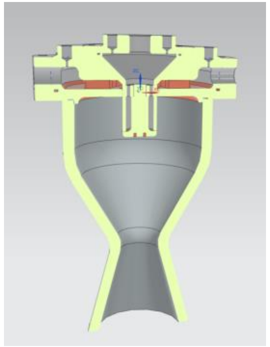
Modelling the Landing Phase of a Rocket

27.01.2025

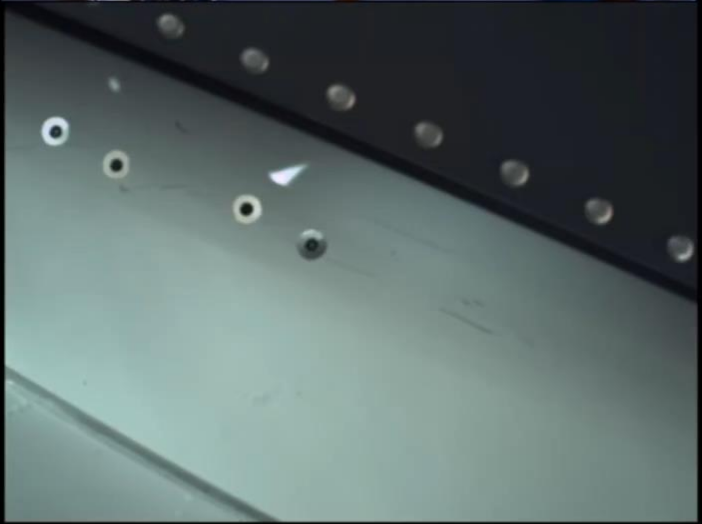
Who we are

- » Lukas Pries
- » PhD in Agile Flight Control
- » Autonomous Aerial Systems

- » Felix Ebert
- » PhD in Rocket Engine Control
- » ASCENT Project Lead



https://www.youtube.com/watch?v=ODY6JWzS8WU&t=732s



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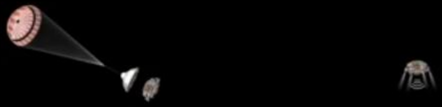


Today's Goal

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x})$$

Structure

1. Why do we need models?
   "and which models do we need to land a rocket?"

2. Translational Dynamics
   "and how to use them for position control?"

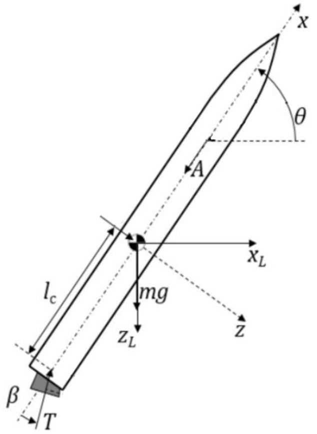
3. Rotational Dynamics
   "and how to stabilize the rockets attitude?"

4. Actuator Dynamics
   "or how to generate the desired thrust?"

Translational Dynamics

Point-mass assumption: $m \in \mathbb{R}, \; r, v \in \mathbb{R}^3$

Newton's Law: $m \, \dot{v} = \sum F$



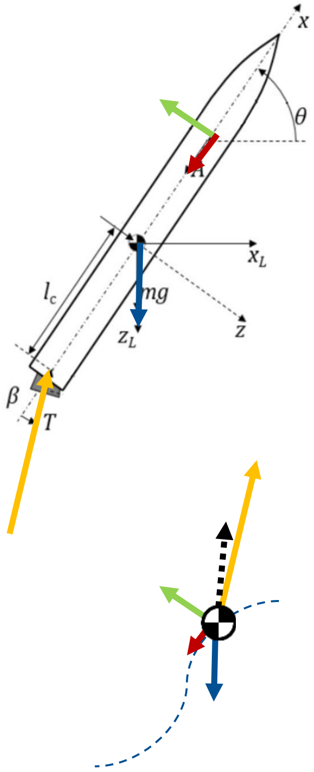
Translational Dynamics

Point-mass assumption: $m \in \mathbb{R},\ r, v \in \mathbb{R}^3$

Newton's Law: $m\,\dot{v} = \sum F$

4 main forces: Weight, Thrust, Lift, Drag

$$\dot{r} = v$$
$$m\dot{v} = F_{\mathrm{g}} + F_{\mathrm{T}} + F_{\mathrm{D}} + F_{\mathrm{L}}$$
$$- mge_3 \quad + T \quad + \frac{1}{2} C_{\mathrm{D}}\, \rho\, v\, ||v|| A_{\mathrm{D}} \quad + \frac{1}{2} C_{\mathrm{L}}\, \rho\, n ||v||^2 A_{\mathrm{L}}$$



Translational Dynamics

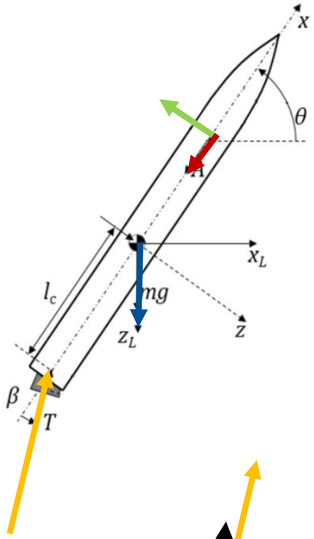
Point-mass assumption: $m \in \mathbb{R},\ r, v \in \mathbb{R}^3$

