Surname: First Name:		Matriculation number:	Matriculation number:		
Subject: Programme: Bachelor Master Lehr		nramt others Signature			
1.	 Please fill in your name above and tick exactly ONE box for the right answer of each question below. 1. We would like to know the unknown probability θ that a phone does NOT break when it is dropped. We assume that the phone thrown onto the ground either breaks or has no damage. In an experiment we have dropped 100 smartphones and obtained 23 broken smartphones. What is the negative log likelihood function f(θ) that we need to minimize in order to obtain the maximum likelihood (ML) estimate of θ? 				
	(a) log(77 θ) - log(23(1 -	- θ))	(b) $23 \log \theta + 77 \log(1-\theta)$	(b) $23\log\theta + 77\log(1-\theta)$	
	(c) $-77 \log \theta - 23 \log(1 - \theta)$	$\theta)$	(d) $\log(23\theta) + \log(77(1-\theta))$	9))	
2.	You are given a pendulum which is ments. Which of the following algo		can be modeled by $y(t) = \theta_1 \cos(\theta_2 t)$ he parameters θ ?	$(t + \theta_3)$, where $y(t)$ are the measure-	
	(a) Recursive Least Squares (RLS)	(b) Maximum a Posteriori Es	stimation (MAP)	
	(c) Linear Least Squares (LL	.S)	(d) Weighted Least Squares	(WLS)	
3.		you use to estimate the parameter	approximated by a model that is lines θ of this linear model without runs data?		
	(a) MAP	(b) ML	(c) RLS	(d) LLS	
4.	You are asked to give a computation model is given as $y_N = \Phi_N \theta + \epsilon_N$ matrix can be approximated by $\Sigma_{\hat{\theta}}$	With $\epsilon_N \sim \mathcal{N}(0, \Sigma_\epsilon), Q_N = \Phi_N^\top$	e covariance of the estimate compute Φ_N and $L(\theta, y_N)$ is the negative log	ed in the previous question $\Sigma_{\hat{\theta}}$. The likelihood function. The covariance	
	(a) $\Phi_N^+ \Sigma_{\epsilon_N} \Phi_N^+^\top$	(b) $\square Q_N^{-1}$	(c) $\Box \nabla^2_{\theta} L(\theta, y_N)$	(d) $\left[\left(\Phi_N^\top \Sigma_{\epsilon_N}^{-1} \Phi_N \right)^{-1} \right]$	
5.			arameter θ , and a set of independent d to solve to get a ML-estimate of θ ?		
	(a) $\sum_{k} \ \theta y(k) \exp(-\theta y(k))\ $	$\ _{2}^{2}$	(b) $\square N \log(\theta) + \theta \sum_k y(k)$	(b) $\square N \log(\theta) + \theta \sum_{k} y(k)$	
	(c) $\Box - \log \left(\sum_{k} \ \theta y(k) \exp(- \theta y(k)) \right)$	$- heta y(k)) \ _2^2 ig)$	(d) $\Box -N\log(\theta) + \theta \sum_{k} y(k)$		
6.	For the problem in the previous que value? The Fisher information matter	estion, what is a lower bound on the rix is defined as $M = \int_{yN} \nabla_{\theta}^2 L(\theta_0)$	covariance for any unbiased estimate $(y_N) \cdot p(y_N \theta_0) dy_N.$	or $\hat{\theta}(y_N)$, assuming that θ_0 is the true	
	(a) N/θ_0^2		(b) $\prod \int_{y_N} N\theta_0^{N-2} \exp[-\theta_0 \sum_k y_k] \mathrm{d}y_N$		
	(c) θ_0^2/N		$ \qquad \qquad$	$\exp[- heta_0 \sum_k y_k] \mathrm{d}y_N$	
7.	Let $\theta_{\rm R}$ denote the <i>regularized</i> LLS	estimator using L_2 regularization.	Which of the following is NOT true?		
	(a) $\theta_{\rm R}$ incorporates prior know	wledge about θ .	(b) $\square \theta_{\rm R}$ can be computed analytically.		
	(c) $\square \theta_{\rm R}$ is asymptotically bias	ed.	(d) $\square \theta_{\rm R}$ is biased.		
8.			tion problem. Which of the following		
	(a) The inverse of the GN He	ssian approximates Σ_{θ} .	(b) The idea of GN is to line	arize the residual function.	
	(c) GN uses a Hessian appro	ximation.	(d) GN finds the global mini	mizer of the objective function.	

9. Given a set of measurements $y_N = [y(1), y(2), \dots, y(N)]^T$ from the linear model $y_N = \Phi \theta$, where $\Phi = [\varphi(1), \varphi(2), \dots, \varphi(N)]^T$, which of the following minimisation problems is solved at each iteration step of the RLS algorithm to estimate the parameter $\hat{\theta}(N+1)$ after N+1 measurements? $\hat{\theta}(N+1) = \arg \min_{a} \frac{1}{2} (\dots)$ [a and c are correct!]

(a) $\ \theta - \hat{\theta}(N) \ _{2}^{2} + \ y(N+1) - \varphi(N+1)^{\top} \theta \ _{Q_{N}}^{2}$	(b) $\ \theta - \hat{\theta}(N) \ _{Q_N}^2 + \ y(N+1) - \varphi(N+1)^\top \theta \ _2^2$
(c) $\qquad \ y_{N+1} - \Phi_{N+1} \cdot \theta\ _2^2$	(d) $\qquad \ y_N - \Phi_N \cdot \theta\ _{Q_N}^2$

10. You suspect your data to contain some outliers, thus you would use ... estimation which assumes your measurement errors follow a ... distribution.

(a) $\Box L_1$, Gaussian	(b) $\Box L_1$, Laplace	(c) $\Box L_2$, Laplace	(d) $\Box L_2$, Gaussian
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11. Assume you solved a nonlinear parameter estimation problem using the Gauss-Newton algorithm. How can you check that your assumptions on the noise distribution are correct?

- 12. For a discrete time LTI system $x_{k+1} = Ax_k + Bu_k$, k = 0, 1, explicitly state the forward simulation map $f_{sim} : \mathbb{R}^{n_x + 2n_u} \to \mathbb{R}^{3n_x}$, $(x_0, u_0, u_1) \mapsto (x_0, x_1, x_2)$.
- 13. Please identify the most general system equation that still is a Auto Regressive Model with Exogenous Inputs (ARX).

