmill wake...* (1979, FFA/TN-AU-1499 part 7, Stockholm, Sweden)

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behind an obstacle

here:
condensation in
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the wake is conserving angular momentum!
(are we convinced?)

Vattenfall 2010, https://www.flickr.com/photos/vattenfall/4270899001

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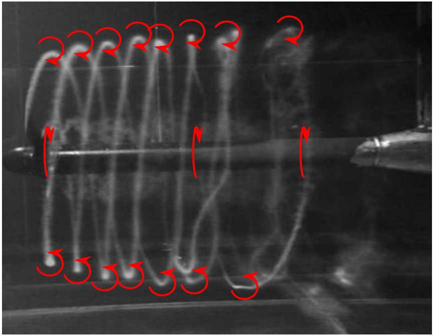
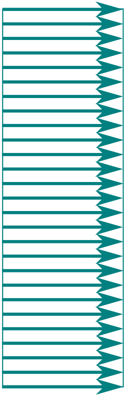
- where to find more practical information

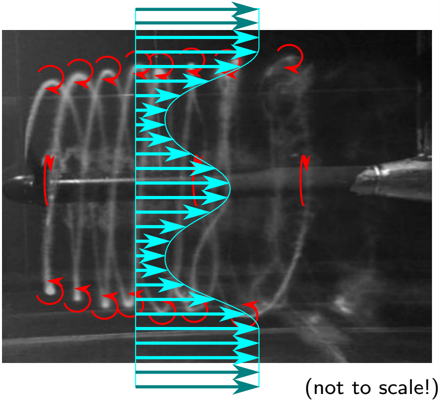
how does the wake affect the turbine?



(not to scale!)

in Okulov et al, 'Wake effect on a uniform flow behind...' in *Wake: J. Phys. Conf. Ser. 625 012011* (2011, IOP, Uppsala, Sweden)

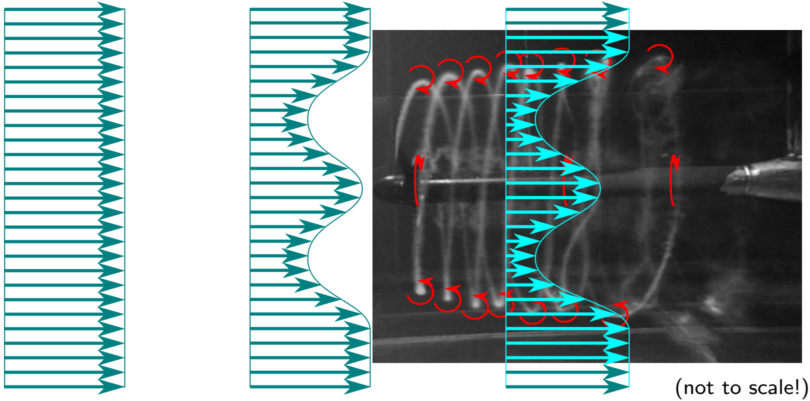




(not to scale!)

in Okulov et al, 'Wake effect on a uniform flow behind...' in *Wake: J. Phys. Conf. Ser. 625 012011* (2011, IOP, Uppsala, Sweden)

the wake decreases the velocity at the rotor!



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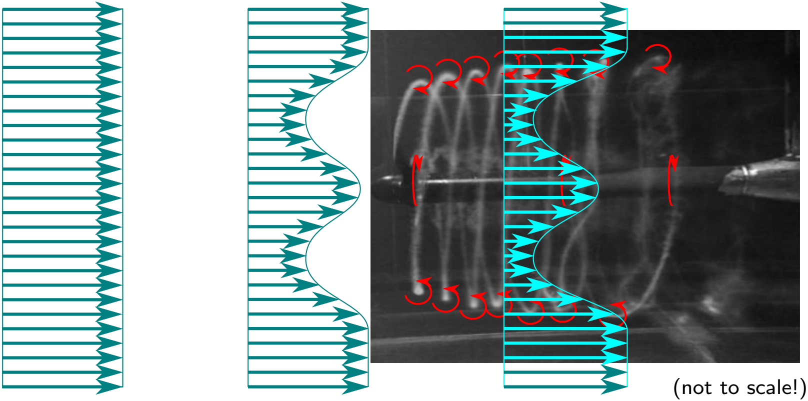

(not to scale!)

remember: $P \propto u^3$!