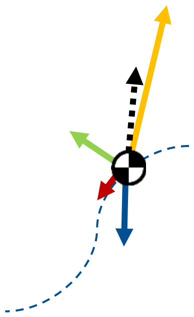
Newton's Law: $m\,\dot{v} = \sum F$

4 main forces: Weight, Thrust, Lift, Drag

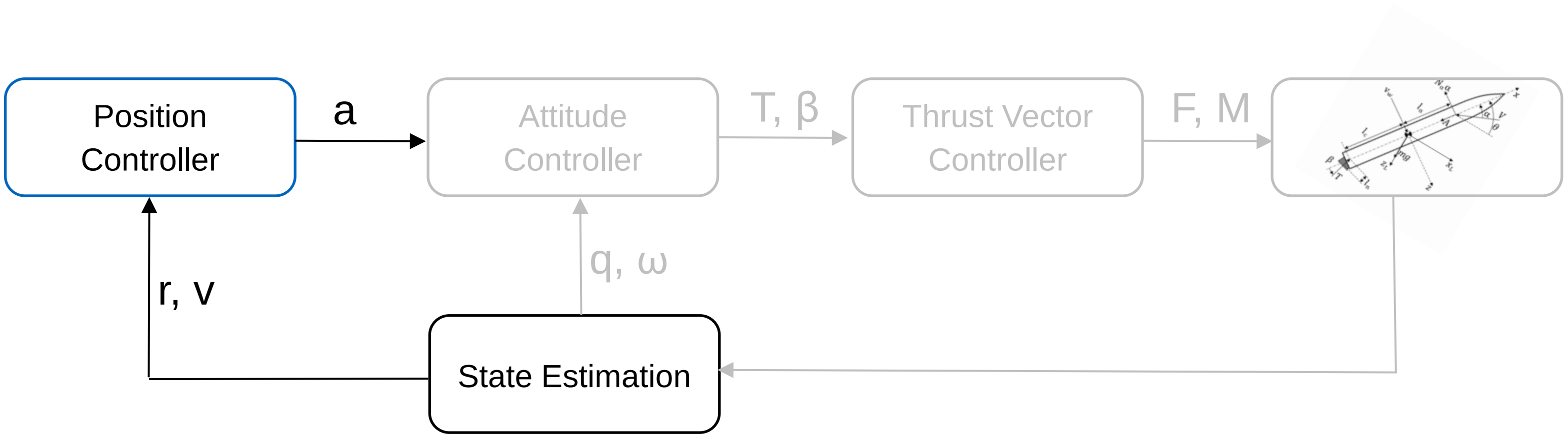
$$\begin{aligned} \dot{r} &= v \\ m\dot{v} &= F_{\mathrm{g}} \quad + F_{\mathrm{T}} \quad + F_{\mathrm{D}} \quad + F_{\mathrm{L}} \\ &\quad - mge_3 \quad + T \quad + \frac{1}{2} C_{\mathrm{D}}\, \rho\, v\, ||v|| A_{\mathrm{D}} \quad + \frac{1}{2} C_{\mathrm{L}}\, \rho\, n ||v||^2 A_{\mathrm{L}} \end{aligned}$$

Changing mass: $\dot{m} = -c||T||$





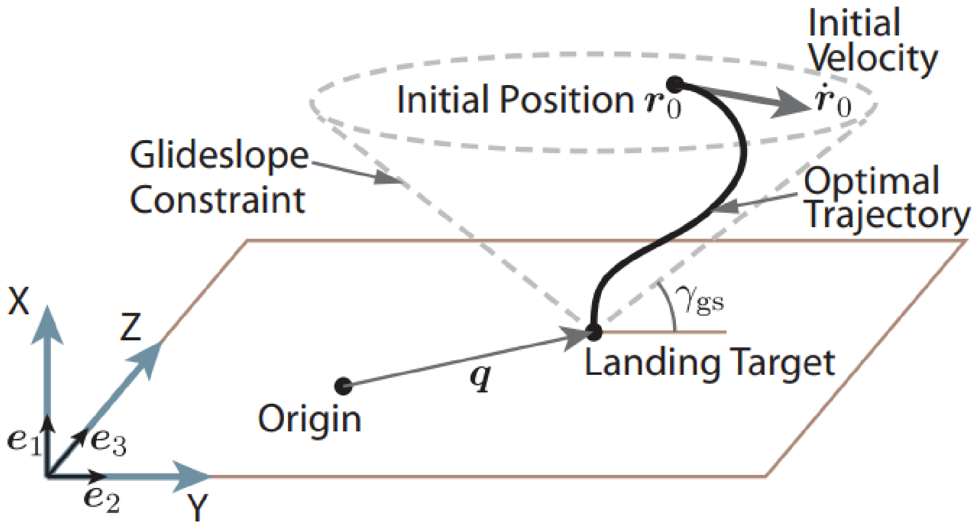
Flight Control Stack



Translational dynamics -> position control, trajectory planning, model-based optimal control

Optimal Control for Rocket Landing

- Precise Landing Position
- Minimum-fuel
- Actuator Constraints



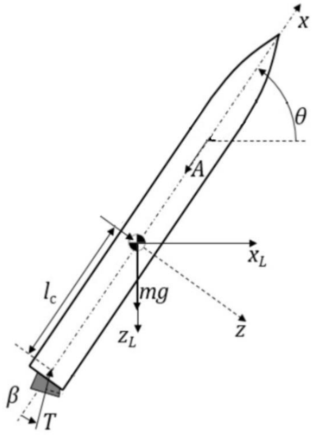
Açıkmeşe, Carson & Blackmore (2013)

Rotational Dynamics

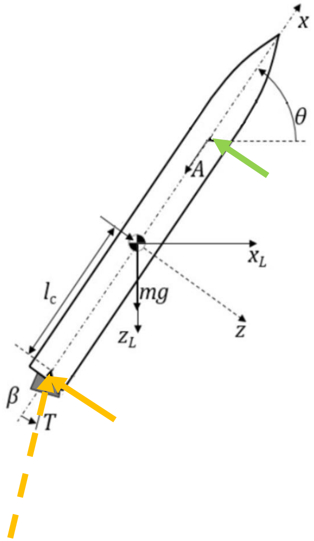
Rigid Body Assumption: $I \in \mathbb{R}^{3x3},\ R \in SO(3),\ \omega \in \mathbb{R}^3$

Euler's Rotational Equation:

$$I\dot{\omega} = M + \omega \times I\omega$$



Rotational Dynamics

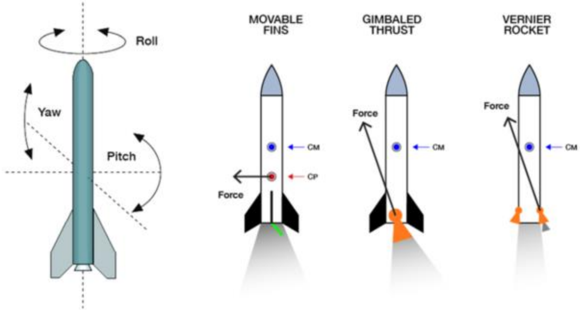


Rigid Body Assumption: $I \in \mathbb{R}^{3x3},\ R \in SO(3),\ \omega \in \mathbb{R}^3$