	(a) $y(k) = -a_1 y(k-1) - \dots - a_{n_a} y(k-n_a)$		(b) $y(k) = b_0 u(k) + \ldots + b_{n_b} u(k - n_b)$	
	(c) $a_0y(k) + a_1y^2(k-1) + \ldots + a_{n_a}y^{a_b}$	$^{n_a+1}(k-n_a) = b_0 u(k) + \ldots + b_0 u(k-n_b)$	(d) $a_0y(k) + a_1y(k-1) + \dots + a_{n_a}y(k-n_a) = b_0u(k) + \dots + b_{n_b}u(k-n_b)$	
14.	Which one of the following statement	nts is NOT true for FIR models:		
	(a) The impulse response is constant.		(b) The output does not depen	nd on previous outputs.
	(c) Output error minimization	is a convex problem.	(d) They are a special class of ARX models	
15.	Which of the following model equat	tions describes a FIR system with in	put u and output y ? $y(k+1) = \dots$	
	(a) $u(k) + \sin(k \cdot \pi)^2$	(b) $\Box u(k) \cdot y(k)$	(c) $u(k) - \sqrt{\pi}u(k-2)$	(d) $u(k+1) + y(k)$
16.	Which of the following dynamic mo	odels with inputs $u(t)$ and outputs y	(t) is NEITHER linear NOR affine.	
	(a) $t\dot{y}(t) = u(t) + \sqrt{2\pi}$	(b) $\qquad \dot{y}(t) + \cos(t) = u(t)$	(c) $\[\dot{y}(t) = u(t) + t \]$	(d) $\qquad \dot{y}(t) = \sqrt{t \cdot u(t)}$
17.	Which of the following models with	input $u(k)$ and output $y(k)$ is NO	f linear-in-the-parameters w.r.t. $\theta \in \mathbb{R}^2$?	
	(a) $y(k) = y(k-1)\theta_1 + \sqrt{\theta}$	$\overline{u^2u(k)}$	(b) $y(k) = \theta_1 u(k)^2 + \theta_2 \exp(u(k))$	
	(c) $y(k) = \exp(y(k-1)) \cdot ($	$(heta_1 + heta_2 u(k))$	(d) $\qquad y(k) = \theta_1 \sqrt{u(k)}$	
18.	Which of the following models is tin	me invariant?		
	(a) $\[\ddot{y}(t)^2 = u(t)^t + e^{u(t)} \]$	(b) $\[\dot{y}(t) = \sqrt{u(t)} + 1 \]$	(c) $\[\dot{y}(t) = -3u(t) + t^2 \]$	(d) $\Box t \cdot \ddot{y}(t) = u(t)^3$
19. With which of the following formulas you can NOT compute the conditional joint distri (y_1, \ldots, y_N) given θ ? $p(y_N \theta) \neq \ldots$			conditional joint distribution of N	independent measurements $y_N =$
	(a) $\int p(y_N x_N, \theta)p(x_N)dx_N$ (c) $\int p(y_N \theta)p(\theta)d\theta$		(b) $\prod_{i} p(y(i) \theta)$	
			(d) $\square \exp\left(\sum_{i=0}^{N} \ln(p(y(i) \theta))\right)$	
20.	Which of the following statements a	bout Maximum A Posteriori (MAP) estimation is NOT true	
	(a) The MAP estimator is bias	ed.	(b) MAP is a generalization of	of ML.
	(c) MAP assumes a linear mo	del.	(d) $\square \hat{\theta}_{MAP} = \arg \min_{\theta \in \mathbb{R}} [-\log(p(y_N \theta)) - \log(p(\theta))]$	

Su	rname: I	First Name:	Matriculation number:	
Su	bject: Program	nme: Bachelor Master Le	hramt others Signatur	e:
1.		d tick exactly ONE box for the righ tion $p_X(x) = \theta x \exp(-\theta x)$, with p	t answer of each question below. parameter θ , and a set of independent	t measurements
	$y_N = [y(1), y(2), \dots, y(N)]^T$, w	hich minimisation problem you nee	ed to solve to get a ML-estimate of θ	? The problem is: $\min_{\theta} \dots$?
	(a) $-N \log(\theta) + \theta \sum_{k} y(k)$)	(b) $\sum_{k} \ \theta y(k) \exp(-\theta y(k))\ $	$ _{2}^{2}$
	(c) $\square N \log(\theta) + \theta \sum_{k} y(k)$		(d) $\Box - \log \left(\sum_{k} \ \theta y(k) \exp \theta y(k) \right)$	$(- heta y(k))\ _2^2 ig)$
2.		estion, what is a lower bound on the trix is defined as $M = \int_{yN} \nabla_{\theta}^2 L(\theta)$		for $\hat{\theta}(y_N)$, assuming that θ_0 is the true
	(a) N/θ_0^2		(b) $\square \theta_0^2/N$	
	(c) $\int_{y_N} N\theta_0^{N-2} \exp[-\theta_0 \sum$	$\sum_k y_k] \mathrm{d} y_N$	(d) $\int_{y_N} N\theta_0^{N-2} \left(\sum_k y(k)\right)$	$)\exp[- heta_0\sum_k y_k]\mathrm{d}y_N$
3.		s by nature a nonlinear system and porithms should you use to estimate		$(t_2 t + \theta_3)$, where $y(t)$ are the measure-
	(a) Linear Least Squares (LI	_S)	(b) Weighted Least Squares	(WLS)
	(c) Recursive Least Squares	(RLS)	(d) Maximum a Posteriori I	Estimation (MAP)
4.	of the following algorithms could		rs θ of this linear model without run	near in the parameters (LIP). Which nning into memory problems or high
	(a) LLS	(b) RLS	(c) ML	(d) MAP
5.		_N with $\epsilon_N \sim \mathcal{N}(0, \Sigma_{\epsilon}), Q_N = \Phi_N^{\top}$		ted in the previous question $\Sigma_{\hat{\theta}}$. The g likelihood function. The covariance
	(a) $\left[(\Phi_N^\top \Sigma_{\epsilon_N}^{-1} \Phi_N)^{-1} \right]$	(b) $\square \Phi_N^+ \Sigma_{\epsilon_N} {\Phi_N^+}^\top$	(c) $\square Q_N^{-1}$	(d) $\Box \nabla^2_{\theta} L(\theta, y_N)$
6.	 Assume you solved a nonlinear parameter estimation problem using the Gauss-Newton algorithm. How can you check that your assumptions the noise distribution are correct? 			n you check that your assumptions on
7.	Let $\theta_{\rm R}$ denote the <i>regularized</i> LLS	S estimator using L_2 regularization.	Which of the following is NOT true	?
	(a) $\[\theta_{\rm R} \]$ is biased.		(b) $\square \theta_{\rm R}$ is asymptotically bia	ised.
	(c) $\theta_{\rm R}$ can be computed and	lytically.	(d) $\theta_{\rm R}$ incorporates prior kr	nowledge about θ .
8.		<u> </u>	tion problem. Which of the followir	g statements is NOT true <i>in general</i> ?
	(a) The idea of GN is to line	arize the residual function.	(b) GN finds the global mir	imizer of the objective function.
	(c) GN uses a Hessian appro	oximation.	(d) The inverse of the GN H	Hessian approximates Σ_{θ} .