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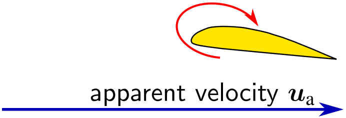
how does the wake affect an airfoil?



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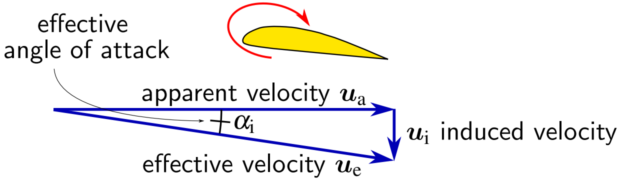

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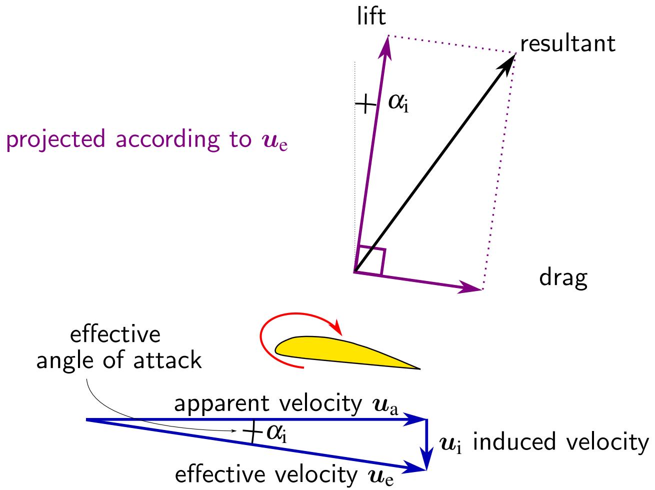