Euler's Rotational Equation:

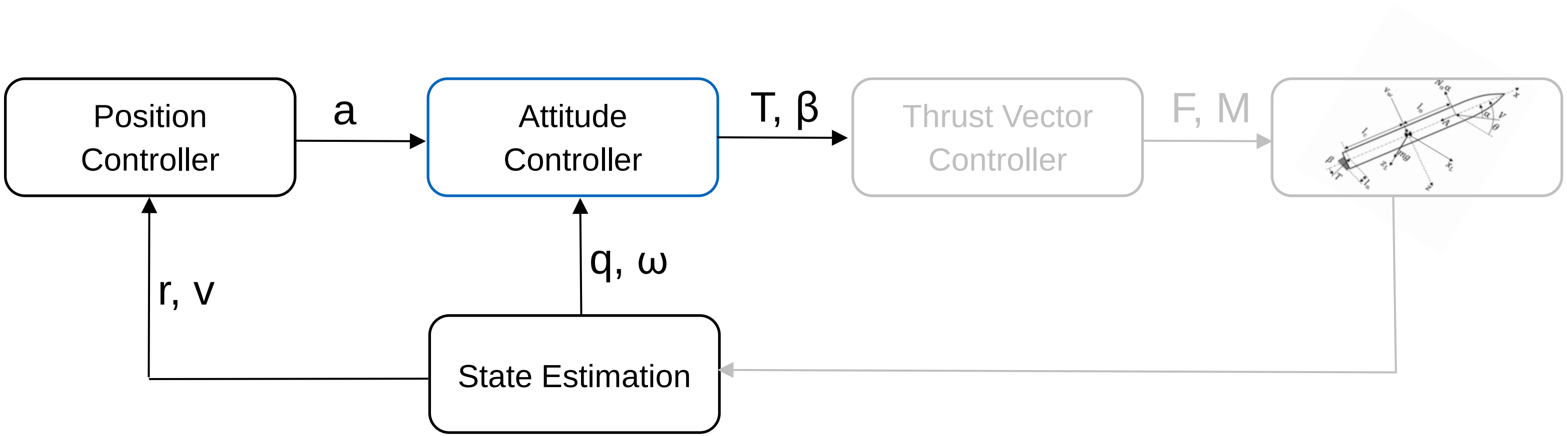
$$I\dot{\omega} = M + \omega \times I\omega$$

$$M = M_{\text{Aero}} + M_{\text{TVC}} + M_{\text{fins}} + M_{\text{thrusters}}$$



https://www.youtube.com/watch?v=ShLC6NDI4Oo

Flight Control Stack



Coupled Dynamics

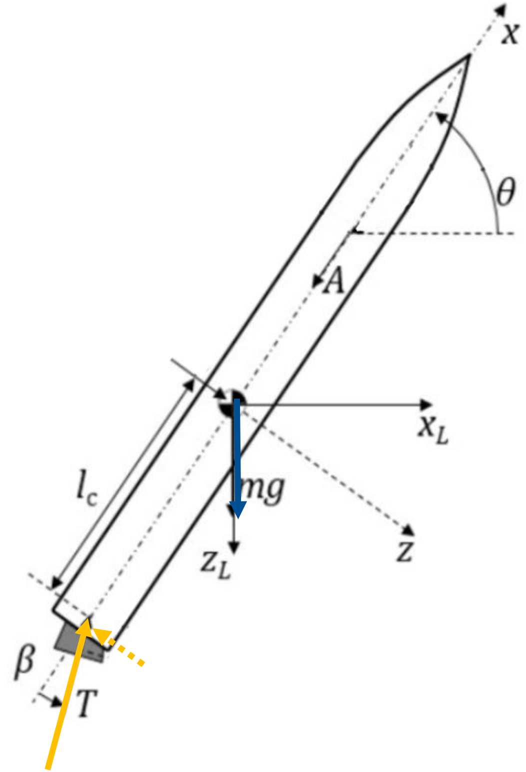
$$\dot{r} = v$$

$$\dot{v} = \begin{bmatrix} 0 \\ -g \end{bmatrix} + R(\theta)\frac{||T||}{m}\begin{bmatrix} \cos\beta \\ \sin\beta \end{bmatrix}$$

