Modeling and System Identification – Microexam 3

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Surname: Name: Matriculation number:

Study: Programm: Bachelor [ ] Master [ ]

Please fill in your name above and tick exactly ONE box for the right answer of each question below. Each question is worth 0.5 points unless otherwise stated. There are no negative points

1. Regard a Kalman filter to estimate the state of a discrete time system $x_{k+1} = A_k x_k + w_k$ and $y_k = Cx_k + v_k$, where we assume i.i.d. process noise w and measurement noise v with variances Σ_w and Σ_v. What choice of entries in matrices Σ_w, Σ_v should we use if we have very accurate measurements of $y(k)$ and trust C, but inaccurate linear system matrix A?

(a) [ ] Pos. semi-definite matrices, small Σ_w, small Σ_v	(b) [x] diagonal matrices, large Σ_w, small Σ_v
(c) [ ] diagonal matrices, large Σ_w, large Σ_v	(d) [ ] negative semi-definite matrices, small Σ_w, large Σ_v

2. Which of the following is associated with the covariance prediction step $P_{[k|k-1]}$ of the Kalman filter, if $x_{k+1} = A_k x_k + w_k$, where w_k is i.i.d. zero mean noise with covariance W_k? $P_{[k|k-1]} = \ldots$

(a) [ ] $\left(A_k^\top \cdot P_{[k-1|k-1]} \cdot A_k + W_k\right)^{-1}$	(b) [ ] $A_{k-1} \cdot P_{[k|k]} \cdot A_{k-1}^\top + W_{k-1}$
(c) [ ] $A_{k-1}^\top \cdot P_{[k|k-1]} \cdot A_{k-1} + W_{k-1}$	(d) [x] $A_{k-1} \cdot P_{[k-1|k-1]} \cdot A_{k-1}^\top + W_{k-1}$

3. Which of the following is **NOT TRUE** about the Extended Kalman Filter (EKF)?

(a) [ ] EKF can be applied to nonlinear systems	(b) [ ] EKF is not optimal due to local linearizations
(c) [x] For a linear system, EKF outperforms KF	(d) [ ] EKF tends to underestimate the true covariance

4. Which of the following is the correct expression for finding the next iterate θ_{k+1} using Gauss Newton Algorithm? Assume $F(\theta) = \frac{1}{2}\|R(\theta)\|_2^2$.

(a) [ ] $\theta_{k+1} = \theta_k - (\frac{\partial R}{\partial \theta}(\theta_k)\frac{\partial R}{\partial \theta}(\theta_k)^{-1})^\top \nabla F(\theta_k)$	(b) [x] $\theta_{k+1} = \theta_k - (\nabla R(\theta_k)\nabla R(\theta_k)^\top)^{-1}\frac{\partial R}{\partial \theta}(\theta_k)^\top R(\theta_k)$
(c) [ ] $\theta_{k+1} = \theta_k - (\nabla R(\theta_k)^\top)^{+}\frac{\partial R}{\partial \theta}(\theta_k)F(\theta_k)$	(d) [ ] $\theta_{k+1} = \theta_k - (\nabla R(\theta_k)^\top)^{+}F(\theta_k)$

5. Which of the following is **NOT TRUE** about the Gauss Newton (GN) Algorithm when used to solve a non-linear problem?

(a) [ ] Its convergence depends on the initial guess	(b) [ ] Not suited for rank deficient Jacobian matrices
(c) [ ] If it converges, it converges to a local minimum	(d) [x] it could move away from a reached stationary point

6. Use Euler's method with step size $h = 0.1$ to find $x(1)$ for the differential equation $\dot{x} = 2 - \exp(-4t) - 2x$ at $t = 0.1$ given that $x(0) = 1$?

(a) [x] $x(0.1) = 0.9$	(b) [ ] $x(0.1) = 1.1$	(c) [ ] $x(0.1) = 1.2$	(d) [ ] $x(0.1) = 0.8$

7. What is the minimum number of hidden layers that could approximate an XOR function?

(a) [x] 1	(b) [ ] 2	(c) [ ] 0	(d) [ ] 4

Points on page (max. 7)

8. While training a network, what is a good indication of an appropriate model complexity to be used?

(a) [ ] Highest training error	(b) [ ] Lowest training error
(c) [ ] Highest testing error	(d) [x] Lowest testing error

9. We have a priori information about a parameter in form of a PDF $g(\theta)$ and know that the PDF to obtain measurements y given θ is given by $f(y,\theta)$. What is the minimization problem using Bayesián estimator?