9.	Which of the following models is time invariant?				
	(a) $\[\dot{y}(t) = \sqrt{u(t)} + 1 \]$ (b) $\[\dot{y}(t) = -3u(t) + t^2 \]$	(c) $[] \ddot{y}(t)^2 = u(t)^t + e^{u(t)}$	(d) $\Box t \cdot \ddot{y}(t) = u(t)^3$		
10.	With which of the following formulas you can NOT compute the (y_1, \ldots, y_N) given θ ? $p(y_N \theta) \neq \ldots$	conditional joint distribution of N	independent measurements $y_N =$		
	(a) $\prod \int p(y_N x_N,\theta)p(x_N)\mathrm{d}x_N$	(b) $\prod_i p(y(i) \theta)$			
	(c) $\int p(y_N \theta)p(\theta)d\theta$	(d) $\Box \exp\left(\sum_{i=0}^{N} \ln(p(y(i) \theta))\right)$			
11.	Which of the following statements about Maximum A Posteriori (MAP) estimation is NOT true			
	(a) The MAP estimator is biased.	(b) MAP is a generalization of	of ML.		
	(c) $\hat{\theta}_{MAP} = \arg\min_{\theta \in \mathbb{R}} [-\log(p(y_N \theta)) - \log(p(\theta))]$	(d) MAP assumes a linear mo	odel.		
12.	Which of the following model equations describes a FIR system with in	put u and output y ? $y(k+1) = \dots$			
	(a) $u(k) + \sin(k \cdot \pi)^2$ (b) $u(k) \cdot y(k)$	(c) $\Box u(k+1) + y(k)$	(d) $u(k) - \sqrt{\pi}u(k-2)$		
13.	Which of the following dynamic models with inputs $u(t)$ and outputs y	(t) is NEITHER linear NOR affine.			
	(a) $\begin{tabular}{ c c c c } \dot{y}(t) = u(t) + t \end{tabular}$ (b) $\begin{tabular}{ c c c } t\dot{y}(t) = u(t) + \sqrt{2\pi} \end{tabular}$	(c) $\[\dot{y}(t) = \sqrt{t \cdot u(t)}\]$	(d) $\boxed{\dot{y}(t) + \cos(t) = u(t)}$		
14.	4. Given a set of measurements $y_N = [y(1), y(2), \dots, y(N)]^T$ from the linear model $y_N = \Phi \theta$, where $\Phi = [\varphi(1), \varphi(2), \dots, \varphi(N)]^T$, who of the following minimisation problems is solved at each iteration step of the RLS algorithm to estimate the parameter $\hat{\theta}(N+1)$ after N measurements? $\hat{\theta}(N+1) = \arg \min_{\alpha} \frac{1}{2} (\dots)$ [a and c are correct!]				
	(a) $\ y_N - \Phi_N \cdot \theta\ _{Q_N}^2$	(b) $\ y_{N+1} - \Phi_{N+1} \cdot \theta \ _2^2$			
	(c) $\ \theta - \hat{\theta}(N)\ _{Q_N}^2 + \ y(N+1) - \varphi(N+1)^\top \theta\ _2^2$	(d) $\ \theta - \hat{\theta}(N) \ _2^2 + \ y(N+1) \ _2^2$	$\ -\varphi(N+1)^{\top}\theta\ _{Q_N}^2$		
15.	Please identify the most general system equation that still is a Auto Reg	gressive Model with Exogenous Inpu	ts (ARX).		
	(a) $y(k) = -a_1 y(k-1) - \dots - a_{n_a} y(k-n_a)$	(b) $y(k) = b_0 u(k) + \ldots + b_{n_b} u(k)$	$u(k-n_b)$		
	(c) $a_0y(k) + a_1y(k-1) + \ldots + a_{n_a}y(k-n_a) = b_0u(k) + \ldots + b_{n_b}u(k-n_b)$	(d) $a_0y(k) + a_1y^2(k-1) + \ldots + a_{n_a}y^{n_a}$	$^{n_a+1}(k-n_a) = b_0 u(k) + \ldots + b_0 u(k-n_b)$		
16.	6. We would like to know the unknown probability θ that a phone does NOT break when it is dropped. We assume that the phone thrown onto ground either breaks or has no damage. In an experiment we have dropped 100 smartphones and obtained 23 broken smartphones. What i negative log likelihood function $f(\theta)$ that we need to minimize in order to obtain the maximum likelihood (ML) estimate of θ ?				
	(a) $23 \log \theta + 77 \log(1-\theta)$	(b) $-77 \log \theta - 23 \log(1 - \theta)$))		
	(c) $\log(23\theta) + \log(77(1-\theta))$	(d) $-\log(77\theta) - \log(23(1 - \log(23))))$	- θ))		
17.	For a discrete time LTI system $x_{k+1} = Ax_k + Bu_k, k = 0, 1$, explicitl $f_{\text{sim}} : \mathbb{R}^{n_x + 2n_u} \to \mathbb{R}^{3n_x}, (x_0, u_0, u_1) \mapsto (x_0, x_1, x_2).$	ly state the forward simulation map			

18. Which of the following models with input u(k) and output y(k) is **NOT** linear-in-the-parameters w.r.t. $\theta \in \mathbb{R}^2$?