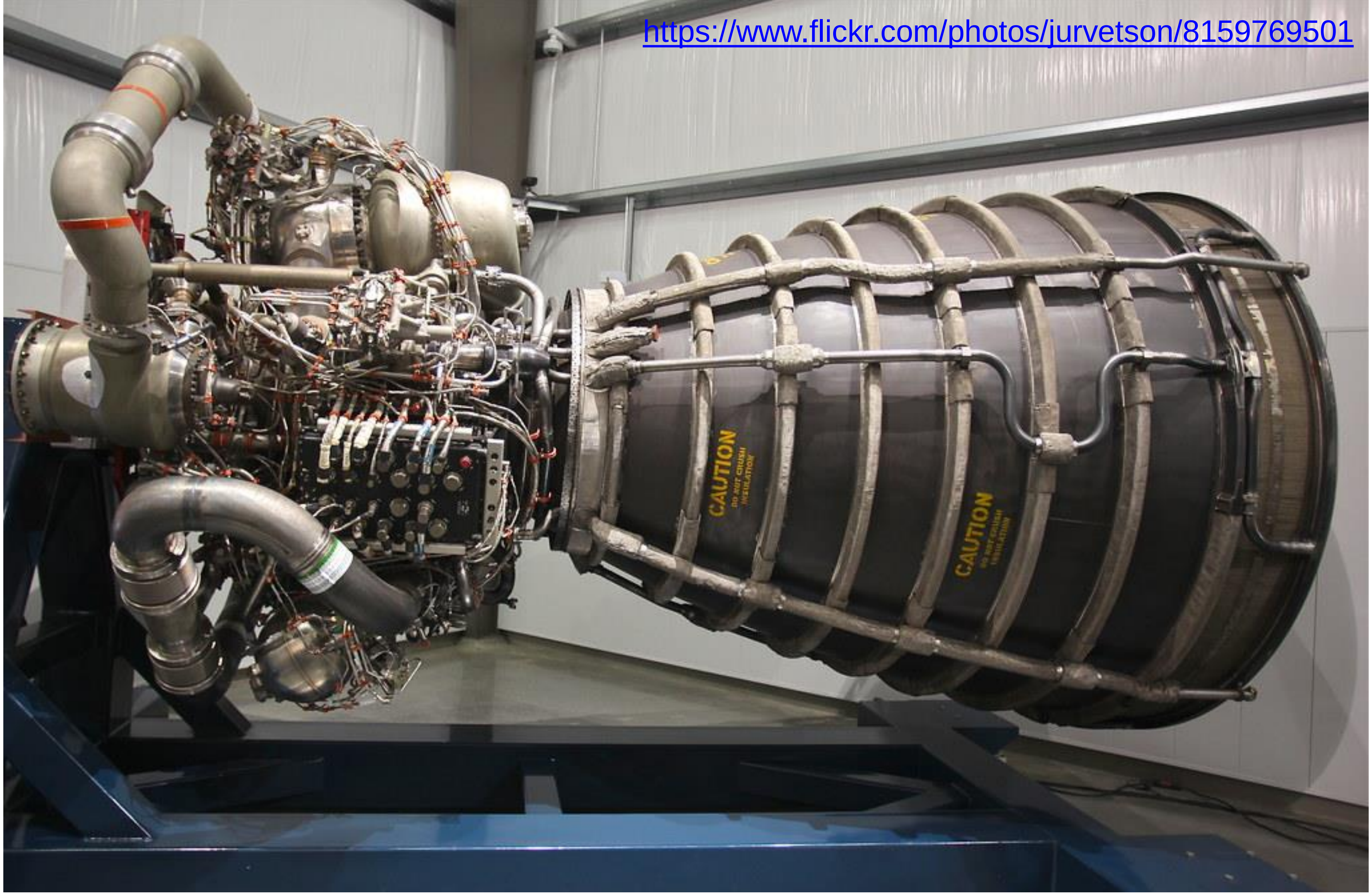
$$\dot{\theta} = \omega$$

$$\dot{\omega} = \frac{1}{I} l_c ||T|| \sin\beta$$

$$\dot{m} = -c||T||$$

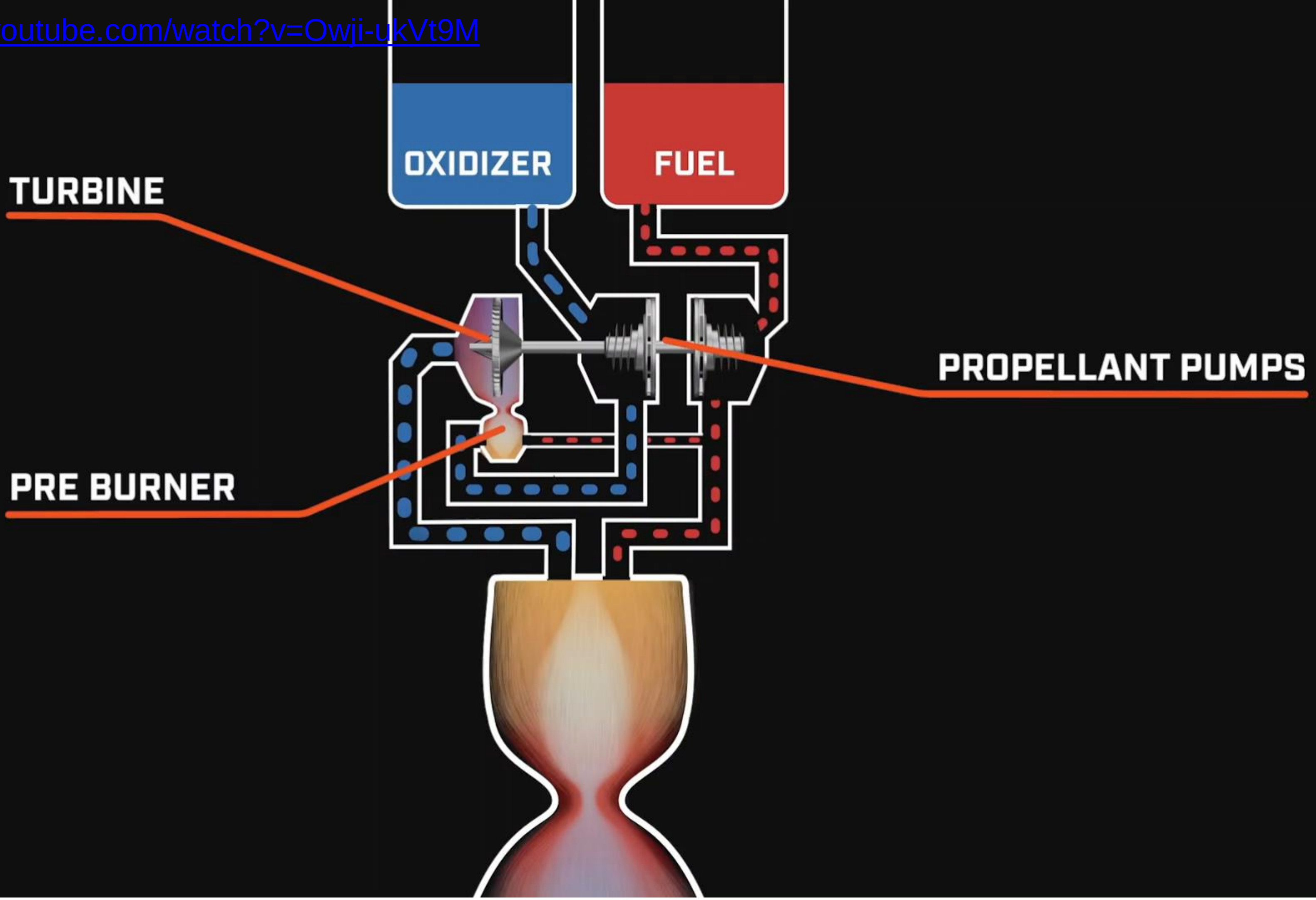


https://www.flickr.com/photos/jurvetson/8159769501



https://www.youtube.com/watch?v=ZkVU-bj9bDk

https://www.youtube.com/watch?v=Owji-ukVt9M



Physics of a Thrust Chamber

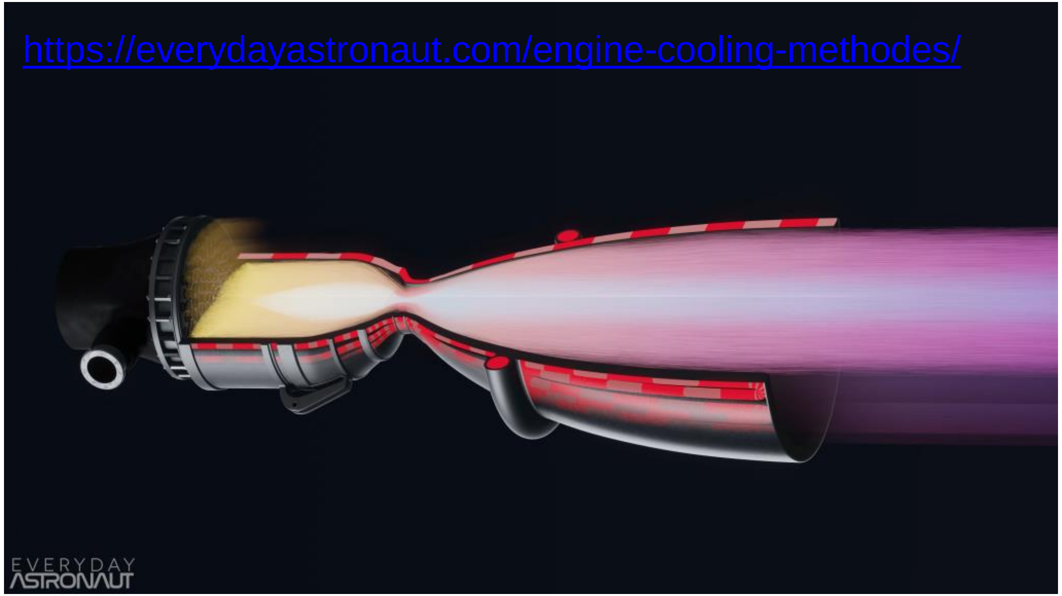
We want to know: Thrust T

$$T = \dot{m} v_e + A_e (p_e - p_{amb})$$

Kinetic Pressure

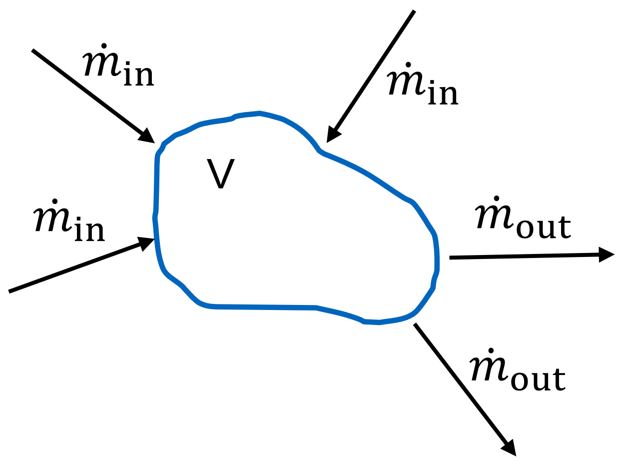
⇩

We have to model: $\dot{m}$ & v_e



General Fluid System Modelling

Capacitance

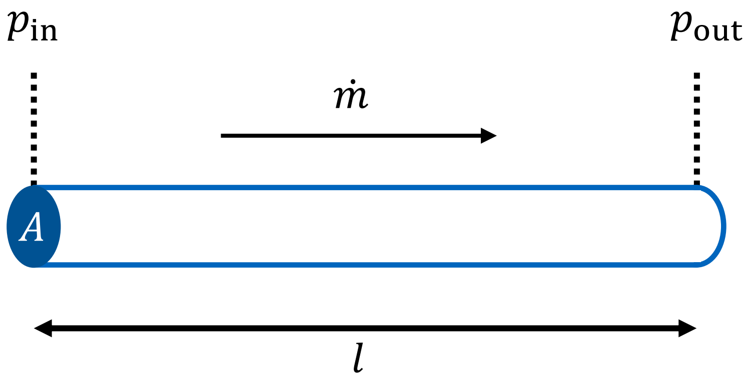


