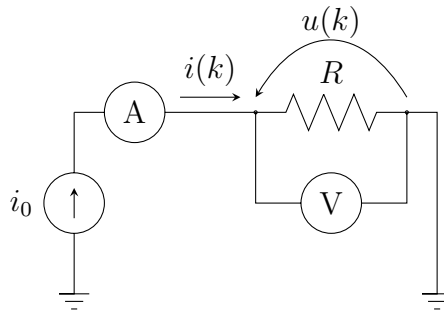# Exercise 1: Resistance Estimation Example
(to be returned before October 28th, 8:15)

Prof. Dr. Moritz Diehl, Katrin Baumgärtner, Jakob Harzer, Dan Wang,
Ashwin Karichannavar, Premnath Srinivasan

In this exercise you investigate some important facts from statistics in numerical experiments.

Exercise Tasks

1. We consider the following experimental setup:

Imagine you are sitting in a class of 200 electrical engineering students and you want to estimate the value of R using Ohm's law. Since the value of the current i_0 flowing through the resistor is not known exactly, an ammeter is used to measure the current $i(k)$ and a voltmeter to measure $u(k)$. Every student is taking 1000 measurements. The measurement number is represented by k. We assume that the measurements are noisy:

$$i(k) = i_0 + n_i(k) \qquad \text{and} \qquad u(k) = u_0 + n_u(k)$$

where $u_0 = 10\,\text{V}$ is the true values of the voltage across the resistor, $i_0 = 5\,\text{A}$ is the true value of the current flowing through the resistor and $n_i(k)$ and $n_u(k)$ are the values of the noise. Please consider the data-set with all measurements of all students provided in the repository.

Let us now investigate the behaviour of the three different estimators which were introduced in the lecture:

$$\hat{R}_{\text{SA}}(N) = \frac{1}{N} \sum_{k=1}^{N} \frac{u(k)}{i(k)} \qquad \hat{R}_{\text{LS}}(N) = \frac{\frac{1}{N} \sum_{k=1}^{N} u(k)i(k)}{\frac{1}{N} \sum_{k=1}^{n} i(k)^2} \qquad \hat{R}_{\text{EV}}(N) = \frac{\frac{1}{N} \sum_{k=1}^{N} u(k)}{\frac{1}{N} \sum_{k=1}^{N} i(k)}$$

We will write code to simulate the behavior of these estimators. For each of the three estimators, carry out the following tasks.

(a) CODE: Compute the result of the function $\hat{R}_*(N)$, for $N = 1, \ldots, N_{\max}$ using your personal measurements (student 1 or experiment 1). Do this for each estimator ($*$ can be either SA, LS or EV). Plot the three curves in one plot.
ON PAPER: Do the estimators converge for $N \to \infty$? (5 points)

(b) CODE: It is good practice to analyze the results of several experiments to cancel noise. Luckily, you get the datasets of all other students. Plot the function $\hat{R}_*(N)$, $N = 1, \ldots, N_{\max}$ for each estimator ($*$ can be either SA, LS or EV). To see the stochastic variations, plot all these functions in one graph per estimator.

ON PAPER: Do you see any difference to the plot from task (b)? (2 points)

(c) CODE: Compute the mean of $\hat{R}_*(N)$ over all experiments (all 200 students) and plot it for N from 1 to $N_{\max}$. (1 point)

(d) CODE: Plot a histogram containing all values of $\hat{R}_*(N_{\max})$.

ON PAPER: Discuss what makes the difference. (2 points)

This sheet gives in total 10 points.