(a) $y(k) = \exp(y(k-1)) \cdot (\theta_1 + \theta_2 u(k))$	(b) $y(k) = \theta_1 u(k)^2 + \theta_2 \exp(u(k))$
(c) $y(k) = \theta_1 \sqrt{u(k)}$	(d) $y(k) = y(k-1)\theta_1 + \sqrt{\theta_2 u(k)}$

19. You suspect your data to contain some outliers, thus you would use ... estimation which assumes your measurement errors follow a ... distribution.

(a) $\Box L_1$, Laplace	(b) $\Box L_1$, Gaussian	(c) $\Box L_2$, Gaussian	(d) $\Box L_2$, Laplace	

20. Which one of the following statements is **NOT** true for FIR models:

(a) They are a special class of ARX models	(b) The impulse response is constant.
(c) Output error minimization is a convex problem.	(d) The output does not depend on previous outputs.

Su	name: Fi	irst Name:	Matriculation number:			
Sul	oject: Program	me: Bachelor Master Leh	ramt others Signatur	re:		
1.	Please fill in your name above and tick exactly ONE box for the right answer of each question below. 1. Let θ_R denote the <i>regularized</i> LLS estimator using L_2 regularization. Which of the following is NOT true?					
	(a) $\theta_{\rm R}$ is biased.	0 - 0	1			
	(c) $\theta_{\rm R}$ can be computed analy	ytically.	(d) $\theta_{\rm R}$ is asymptotically bia	ased.		
2.	We use the Gauss-Newton (GN) alg	gorithm to solve a nonlinear estimat	ion problem. Which of the followin	ng statements is NOT true in general?		
	(a) GN uses a Hessian approx	timation.	(b) The idea of GN is to lin	earize the residual function.		
	(c) GN finds the global minin	mizer of the objective function.	(d) The inverse of the GN I	Hessian approximates Σ_{θ} .		
3.	Please identify the most general sys	stem equation that still is a Auto Reg	gressive Model with Exogenous In	puts (ARX).		
	(a) $y(k) = b_0 u(k) + \ldots + b_{n_b} u(k)$	$u(k-n_b)$	(b) $y(k) = -a_1 y(k-1) - \dots$	$\dots - a_{n_a} y(k - n_a)$		
	(c) $a_0y(k) + a_1y(k-1) + \ldots + a_{n_a}y(k-1)$	$(k-n_a) = b_0 u(k) + \ldots + b_{n_b} u(k-n_b)$	(d) $a_0y(k) + a_1y^2(k-1) + \ldots + a_n$	$a_{n_a}y^{n_a+1}(k-n_a) = b_0u(k) + \ldots + b_0u(k-n_b)$		
4.	Which of the following model equa	tions describes a FIR system with in	nput u and output y ? $y(k+1) = .$			
	(a) $u(k) - \sqrt{\pi}u(k-2)$	(b) $\Box u(k) \cdot y(k)$	(c) $u(k) + \sin(k \cdot \pi)^2$	(d) $u(k+1) + y(k)$		
	You suspect your data to contain so bution.	ome outliers, thus you would use	. estimation which assumes your n	neasurement errors follow a distri-		
	(a) $\Box L_1$, Gaussian	(b) $\Box L_2$, Laplace	(c) $\Box L_1$, Laplace	(d) $\Box L_2$, Gaussian		
6.	Which one of the following stateme	ents is NOT true for FIR models:				
	(a) Output error minimization	n is a convex problem.	(b) They are a special class	of ARX models		
	(c) The output does not depe	nd on previous outputs.	(d) The impulse response is	s constant.		
	7. We would like to know the unknown probability θ that a phone does NOT break when it is dropped. We assume that the phone thrown onto the ground either breaks or has no damage. In an experiment we have dropped 100 smartphones and obtained 23 broken smartphones. What is the negative log likelihood function $f(\theta)$ that we need to minimize in order to obtain the maximum likelihood (ML) estimate of θ ?					
	(a) $-77\log\theta - 23\log(1-\theta)$))	(b) $\log(23\theta) + \log(77(1 - \theta))$	- <i>θ</i>))		
	(c) $-\log(77\theta) - \log(23(1 - 1)))$	- θ))	(d) $23 \log \theta + 77 \log(1 - \theta)$	9)		
	Given a set of measurements $y_N =$ of the following minimisation prob measurements? $\hat{\theta}(N+1) = \arg m$	lems is solved at each iteration step	e linear model $y_N = \Phi \theta$, where Φ o of the RLS algorithm to estimate	$\hat{\Phi} = [\varphi(1), \varphi(2), \dots, \varphi(N)]^T$, which the parameter $\hat{\theta}(N+1)$ after $N+1$		
	(a) $\ \theta - \hat{\theta}(N)\ _{Q_N}^2 + \ y(N +$	$1) - \varphi(N+1)^\top \theta \ _2^2$	(b) $\ y_N - \Phi_N \cdot \theta \ _{Q_N}^2$			
	(c) $\ \theta - \hat{\theta}(N)\ _2^2 + \ y(N+1)\ _2^2$	$) - \varphi(N+1)^{\top} \theta \ _{Q_N}^2$	(d) $\ y_{N+1} - \Phi_{N+1} \cdot \theta \ _2^2$			

- 9. Assume you solved a nonlinear parameter estimation problem using the Gauss-Newton algorithm. How can you check that your assumptions on the noise distribution are correct?
- 10. For a discrete time LTI system $x_{k+1} = Ax_k + Bu_k, k = 0, 1$, explicitly state the forward simulation map $f_{sim} : \mathbb{R}^{n_x + 2n_u} \to \mathbb{R}^{3n_x}, (x_0, u_0, u_1) \mapsto (x_0, x_1, x_2).$