$$\dot{p} = \frac{a^2}{V}\left(\sum \dot{m}_{\text{in}} - \sum \dot{m}_{\text{out}}\right)$$

Manfletti PhD Thesis (2010)

General Fluid System Modelling

Inductance



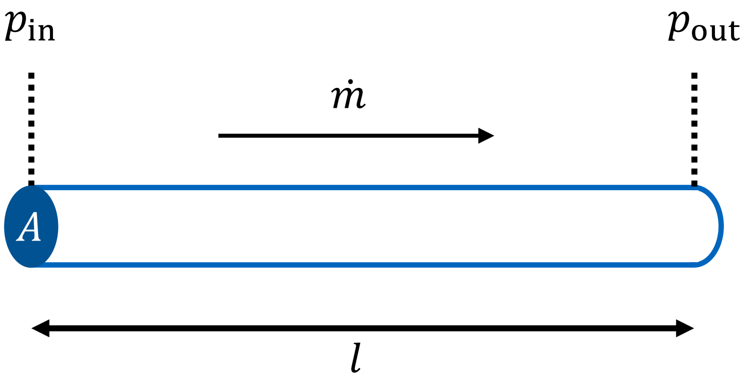
$$\dot{p} = \frac{a^2}{V}\left(\sum \dot{m}_{\text{in}} - \sum \dot{m}_{\text{out}}\right)$$

$$\frac{d\dot{m}}{dt} = \frac{A}{l}\left(p_{\text{in}} - p_{\text{out}} - \frac{\zeta}{\rho}\,\dot{m}\,|\dot{m}|\right)$$

Manfletti PhD Thesis (2010)

General Fluid System Modelling

Resistance

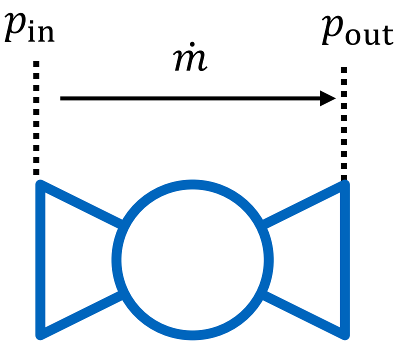


$$\frac{d\dot{m}}{dt} = \frac{A}{l}\left(p_{\text{in}} - p_{\text{out}} - \frac{\zeta}{\rho}\,\dot{m}\,|\dot{m}|\right)$$

