Exercises for Course on State-Space Control Systems (SSC)
Albert-Ludwigs-Universität Freiburg – Summer Term 2019

Exercise 4: Eigenvalue Assignment

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Exercises

1. Consider the predator-prey model introduced in Example 5.16 of *Feedback Systems*. The dynamics for the system are given by

$$\frac{\mathrm{d}H}{\mathrm{d}t} = (r+u)H\left(1+\frac{H}{k}\right) - \frac{aHL}{c+H}$$
$$\frac{\mathrm{d}L}{\mathrm{d}t} = +\frac{aHL}{c+H} - dL$$

We choose the following nominal parameters for the system

$$a = 3.2,\ b = 0.6,\ c = 50,\ d = 0.56,\ k = 125,\ r = 1.6.$$

We take the parameter r, corresponding to the growth rate for hares, as the input to the system, which we might modulate by controlling a food source for the hares. This is reflected in our model by the term $(r+u)$ in the first equation. We choose the number of lynxes L as the output of our system. To control this system, we first linearize the system around the equilibrium point (H_e, L_e), which can be determined numerically to be $x_e \approx (20.6, 29.5)$. This yields a linear dynamical system

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} 0.13 & -0.93 \\ 0.57 & 0 \end{bmatrix}\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} 17.2 \\ 0 \end{bmatrix} u, \quad y = \begin{bmatrix} 0 & 1 \end{bmatrix}\begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$$

where $z_1 = H - H_e$ and $z_2 = L - L_e$. We introduce the feedback law,

$$u = -Kz + k_f L_r$$

where L_r is the desired number of lynxes.

- Compute the characteristic polynomial of the closed-loop system.
- How do you have to choose K for the closed-loop system to have the eigenvalues $\lambda_1 = -0.1$, $\lambda_2 = -0.2$.
- Check your results by solving the Ackerman equation. This can be done also with Matlab using `acker` or `place`.
- How do you have to choose k_f to obtain the desired output?
- Simulate the closed-loop system for multiple initial conditions.

2. Consider the system

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u, \qquad y = \begin{bmatrix} 1 & 0 \end{bmatrix} x,$$

with the control law $u = -k_1 x_1 - k_2 x_2 + k_f r$. Show that the eigenvalues of the system cannot be assigned to arbitrary values. Is the system controllable?

3. Consider the closed-loop system described by

$$\frac{\mathrm{d}x}{\mathrm{d}t} = Ax + Bu,$$
$$y = Cx + Du,$$
$$u = -Kx + k_f r.$$

For $D \neq 0$, how do you have to choose k_f to obtain the steady-state output y_e?