	Given the probability density function $p_X(x) = \theta x \exp(-\theta x)$, with parameter θ , and a set of independent measurements $y_N = [y(1), y(2), \dots, y(N)]^T$, which minimisation problem you need to solve to get a ML-estimate of θ ? The problem is: $\min_{\theta} \dots$?				
	(a) $N \log(\theta) + \theta \sum_{k} y(k)$		(b) $\left[-\log\left(\sum_{k} \ \theta y(k) \exp(-\theta y(k))\ _{2}^{2}\right) \right]$		
	(c) $\sum_{k} \ \theta y(k) \exp(-\theta y(k))\ _2^2$		(d) $\Box -N\log(\theta) + \theta \sum_{k} y(k)$		
12.		stion, what is a lower bound on the c ix is defined as $M = \int_{yN} \nabla_{\theta}^2 L(\theta_0,$	vovariance for any unbiased estimato $y_N \cdot p(y_N \theta_0) dy_N.$	$r \hat{ heta}(y_N)$, assuming that $ heta_0$ is the true	
	(a) $\square N/\theta_0^2$		(b) $\prod \int_{y_N} N\theta_0^{N-2} \left(\sum_k y(k)\right) \exp\left[-\theta_0 \sum_k y_k\right] dy_N$		
	(c) $\square \theta_0^2/N$		(d) $\int_{y_N} N\theta_0^{N-2} \exp[-\theta_0 \sum$	$_{k}y_{k}]\mathrm{d}y_{N}$	
13.	Which of the following dynamic mo	odels with inputs $u(t)$ and outputs y	(t) is NEITHER linear NOR affine		
	(a) $\boxed{\dot{y}(t) = u(t) + t}$	(b) $\Box t\dot{y}(t) = u(t) + \sqrt{2\pi}$	(c) $\boxed{\dot{y}(t) + \cos(t) = u(t)}$	(d) $\[\dot{y}(t) = \sqrt{t \cdot u(t)}\]$	
14.		by nature a nonlinear system and ca rithms should you use to estimate th	In be modeled by $y(t) = \theta_1 \cos(\theta_2 t)$ be parameters θ ?	$(+ \theta_3)$, where $y(t)$ are the measure-	
	(a) Linear Least Squares (LLS	5)	(b) Recursive Least Squares ((RLS)	
	(c) Weighted Least Squares (WLS)	(d) Maximum a Posteriori Es	timation (MAP)	
15.	of the following algorithms could		pproximated by a model that is linear in the parameters (LIP). Which θ of this linear model without running into memory problems or high data?		
	(a) RLS	(b) ML	(c) MAP	(d) LLS	
		with $\epsilon_N \sim \mathcal{N}(0, \Sigma_{\epsilon}), Q_N = \Phi_N^{\top} \Phi$	covariance of the estimate compute b_N and $L(\theta, y_N)$ is the negative log		
	(a) $\square Q_N^{-1}$	(b) $\Phi_N^+ \Sigma_{\epsilon_N} \Phi_N^+^\top$	(c) $\left[\left(\Phi_N^\top \Sigma_{\epsilon_N}^{-1} \Phi_N \right)^{-1} \right]$	(d) $\Box \nabla^2_{\theta} L(\theta, y_N)$	
17.	Which of the following models is ti	me invariant?			
	(a) $\qquad \dot{y}(t) = \sqrt{u(t)} + 1$	(b) $\qquad \ddot{y}(t)^2 = u(t)^t + e^{u(t)}$	(c) $t \cdot \ddot{y}(t) = u(t)^3$	(d) $\[\dot{y}(t) = -3u(t) + t^2 \]$	
18.	With which of the following form (y_1, \ldots, y_N) given θ ? $p(y_N \theta) \neq .$		conditional joint distribution of N	independent measurements $y_N =$	
	(a) $\int p(y_N x_N,\theta)p(x_N)\mathrm{d}x_N$		(b) $\prod_{i} p(y(i) \theta)$		
	(c) $\Box \exp\left(\sum_{i=0}^{N} \ln(p(y(i) \theta))\right)$		$ (\mathbf{d}) \prod \int p(y_N \theta) p(\theta) \mathbf{d}\theta $		
19.	Which of the following statements a	about Maximum A Posteriori (MAP) estimation is NOT true		
	(a) MAP is a generalization of	f ML.	(b) MAP assumes a linear model.		
	(c) The MAP estimator is bia	sed.	$ (\mathbf{d}) \qquad \hat{\theta}_{\mathrm{MAP}} = \arg \min_{\theta \in \mathbb{R}} \left[-\log(p(y_N \theta)) - \log(p(\theta)) \right] $		
20.	Which of the following models with	in input $u(k)$ and output $y(k)$ is NO	Γ linear-in-the-parameters w.r.t. $\theta \in$	\mathbb{R}^2 ?	
	(a) $y(k) = y(k-1)\theta_1 + \sqrt{\theta_2 u(k)}$		(b) $y(k) = \exp(y(k-1)) \cdot (\theta_1 + \theta_2 u(k))$		
	(c) $y(k) = \theta_1 u(k)^2 + \theta_2 \exp(k)$	p(u(k))	(d) $\qquad y(k) = \theta_1 \sqrt{u(k)}$		

Points on page (max. 11)

Prof. Dr. Moritz Diehl, IMTEK, Universität Freiburg December 19, 2018, 14:10-14:55, Freiburg