Manfletti PhD Thesis (2010)

General Fluid System Modelling

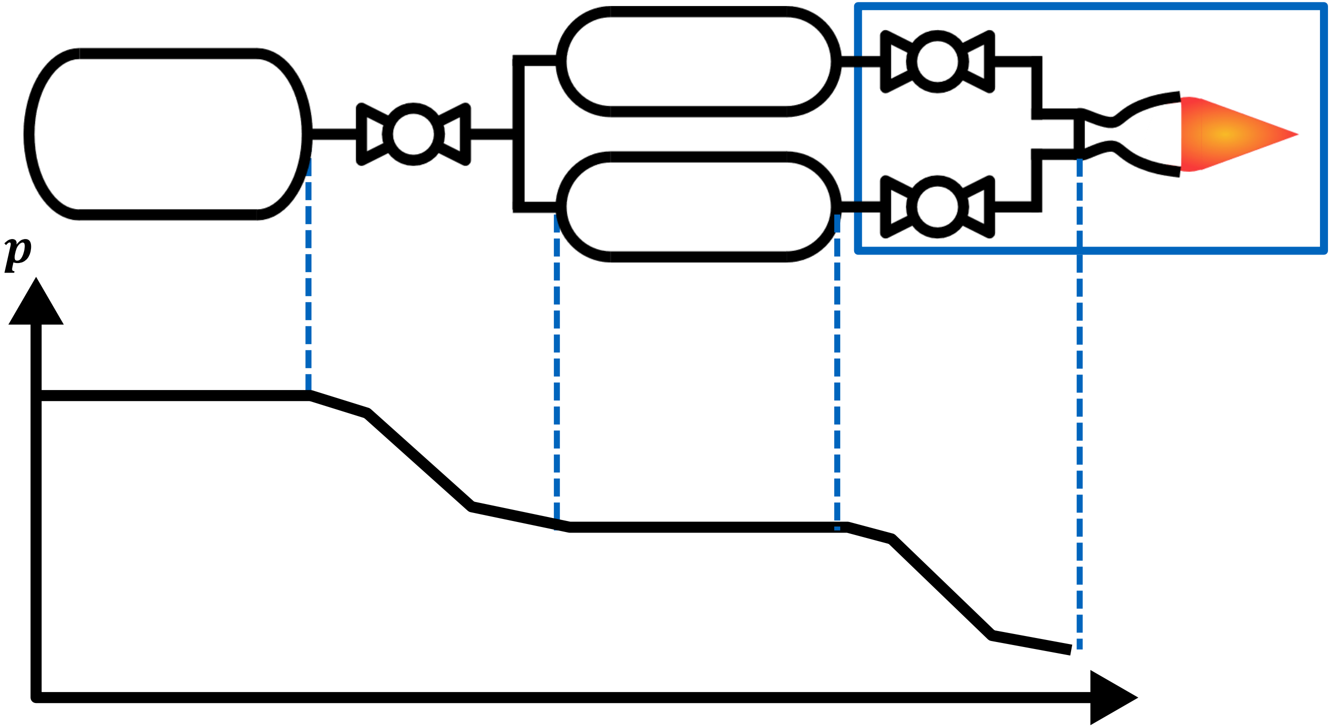
Valve – Special Formulation of Resistance



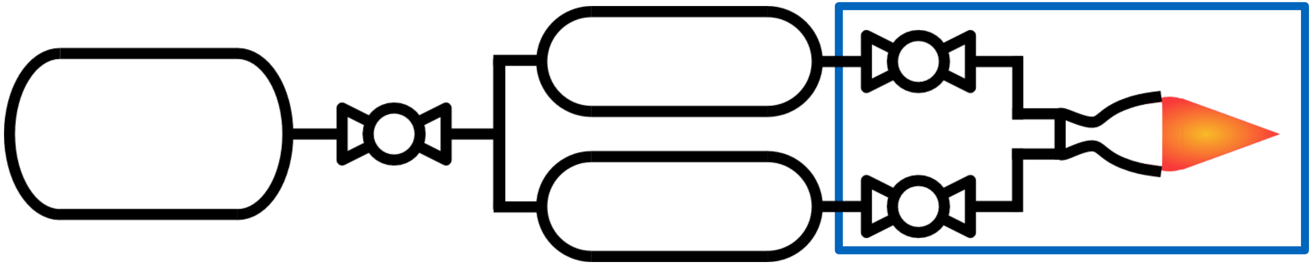
$$\dot{m} = \sqrt{\frac{|p_{\mathrm{in}} - p_{\mathrm{out}}|}{3600^2 \cdot 1e5}} \cdot \rho \cdot k_{\mathrm{v}} \cdot sign(p_{\mathrm{in}} - p_{\mathrm{out}})$$

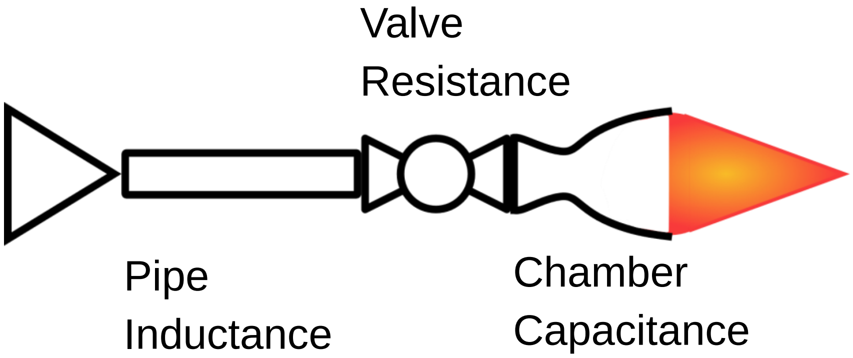
$$\dot{k_{\mathrm{v}}} = \frac{k_{\mathrm{v,set}} - k_{\mathrm{v}}}{T}$$

Hopper Propulsion System



Hopper Propulsion System

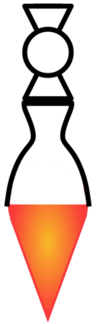




Coupled Fluid System

$$\dot{p}_{\text{chamber}} = \frac{a^2}{V}(\dot{m}_{\text{in}} - \dot{m}_{\text{nozzle}}) \qquad \dot{m}_{\text{nozzle}} = k_{\text{chamber}} \cdot p_{\text{chamber}} \qquad v_{\text{e}} = f(T_{\text{c}}, M_{\text{c}}, p_{\text{c}}, p_{\text{e}})$$

