

## Numerical Optimal Control of Hybrid Systems with Linear Subsystems

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There exist many classes of formalisms for hybrid dynamical systems including Piecewise Affine systems (PWA), Mixed Logical Dynamical systems (MLD), and Linear Complementarity Systems, a subclass of dynamic complementarity systems. All these systems encompass linear subsystems, and previous work has developed equivalences between these formalisms which allow for the sharing of tools between the formalisms via reformulation [1]. These systems were traditionally studied in a discrete-time setting and computations are based on linear and quadratic mixed-integer formulations [2].

The Finite Elements with Switch Detection (FESD) method is an accurate discretization method for non-smooth systems [3]. Taking a continuous-time viewpoint in PWA, MLD and extending FESD to these models, using complementarity instead of integer formulations, has the potential to lead to more computationally efficient methods for hybrid systems.

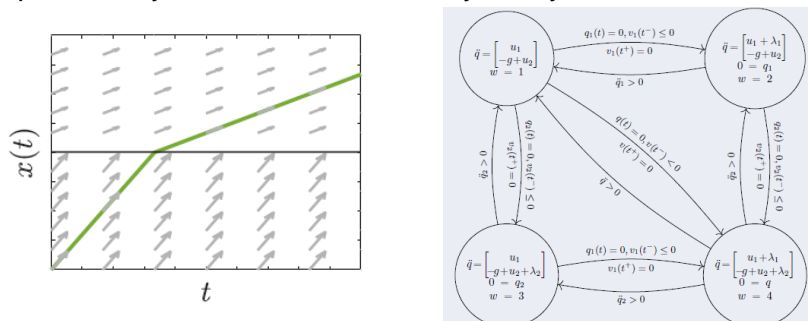


Figure 1: Examples of: a PWA system trajectory (left), hybrid automaton (right).

**Master topic:** The primary focus would be identifying classes of PWA and MLD which can be reformulated into linear complementarity systems, and developing an extension to the FESD method to handle such systems. This would be then implemented in the open-source software package *nosnoc*. Application fields for MLD should be explored and illustrated.

**Your skills:** A strong background in systems, control and optimization. A primary focus of the work will be a python implementation within the open-source tool *nosnoc/nosnoc\_py*.

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[2] Bemporad, A., & Morari, M. (1999). Control of systems integrating logic, dynamics, and constraints. *Automatica*, 35(3), 407-427.  
[3] Nurkanović, A., Sperl, M., Albrecht, S., & Diehl, M. (2024). Finite elements with switch detection for direct optimal control of nonsmooth systems. *Numerische Mathematik*, 1-48.