Su	rname: Fi	rst Name:	Matriculation number:		
Su	bject: Program	me: Bachelor Master Lehr	ramt others Signature:		
1.	Please fill in your name above and tick exactly ONE box for the right answer of each question below. 1. Which of the following models is time invariant?				
	(a) $\Box t \cdot \ddot{y}(t) = u(t)^3$	(b) $\[\dot{y}(t) = -3u(t) + t^2 \]$	(c) $\[\dot{y}(t) = \sqrt{u(t)} + 1 \]$	(d) $\qquad \ddot{y}(t)^2 = u(t)^t + e^{u(t)}$	
2.	With which of the following form (y_1, \ldots, y_N) given θ ? $p(y_N \theta) \neq 0$	-	conditional joint distribution of N	independent measurements $y_N =$	
	(a) $\Box \exp\left(\sum_{i=0}^{N} \ln(p(y(i) \theta))\right)$)	(b) $\prod \int p(y_N \theta)p(\theta)\mathrm{d}\theta$		
	(c) $\prod_i p(y(i) \theta)$		(d) $\int p(y_N x_N,\theta)p(x_N)dx_N$	V	
3.	Which of the following statements	about Maximum A Posteriori (MAP) estimation is NOT true		
	(a) MAP is a generalization o	f ML.	(b) The MAP estimator is bia	ised.	
	(c) MAP assumes a linear mo	odel.	(d) $\hat{\theta}_{MAP} = \arg \min_{\theta \in \mathbb{R}} [-1]$	$\log(p(y_N \theta)) - \log(p(\theta))]$	
		by nature a nonlinear system and ca rithms should you use to estimate th	an be modeled by $y(t) = \theta_1 \cos(\theta_2 t)$ ne parameters θ ?	$(+ \theta_3)$, where $y(t)$ are the measure-	
	(a) Maximum a Posteriori Est	imation (MAP)	(b) Linear Least Squares (LL	S)	
	(c) Recursive Least Squares	(RLS)	(d) Weighted Least Squares (WLS)	
	of the following algorithms could		pproximated by a model that is lin θ of this linear model without runr data?		
	(a) LLS	(b) MAP	(c) RLS	(d) ML	
	You are asked to give a computation model is given as $y_N = \Phi_N \theta + \epsilon_N$ matrix can be approximated by $\Sigma_{\hat{\theta}}$	with $\epsilon_N \sim \mathcal{N}(0, \Sigma_{\epsilon}), Q_N = \Phi_N^\top \Phi_N$	covariance of the estimate compute Φ_N and $L(\theta, y_N)$ is the negative log	d in the previous question $\Sigma_{\hat{\theta}}$. The likelihood function. The covariance	
	(a) $\square Q_N^{-1}$	(b) $\square \Phi_N^+ \Sigma_{\epsilon_N} \Phi_N^+^\top$	(c) $\Box \nabla^2_{\theta} L(\theta, y_N)$	(d) $\left[\left(\Phi_N^\top \Sigma_{\epsilon_N}^{-1} \Phi_N \right)^{-1} \right]$	
	7. We would like to know the unknown probability θ that a phone does NOT break when it is dropped. We assume that the phone thrown onto the ground either breaks or has no damage. In an experiment we have dropped 100 smartphones and obtained 23 broken smartphones. What is the negative log likelihood function $f(\theta)$ that we need to minimize in order to obtain the maximum likelihood (ML) estimate of θ ?			23 broken smartphones. What is the	
	(a) $23 \log \theta + 77 \log(1-\theta)$		(b) $\log(23\theta) + \log(77(1-\theta))$)))	
	(c) $-\log(77\theta) - \log(23(1 - 1)))$	- <i>θ</i>))	(d) $-77 \log \theta - 23 \log(1 - \theta)$	9)	
	8. Given a set of measurements $y_N = [y(1), y(2), \dots, y(N)]^T$ from the linear model $y_N = \Phi\theta$, where $\Phi = [\varphi(1), \varphi(2), \dots, \varphi(N)]^T$, which of the following minimisation problems is solved at each iteration step of the RLS algorithm to estimate the parameter $\hat{\theta}(N+1)$ after $N+1$ measurements? $\hat{\theta}(N+1) = \arg \min_{\alpha} \frac{1}{2} (\dots)$ [a and c are correct!]				
	(a) $\ y_{N+1} - \Phi_{N+1} \cdot \theta\ _2^2$		(b) $\ y_N - \Phi_N \cdot \theta\ _{Q_N}^2$		
	(c) $\ \theta - \hat{\theta}(N)\ _{Q_N}^2 + \ y(N + \theta)\ _{Q_N}^2$	$1) - \varphi(N+1)^\top \theta \ _2^2$	(d) $\ \theta - \hat{\theta}(N) \ _{2}^{2} + \ y(N+1) \ _{2}^{2}$	$ - \varphi(N+1)^{\top} \theta \ _{Q_N}^2 $	

9. Which of the following dynamic models with inputs u(t) and outputs y(t) is **NEITHER** linear **NOR** affine.

	(a) $\Box t\dot{y}(t) = u(t) + \sqrt{2\pi}$	(b) $\qquad \dot{y}(t) + \cos(t) = u(t)$	(c) $\[\dot{y}(t) = u(t) + t \]$	(d) $\boxed{\dot{y}(t)} = \sqrt{t \cdot u(t)}$
10. Which of the following models with input $u(k)$ and output $y(k)$ is NOT linear-in-the-parameters w.r.t. $\theta \in \mathbb{R}^2$?				\mathbb{R}^2 ?
	(a) $y(k) = y(k-1)\theta_1 + \sqrt{\theta_2 u(k)}$		(b) $\qquad y(k) = \theta_1 \sqrt{u(k)}$	
	(c) $y(k) = \exp(y(k-1)) \cdot ($	$(heta_1 + heta_2 u(k))$	(d) $[] y(k) = \theta_1 u(k)^2 + \theta_2 \exp(u(k))$	
11.	Which of the following model equat	tions describes a FIR system with in	put u and output y ? $y(k+1) = \dots$	
	(a) $u(k) - \sqrt{\pi}u(k-2)$	(b) $u(k+1) + y(k)$	(c) $u(k) \cdot y(k)$	(d) $u(k) + \sin(k \cdot \pi)^2$
12.	12. Let $\theta_{\rm R}$ denote the <i>regularized</i> LLS estimator using L_2 regularization. Which of the following is NOT true?			
	(a) $\square \theta_{\rm R}$ can be computed analytically.		(b) $\square \theta_{\rm R}$ is biased.	
	(c) $\theta_{\rm R}$ incorporates prior know	wledge about θ .	(d) $\square \theta_{\rm R}$ is asymptotically biase	ed.
13.	3. We use the Gauss-Newton (GN) algorithm to solve a nonlinear estimation problem. Which of the following statements is NOT true <i>in gen</i>			statements is NOT true in general?
	(a) The idea of GN is to linearize the residual function.		(b) The inverse of the GN Hessian approximates Σ_{θ} .	
	(c) GN uses a Hessian approximation.		(d) GN finds the global minimizer of the objective function.	
14.	. Which one of the following statements is NOT true for FIR models:			
	(a) They are a special class of ARX models		(b) Output error minimization is a convex problem.	
	(c) The output does not depend on previous outputs.		(d) The impulse response is constant.	

15. For a discrete time LTI system $x_{k+1} = Ax_k + Bu_k$, k = 0, 1, explicitly state the forward simulation map $f_{sim} : \mathbb{R}^{n_x + 2n_u} \to \mathbb{R}^{3n_x}$, $(x_0, u_0, u_1) \mapsto (x_0, x_1, x_2)$.

16. You suspect your data to contain some outliers, thus you would use ... estimation which assumes your measurement errors follow a ... distribution.