Coupled Fluid System



$$\dot{p}_{\mathrm{chamber}} = \frac{a^2}{V}\left(\dot{m}_{\mathrm{valve}} - \dot{m}_{\mathrm{nozzle}}\right) \qquad \dot{m}_{\mathrm{nozzle}} = k_{\mathrm{chamber}} \cdot p_{\mathrm{chamber}} \qquad v_e = f(T_{\mathrm{c}}, M_{\mathrm{c}}, p_{\mathrm{c}}, p_{\mathrm{e}})$$

$$\dot{k_{\mathrm{v}}} = \frac{k_{\mathrm{v,set}} - k_{\mathrm{v}}}{T} \qquad \dot{m}_{\mathrm{valve}} = \sqrt{\frac{|p_{\mathrm{in}} - p_{\mathrm{chamber}}|}{3600^2 \cdot 1e5}} \cdot \rho \cdot k_{\mathrm{v}} \cdot sign(p_{\mathrm{in}} - p_{\mathrm{chamber}})$$

Coupled Fluid System

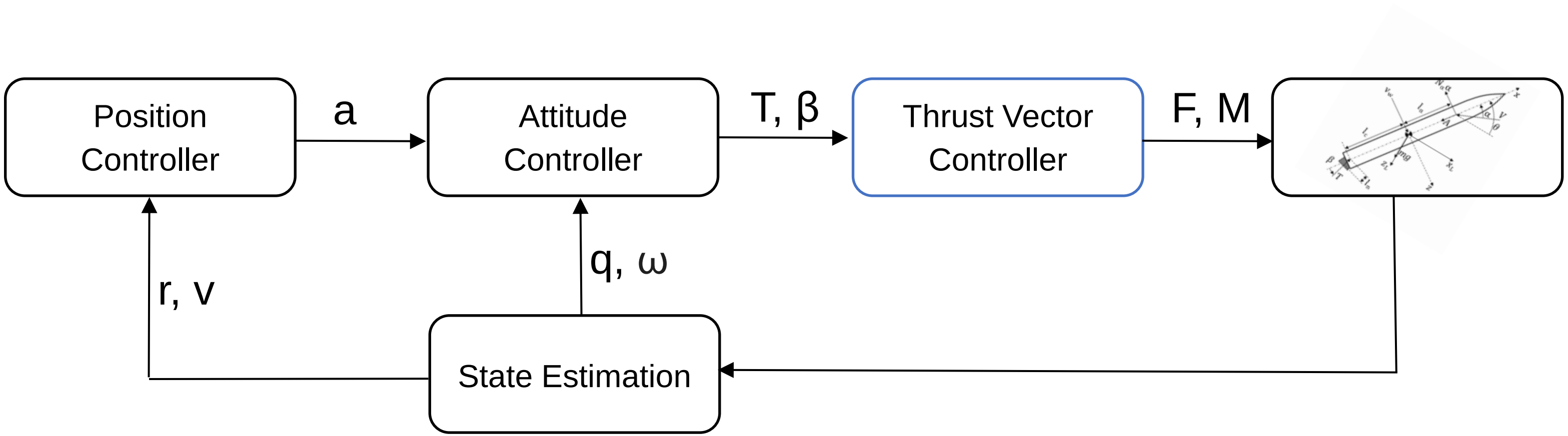
$$\dot{p}_{\text{chamber}} = \frac{a^2}{V}\left(\dot{m}_{\text{valve}} - \dot{m}_{\text{nozzle}}\right) \qquad \dot{m}_{\text{nozzle}} = k_{\text{chamber}} \cdot p_{\text{chamber}} \qquad v_e = f\left(T_{\text{c}}, M_{\text{c}}, p_{\text{c}}, p_{\text{e}}\right)$$

$$\dot{k_{\text{v}}} = \frac{k_{\text{v,set}} - k_{\text{v}}}{T} \qquad \dot{m}_{\text{valve}} = \sqrt{\frac{\left|p_{\text{pipe}} - p_{\text{chamber}}\right|}{3600^2 \cdot 1e5}} \cdot \rho \cdot k_{\text{v}} \cdot sign\left(p_{\text{pipe}} - p_{\text{chamber}}\right)$$

$$\frac{d\dot{m}_{\text{pipe}}}{dt} = \frac{A_{\text{pipe}}}{l_{\text{pipe}}}\left(p_{\text{BC}} - p_{\text{pipe}} - \frac{\zeta_{\text{pipe}}}{\rho}\,\dot{m}_{\text{pipe}}\left|\dot{m}_{\text{pipe}}\right|\right) \qquad \zeta_{\text{pipe}} = f\left(\dot{m}_{\text{pipe}}\right)$$

$$\dot{p}_{\text{pipe}} = \frac{a^2}{V}\left(\dot{m}_{\text{pipe}} - \dot{m}_{\text{valve}}\right)$$

Flight Control Stack

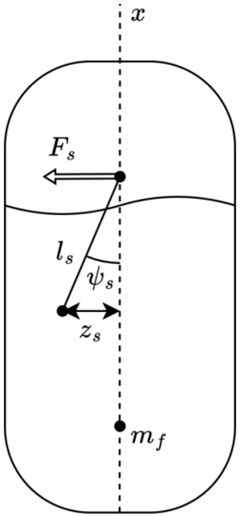


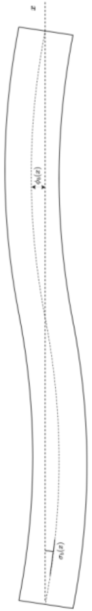
Final Set of Equations

$$\dot{x} = f(x,z) = \begin{pmatrix} \dot{x}_{\text{translational}} \\ \dot{x}_{\text{rotational}} \\ \dot{p}_{\text{pipe}} = \dfrac{a^2}{V}\left(\dot{m}_{\text{pipe}} - \dot{m}_{\text{valve}}\right) \\ \dfrac{d\dot{m}_{\text{pipe}}}{dt} = \dfrac{A_{\text{pipe}}}{l_{\text{pipe}}}\left(p_{\text{BC}} - p_{\text{pipe}} - \dfrac{\zeta_{\text{pipe}}}{\rho}\,\dot{m}_{\text{pipe}}\left|\dot{m}_{\text{pipe}}\right|\right) \\ \dot{k}_{\text{v}} = \dfrac{k_{\text{v,set}} - k_{\text{v}}}{T} \\ \dot{p}_{\text{chamber}} = \dfrac{a^2}{V}\left(\dot{m}_{\text{valve}} - \dot{m}_{\text{nozzle}}\right) \end{pmatrix}$$