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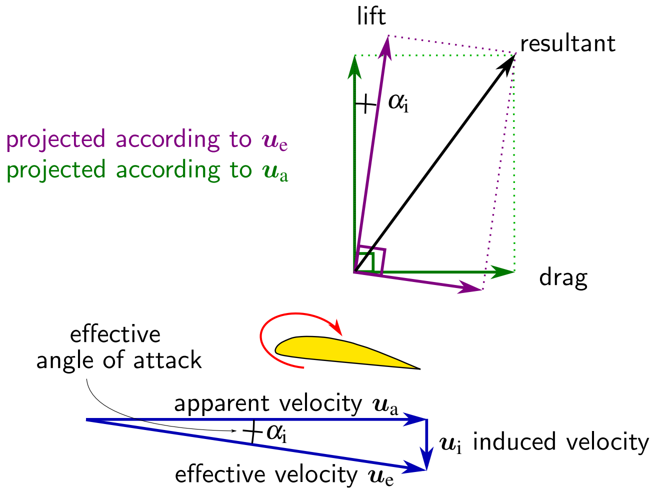
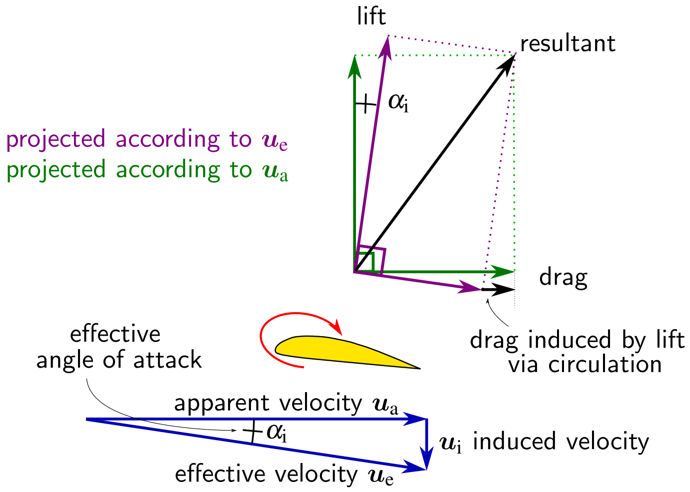
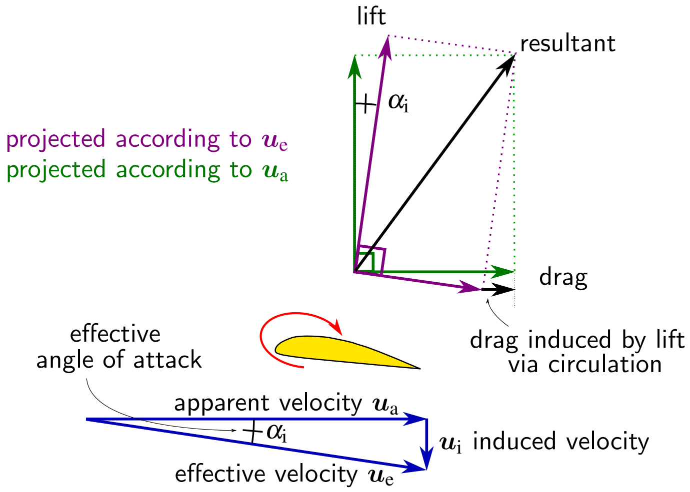


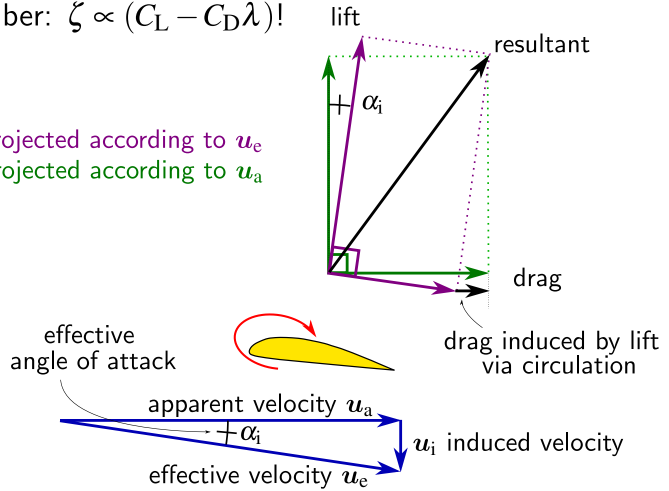
how does the wake affect an airfoil?




lift
resultant
α_i
projected according to $\boldsymbol{u}_e$
projected according to $\boldsymbol{u}_a$
drag
drag induced by lift via circulation
effective angle of attack
apparent velocity $\boldsymbol{u}_a$
α_i
$\boldsymbol{u}_i$ induced velocity
effective velocity $\boldsymbol{u}_e$

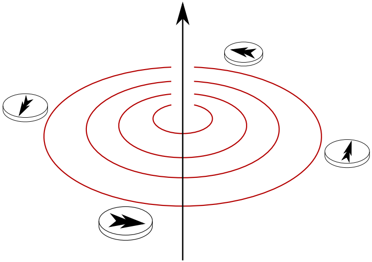
remember: $\zeta \propto (C_{\mathrm{L}} - C_{\mathrm{D}}\lambda)$!

projected according to $\boldsymbol{u}_{\mathrm{e}}$
projected according to $\boldsymbol{u}_{\mathrm{a}}$



what's in here?