	(a) $\Box L_2$, Gaussian	(b) $\Box L_1$, Laplace	(c) $\Box L_2$, Laplace	(d) $\Box L_1$, Gaussian
17.	Please identify the most general sys	stem equation that still is a Auto Reg	ressive Model with Exogenous Inpu	ts (ARX).
	(a) $y(k) = b_0 u(k) + \ldots + b_{n_b} u(k - n_b)$		(b) $y(k) = -a_1 y(k-1) - \dots - a_{n_a} y(k-n_a)$	
	(c) $a_0y(k) + a_1y(k-1) + \ldots + a_{n_a}y(k-1) $	$(k - n_a) = b_0 u(k) + \ldots + b_{n_b} u(k - n_b)$	(d) $a_0y(k) + a_1y^2(k-1) + \ldots + a_{n_a}y^{n_a}$	$a_{a+1}(k - n_a) = b_0 u(k) + \ldots + b_0 u(k - n_b)$

18. Given the probability density function $p_X(x) = \theta x \exp(-\theta x)$, with parameter θ , and a set of independent measurements $y_N = [y(1), y(2), \dots, y(N)]^T$, which minimisation problem you need to solve to get a ML-estimate of θ ? The problem is: $\min_{\theta} \dots$?

(a) $\Box - \log \left(\sum_{k} \ \theta y(k) \exp(-\theta y(k)) \ _{2}^{2} \right)$	(b) $\square N \log(\theta) + \theta \sum_{k} y(k)$
(c) $\sum_{k} \ \theta y(k) \exp(-\theta y(k))\ _2^2$	(d) $\square -N\log(\theta) + \theta \sum_{k} y(k)$

19. For the problem in the previous question, what is a lower bound on the covariance for any unbiased estimator $\hat{\theta}(y_N)$, assuming that θ_0 is the true value? The Fisher information matrix is defined as $M = \int_{y_N} \nabla_{\theta}^2 L(\theta_0, y_N) \cdot p(y_N | \theta_0) dy_N$.

(a) $\prod \int_{y_N} N\theta_0^{N-2} \exp[-\theta_0 \sum_k y_k] \mathrm{d}y_N$	(b) $\square \theta_0^2/N$	
(c) $\square N/\theta_0^2$	(d) $\prod \int_{y_N} N\theta_0^{N-2} \left(\sum_k y(k)\right) \exp[-\theta_0 \sum_k y_k] dy_N$	

20. Assume you solved a nonlinear parameter estimation problem using the Gauss-Newton algorithm. How can you check that your assumptions on the noise distribution are correct?

Surname:		rst Name:	Matriculation number:	
Subject: Programme: Bachelor Master Lehram		ramt others Signatu	re:	
	Please fill in your name above and	tick exactly ONE box for the right	answer of each question below.	
1.	For a discrete time LTI system x_{k+1}	$A_1 = Ax_k + Bu_k, k = 0, 1,$ explicit	ly state the forward simulation ma	р
	$f_{\rm sim}: \mathbb{R}^{n_x+2n_u} \to \mathbb{R}^{3n_x}, (x_0, u_0, u_0)$	$u_1)\mapsto (x_0,x_1,x_2).$		
2.	Please identify the most general sys	tem equation that still is a Auto Re	gressive Model with Exogenous In	puts (ARX).
	(a) $y(k) = b_0 u(k) + \ldots + b_{n_b} u$	$(k - n_b)$	(b) $a_0y(k) + a_1y^2(k-1) + \ldots + a_k$	$a_{n_a}y^{n_a+1}(k-n_a) = b_0u(k) + \ldots + b_0u(k-n_b)$
	(c) $y(k) = -a_1y(k-1) - \dots$	$-a_{n_a}y(k-n_a)$	(d) $a_0y(k) + a_1y(k-1) + \ldots + a_n$	$a_{a}y(k-n_{a}) = b_{0}u(k) + \ldots + b_{n_{b}}u(k-n_{b})$
3.	Let $\theta_{\rm R}$ denote the <i>regularized</i> LLS	estimator using L_2 regularization.	Which of the following is NOT tru	e?
	(a) $\theta_{\rm R}$ is asymptotically biase	d.	(b) $\square \theta_{\rm R}$ incorporates prior k	nowledge about θ .
	(c) $\theta_{\rm R}$ is biased.		(d) $\theta_{\rm R}$ can be computed an	alytically.
4.	We use the Gauss-Newton (GN) alg	orithm to solve a nonlinear estimat	ion problem. Which of the followi	ng statements is NOT true <i>in general</i> ?
(a) GN finds the global minimizer of the objective function. (b) GN uses a Hessian approximation.			roximation.	
	(c) The inverse of the GN He	ssian approximates Σ_{θ} .	(d) The idea of GN is to lin	hearize the residual function.
5.	Which of the following models is ti	me invariant?		
	(a) $\qquad \dot{y}(t) = -3u(t) + t^2$	(b) $\Box t \cdot \ddot{y}(t) = u(t)^3$	(c) $\dot{y}(t) = \sqrt{u(t)} + 1$	(d) $\qquad \ddot{y}(t)^2 = u(t)^t + e^{u(t)}$
6.	With which of the following form (y_1, \ldots, y_N) given θ ? $p(y_N \theta) \neq \ldots$		the conditional joint distribution of N independent measurements $y_N =$	
	(a) $\int p(y_N \theta)p(\theta)d\theta$		(b) $\prod_{i} p(y(i) \theta)$	
	(c) $\Box \exp\left(\sum_{i=0}^{N} \ln(p(y(i) \theta))\right)$		(d) $\int p(y_N x_N,\theta)p(x_N)dt$	x_N
7.	Which of the following statements a	about Maximum A Posteriori (MAF	P) estimation is NOT true	
	(a) The MAP estimator is bias	sed.	(b) $\widehat{\theta}_{MAP} = \arg \min_{\theta \in \mathbb{R}} [-\log(p(y_N \theta)) - \log(p(\theta))]$	
	(c) MAP assumes a linear mo	r model. (d) MAP is a generalization of ML.		n of ML.
8.	Which of the following model equa	tions describes a FIR system with in	nput u and output y ? $y(k+1) =$	
	(a) $u(k) + \sin(k \cdot \pi)^2$	(b) $u(k+1) + y(k)$	(c) $u(k) - \sqrt{\pi}u(k-2)$	(d) $\Box u(k) \cdot y(k)$
9.	Which of the following dynamic mo	odels with inputs $u(t)$ and outputs y	y(t) is NEITHER linear NOR affi	ne.
	(a) $t\dot{y}(t) = u(t) + \sqrt{2\pi}$	(b) $\begin{tabular}{ll} \dot{y}(t) = \sqrt{t \cdot u(t)} \end{tabular}$	(c) $\[\dot{y}(t) + \cos(t) = u(t)\]$	(d) $\qquad \dot{y}(t) = u(t) + t$
10.	10. We would like to know the unknown probability θ that a phone does NOT break when it is dropped. We assume that the phone thrown ground either breaks or has no damage. In an experiment we have dropped 100 smartphones and obtained 23 broken smartphones. W negative log likelihood function $f(\theta)$ that we need to minimize in order to obtain the maximum likelihood (ML) estimate of θ ?		d 23 broken smartphones. What is the	
	$(-)$ 771- -0 921- $-\pi/1$ 0	\	(1) (2)	0)