$$0 = g(x,z) = \begin{pmatrix} \dot{m}_{\text{nozzle}} = k_{\text{chamber}} \cdot p_{\text{chamber}} \\ v_e = f(T_{\text{c}}, M_{\text{c}}, p_{\text{c}}, p_{\text{e}}) \\ \zeta_{\text{pipe}} = f\left(\dot{m}_{\text{pipe}}\right) \end{pmatrix}$$

Sloshing and Bending





https://www.youtube.com/watch?v=s0UYNoTPdNs

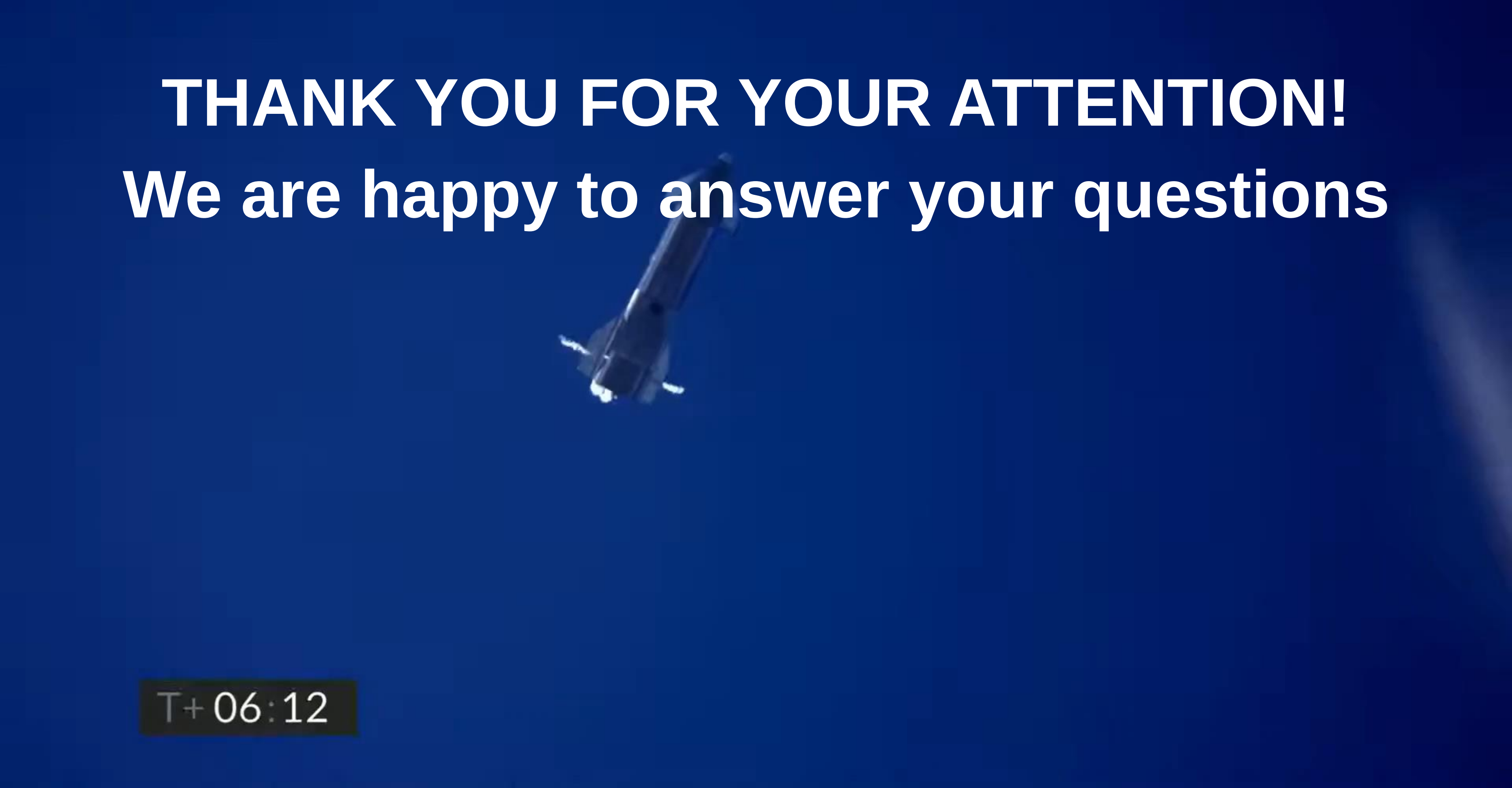
Saturn V Rocket

https://www.youtube.com/watch?v=2qUCRbVm5xg&t=135s

https://www.youtube.com/watch?v=_zZ7fIkpBgs&t=718s

THANK YOU FOR YOUR ATTENTION!

We are happy to answer your questions



Give Aways

- Optimal Control for Rocket Landing
  - *„Lossless Convexification of Nonconvex Control Bound and Pointing Constrains of the Soft Landing Optimal Control Problem“* http://www.larsblackmore.com/iee_tcst13.pdf
  - Zac Manchester Lecture https://www.youtube.com/watch?v=gwdcIxzp2N4&list=PLZnJoM76RM6KugDT9sw5zhAmqKnGeoLRa&index=33
  - High level explanation https://www.youtube.com/watch?v=Kk5J-wOYOFM&list=PLhaBM0eoTJ-k-M9-8gC8U5t1ou4Jclmvu

- Insights on Rocketry and Rocket Engines (Everyday Astronaut)
  - Rocket Engine Cycles https://www.youtube.com/watch?v=Owji-ukVt9M&list=PLWzKfs3icbT6yhDTpO1GyDlz9AXdWSiGr&index=15
  - Engine Cooling https://www.youtube.com/watch?v=he_BL6Q5u1Y&list=PLWzKfs3icbT6yhDTpO1GyDlz9AXdWSiGr&index=5

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