- what is the wake doing?
  - → some philosophical implications of lift
  - → an angular momentum detour
  - → some experiments
- how does the wake affect wind turbines
- one conceptual way to model the wake
- where to find more practical information



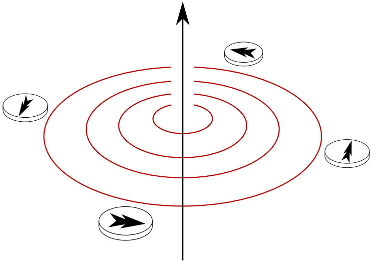
there is an invisible state

conducted in a structure

and produces a force field

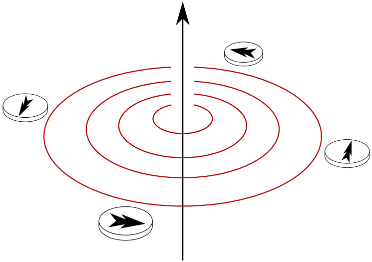
circulates certain particles

imagine the following...



there is an invisible state	current
conducted in a structure	wire
and produces a force field	electromagnetic
circulates certain particles	magnetic

imagine the following...



there is an invisible state	current	vorticity
conducted in a structure	wire	vortex
and produces a force field	electromagnetic	pressure
circulates certain particles	magnetic	fluid

the analogy in equations

electromagnetism

Poisson equation for electrostatics:

$$\underbrace{\nabla^2 \boldsymbol{A} = -\mu_0 \boldsymbol{J}}$$

$\boldsymbol{A}$	vector potential
$\boldsymbol{J}$	current density

Ampère's law:

$$\underbrace{\boldsymbol{J} = \nabla \times \boldsymbol{H}}$$

$\boldsymbol{H}$	magnetic field intensity

aerodynamics

for incompressible flows:

$$\underbrace{\nabla^2 \boldsymbol{\psi} = -\boldsymbol{\omega}}$$

$\boldsymbol{\psi}$	vector potential
$\boldsymbol{\omega}$	vorticity

definition of vorticity:

$$\underbrace{\boldsymbol{\omega} := \nabla \times \boldsymbol{u}}$$

$\boldsymbol{u}$	flow velocity, s.t. $\boldsymbol{u} = \boldsymbol{u}_\infty + \nabla \times \boldsymbol{\psi}$

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define 'circulation' as flux of vorticity through surface S

$$\Gamma := \int_S \boldsymbol{\omega} \cdot \hat{\boldsymbol{n}} \, dS$$

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then, Kutta-Joukowski expression says:

'lift per unit span is proportional to circulation' $(L' = \rho u \Gamma)$

electromagnetism

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solution:

$$\boldsymbol{u}(\boldsymbol{x}) = \boldsymbol{u}_\infty - \frac{1}{4\pi} \int_V \frac{(\boldsymbol{x} - \boldsymbol{\xi})}{||\boldsymbol{x} - \boldsymbol{\xi}||_2^3} \times \boldsymbol{\omega}(\boldsymbol{\xi}) \; d\boldsymbol{\xi}$$

electromagnetism

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YUCK!

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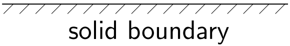
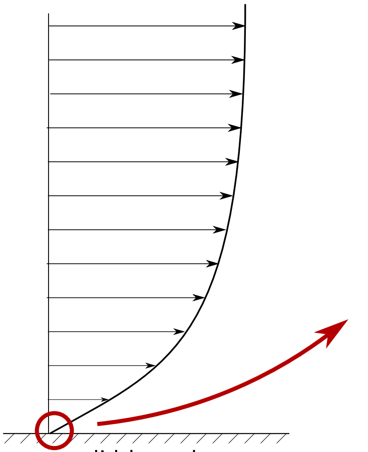
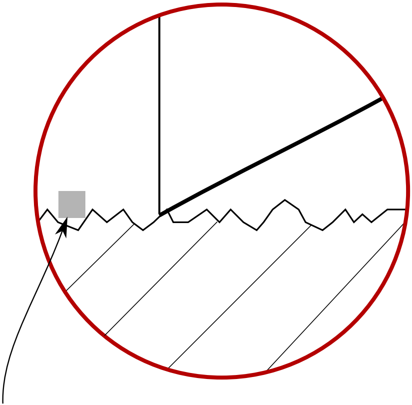

solid boundary

the boundary layer...

freestream u_∞

the boundary layer...