(a) [ ] $argmin_\theta\ \log g(\theta) - \log f(y,\theta)$	(b) [x] $argmin_\theta\ -\log g(\theta) - \log f(y,\theta)$
(c) [ ] $argmin_\theta\ g(\theta) + f(y,\theta)$	(d) [ ] $argmin_\theta\ -g(\theta) + f(y,\theta)$

10. Which of the following model equations describes a FIR system with input u and output y? $y(k+1) = \dots$

(a) [ ] $u(k) + \sin(k \cdot \pi)$	(b) [x] $u(k) - 5 \cdot u(k-1)$	(c) [ ] $u(k) \cdot y(k) + y(k+1)$	(d) [ ] $u(k+1) + y(k)$

11. Given a linear state space systems with the equations $x_{k+1} = Ax_k + Bu_k$ and $y_k = Cx_k$, if you know that the state dimensions are $x_k \in \mathbb{R}^{n_x}$, the output dimensions are $y_k \in \mathbb{R}^{n_y}$ and the control input dimensions $u_k \in \mathbb{R}^{n_u}$, what are the dimensions of A and B respectively?

(a) [x] $(n_x \times n_x)$, $(n_x \times n_u)$	(b) [ ] $(1 \times n_x)$, $(n_y \times n_u)$	(c) [ ] $(n_y \times n_x)$, $(n_y \times n_u)$	(d) [ ] $(n_x \times n_x)$, $(n_y \times n_u)$

12. Which of the following dynamic models with inputs $u(t)$ and outputs $y(t)$ is **neither** linear **nor** affine.

(a) [ ] $t^3 \ddot{y}(t) = u(t)$	(b) [ ] $\ddot{y}(t) = t^3 u(t)$	(c) [ ] $\dot{y}(t) = u(t) + \cos(t)$	(d) [x] $\dot{y}(t)^3 = u(t)$

13. Which of the following models is time invariant?

(a) [ ] $\dot{y}(t) = u(t) + u(t)^t$	(b) [ ] $t \cdot \ddot{y}(t) = u(t)^3 + e^{u(t)}$	(c) [x] $\dot{y}(t) = \sqrt{u(t)} + 1$	(d) [ ] $\ddot{y}(t)^2 = u(t)^t + e^{5u(t)}$

14. Which statement is **NOT TRUE** about Moving Horizon Estimation (MHE) with horizon length N? Here KF denotes the regular Kalman Filter, EKF denotes the extended Kalman Filter.

(a) [ ] MHE can be applied to nonlinear systems.	(b) [ ] Computing the MHE estimate at time N is as expensive as computing the MHE estimate at time $2N$.
(c) [x] MHE is computationally cheaper than EKF.	(d) [ ] MHE is equiv. to KF in unconstrained linear case.

15. Consider a problem where a moving horizon estimator is used for state estimation of a non-linear system. How do the covariance and the computation time change when the horizon length increases? (1 point)

$f_k(\theta) = y(k) - \theta_1 y(k-1) + \theta_2 u(k-1), \quad c_k = \frac{1}{\sigma^2}$

16. Which one of the following statements is **NOT TRUE** for FIR models:

(a) [ ] The output does not depend on previous outputs.	(b) [ ] Output error minimization is a convex problem.
(c) [x] The impulse response is constant.	(d) [ ] They are a special class of ARX models

17. Which of the following models with input $u(k)$ and output $y(k)$ is **NOT** LIP with respect to $\theta \in \mathbb{R}^2$?

(a) [x] $y(k) = \theta_1 \exp(\theta_2 u(k))$	(b) [ ] $y(k) = y(k-1) \cdot (\theta_1 + \theta_2 u(k))$
(c) [ ] $y(k) = \theta_1 \sqrt{u(k)} + \theta_2 u(k)$	(d) [ ] $y(k) = \theta_1 u(k)^2 + \theta_2 \exp(u(k))$

18. Consider the scalar ARX model $y(k) = \theta_1 y(k-1) + \theta_2 u(k-1) + w_k$ where $w_k \sim \mathcal{N}(0, \sigma^2)$. Given measurements $y(k)$ and controls $u(k)$, $k = 0, \dots, N$, specify functions $f_k(\theta)$ and weighing factors c_k (that account for the noise variance) such that the parameter estimate $\theta^* = [\theta_1^*, \theta_2^*]^\top$ is given by $\theta^* = \arg\min_{\theta \in \mathbb{R}^2} \sum_{k=1}^{N} c_k \|f_k(\theta)\|_2^2$ (2 points)

The covariance decreases and the computation time increases with increasing horizon length.

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