(a) $-77\log\theta - 23\log(1-\theta)$	(b) $23 \log \theta + 77 \log(1-\theta)$
(c) $-\log(77\theta) - \log(23(1-\theta))$	(d) $\log(23\theta) + \log(77(1-\theta))$

Points on page (max. 0)

11. Given the probability density function $p_X(x) = \theta x \exp(-\theta x)$, with parameter θ , and a set of independent measurements $y_N = [y(1), y(2), \dots, y(N)]^T$, which minimisation problem you need to solve to get a ML-estimate of θ ? The problem is: $\min \dots$?

(a) $-\log\left(\sum_{k} \ \theta y(k) \exp(-\theta y(k))\ _{2}^{2}\right)$	(b) $\sum_{k} \ \theta y(k) \exp(-\theta y(k))\ _2^2$
(c) $N \log(\theta) + \theta \sum_{k} y(k)$	(d) $\Box -N\log(\theta) + \theta \sum_{k} y(k)$

12. For the problem in the previous question, what is a lower bound on the covariance for any unbiased estimator $\hat{\theta}(y_N)$, assuming that θ_0 is the true value? The Fisher information matrix is defined as $M = \int_{uN} \nabla_{\theta}^2 L(\theta_0, y_N) \cdot p(y_N | \theta_0) dy_N$.

(a) $\prod \int_{y_N} N\theta_0^{N-2} \left(\sum_k y(k)\right) \exp[-\theta_0 \sum_k y_k] dy_N$	(b) $\square N/\theta_0^2$	
(c) $\square \theta_0^2/N$	(d) $\prod \int_{y_N} N\theta_0^{N-2} \exp[-\theta_0 \sum_k y_k] \mathrm{d}y_N$	

13. Given a set of measurements $y_N = [y(1), y(2), \dots, y(N)]^T$ from the linear model $y_N = \Phi \theta$, where $\Phi = [\varphi(1), \varphi(2), \dots, \varphi(N)]^T$, which of the following minimisation problems is solved at each iteration step of the RLS algorithm to estimate the parameter $\hat{\theta}(N+1)$ after N+1 measurements? $\hat{\theta}(N+1) = \arg \min_{\theta} \frac{1}{2} (\dots)$ [a and c are correct!]

(a) $\ \theta - \hat{\theta}(N)\ _2^2 + \ y(N+1) - \varphi(N+1)^\top \theta\ _{Q_N}^2$	(b) $\qquad \ y_{N+1} - \Phi_{N+1} \cdot \theta\ _2^2$
(c) $ \ \theta - \hat{\theta}(N) \ _{Q_N}^2 + \ y(N+1) - \varphi(N+1)^\top \theta \ _2^2 $	(d) $\qquad \ y_N - \Phi_N \cdot \theta\ _{Q_N}^2$

14. Which one of the following statements is NOT true for FIR models:

(a) They are a special class of ARX models	(b) The impulse response is constant.	
(c) The output does not depend on previous outputs.	(d) Output error minimization is a convex problem.	

15. You are given a pendulum which is by nature a nonlinear system and can be modeled by $y(t) = \theta_1 \cos(\theta_2 t + \theta_3)$, where y(t) are the measurements. Which of the following algorithms should you use to estimate the parameters θ ?

(a) Weighted Least Squares (WLS)	(b) Recursive Least Squares (RLS)	
(c) Maximum a Posteriori Estimation (MAP)	(d) Linear Least Squares (LLS)	

16. Suppose now that the system given in the previous question can be approximated by a model that is linear in the parameters (LIP). Which of the following algorithms could you use to estimate the parameters θ of this linear model without running into memory problems or high computational costs for a continuous and infinite flow of measurement data?

(a) ML	(b) LLS	(c) MAP	(d) RLS
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17. You are asked to give a computationally efficient approximation of the covariance of the estimate computed in the previous question $\Sigma_{\hat{\theta}}$. The model is given as $y_N = \Phi_N \theta + \epsilon_N$ with $\epsilon_N \sim \mathcal{N}(0, \Sigma_{\epsilon})$, $Q_N = \Phi_N^{\top} \Phi_N$ and $L(\theta, y_N)$ is the negative log likelihood function. The covariance matrix can be approximated by $\Sigma_{\hat{\theta}} \approx \ldots$

(a) $\nabla^2_{\theta} L(\theta, y_N)$	(b) $\square Q_N^{-1}$	(c) $\left[\left(\Phi_N^\top \Sigma_{\epsilon_N}^{-1} \Phi_N \right)^{-1} \right]$	(d) $\square \Phi_N^+ \Sigma_{\epsilon_N} \Phi_N^+^\top$
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18. You suspect your data to contain some outliers, thus you would use ... estimation which assumes your measurement errors follow a ... distribution.

	(a) $\Box L_1$, Laplace	(b) $\Box L_1$, Gaussian	(c) $\Box L_2$, Laplace	(d) $\Box L_2$, Gaussian			
19.	9. Which of the following models with input $u(k)$ and output $y(k)$ is NOT linear-in-the-parameters w.r.t. $\theta \in \mathbb{R}^2$?						
	(a) $\qquad y(k) = \theta_1 \sqrt{u(k)}$		(b) $y(k) = y(k-1)\theta_1 + \sqrt{\theta_2 u(k)}$				
	(c) $y(k) = \theta_1 u(k)^2 + \theta_2 \exp(u(k))$		(d) $y(k) = \exp(y(k-1)) \cdot (\theta_1 + \theta_2 u(k))$				

20. Assume you solved a nonlinear parameter estimation problem using the Gauss-Newton algorithm. How can you check that your assumptions on the noise distribution are correct?