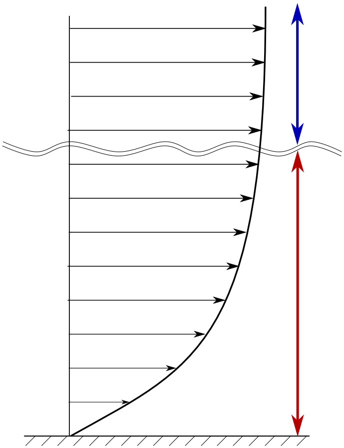


fluid elements get stuck

→ 'no-slip' condition

(zero relative surface velocity)

the boundary layer...

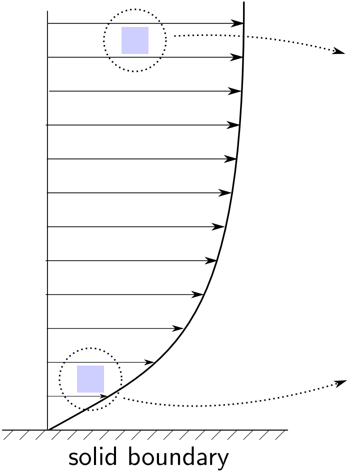


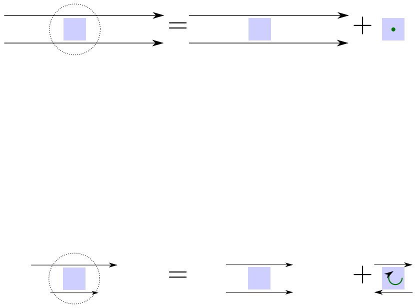
the 'external flow':
shear gradient ≈ 0

$$u(\delta) = \kappa \, u_\infty$$

the 'boundary layer':
shear gradient strong!

the boundary layer...





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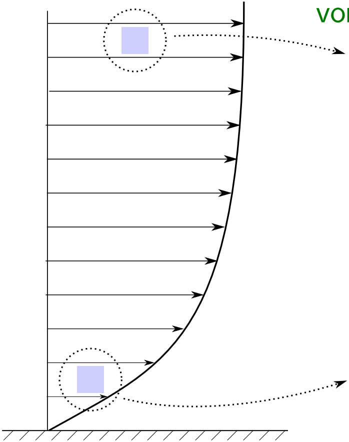
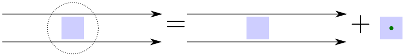
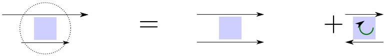


solid boundary

vorticity is related to viscous shear...





... and might be negligible*
far from solid bodies

∗ certain conditions apply

the boundary layer...

$$\left(\text{remember: trying to solve } \boldsymbol{u}(\boldsymbol{x}) = \boldsymbol{u}_\infty - \frac{1}{4\pi}\int_V \frac{(\boldsymbol{x}-\boldsymbol{\xi})}{||\boldsymbol{x}-\boldsymbol{\xi}||_2^3} \times \boldsymbol{\omega}(\boldsymbol{\xi})\ d\boldsymbol{\xi}\right)$$

... separates V into two parts:

boundary layer Ω

vorticity $\boldsymbol{\omega}(\boldsymbol{\xi}) \neq 0$

$\Downarrow$

local circulating motion of fluid elements due to viscous shear stress

external flow Ω'

vorticity $\boldsymbol{\omega}(\boldsymbol{\xi}) \approx 0$ $*$

$\Downarrow$

drops out of integral

$*$ for 'potential' external flow
$\approx$ incompressible
$+$ inviscid
$+$ irrotational.

the boundary layer saves a lot of effort!

$$\left(\text{remember: trying to solve } \boldsymbol{u}(\boldsymbol{x}) = \boldsymbol{u}_\infty - \frac{1}{4\pi}\int_V \frac{(\boldsymbol{x}-\boldsymbol{\xi})}{||\boldsymbol{x}-\boldsymbol{\xi}||_2^3} \times \boldsymbol{\omega}(\boldsymbol{\xi})\ d\boldsymbol{\xi}\right)$$

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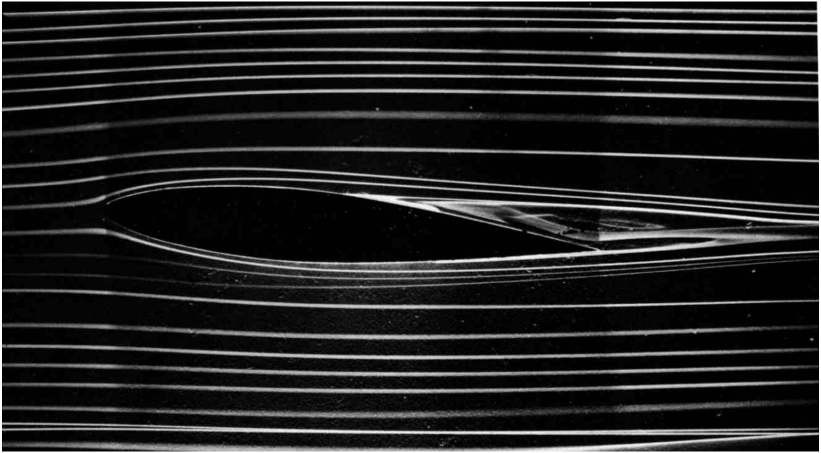

Werlé 1974, in Van Dyke's *Album of Fluid Motion* (1982, Parabolic, Stanford, CA, USA)

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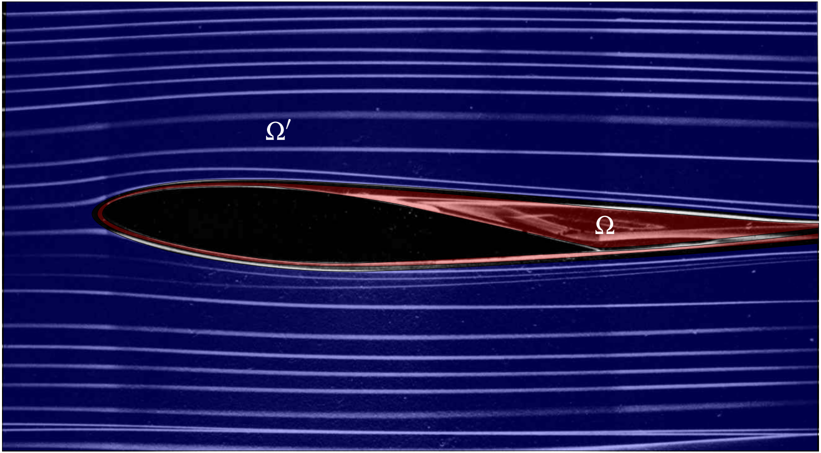

$$\boldsymbol{u}(\boldsymbol{x}) \approx \boldsymbol{u}_\infty - \frac{1}{4\pi}\left(\int_{\Omega'} \mathbf{0}\ d\boldsymbol{\xi} + \int_{\Omega} \frac{(\boldsymbol{x}-\boldsymbol{\xi})}{||\boldsymbol{x}-\boldsymbol{\xi}||_2^3} \times \boldsymbol{\omega}(\boldsymbol{\xi})\ d\boldsymbol{\xi}\right)$$

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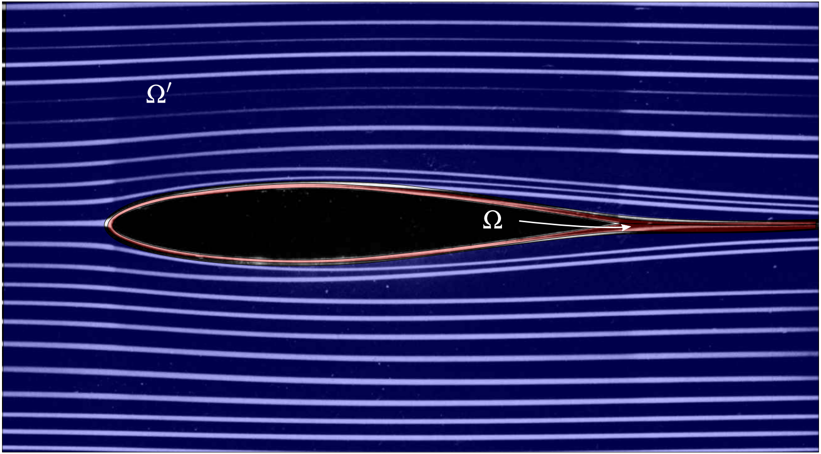


$$\boldsymbol{u}(\boldsymbol{x}) \approx \boldsymbol{u}_\infty - \frac{1}{4\pi}\left(\int_{\Omega'} \mathbf{0} \; d\boldsymbol{\xi} + \int_{\Omega} \frac{(\boldsymbol{x}-\boldsymbol{\xi})}{||\boldsymbol{x}-\boldsymbol{\xi}||_2^3} \times \boldsymbol{\omega}(\boldsymbol{\xi}) \; d\boldsymbol{\xi} \right)$$

vortex methods discretize the vorticity

$$\boldsymbol{u}(\boldsymbol{x}) \approx \boldsymbol{u}_\infty - \frac{1}{4\pi} \int_\Omega \frac{(\boldsymbol{x}-\boldsymbol{\xi})}{||\boldsymbol{x}-\boldsymbol{\xi}||_2^3} \times \boldsymbol{\omega}(\boldsymbol{\xi}) \; d\boldsymbol{\xi}$$

discretize $\boldsymbol{\omega}$ into vortices* with known analytical integrals...

points

curves like line-segments or helices

surfaces like rectangles

volumes like spheres

...then sum so that given boundary conditions are satisfied!

$$\boldsymbol{u}(\boldsymbol{x}) \approx \boldsymbol{u}_\infty + \sum_i^N \boldsymbol{u}_i(\boldsymbol{x}, \boldsymbol{\omega}_i)$$

$*$ overlapping vortices allowed due to linearity of ∇^2 operator.

more practical information can be found in...

Fizzy's guide to vorticity and vortex methods

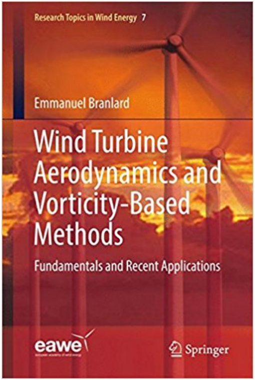
R. Leuthold

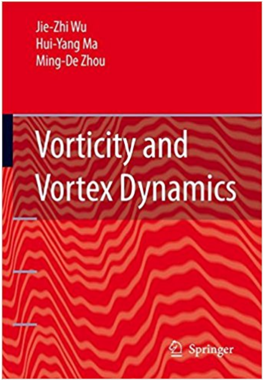
April 30, 2018

I'd rather study fluids than swim in them.

-Fizzy

Contents





...in order of increasing thoroughness + rigorousness

thank you for your attention!

... any questions?