Optimization of Long Trajectories of Dual-Wing AWE Systems with Many Cycles

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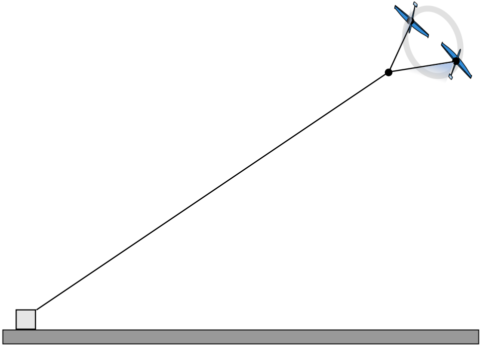
University of Freiburg

AWEC, April 26, 2024

Motivation



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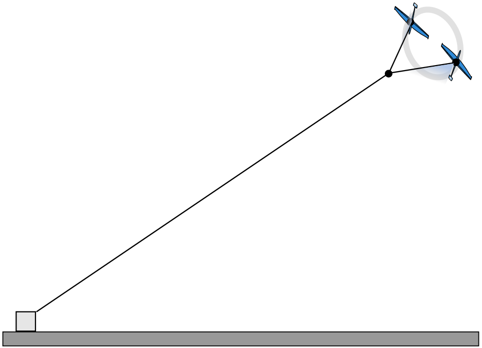

- Optimize a single pumping cycle of a dual-kite AWE system

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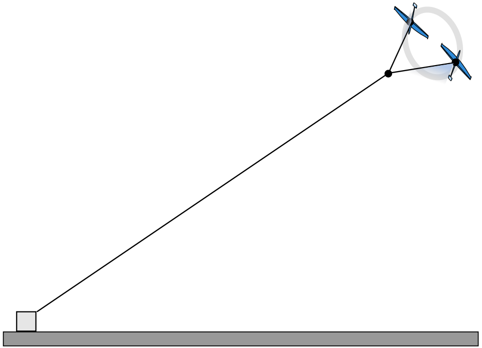

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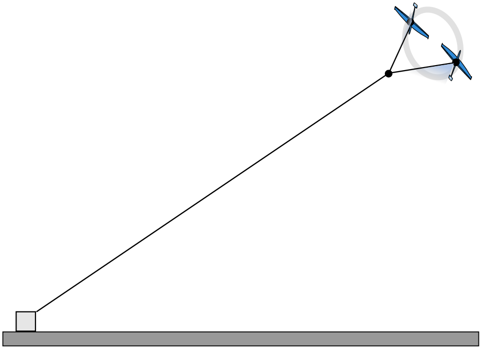

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$$\begin{aligned}
\max_{\substack{x(\cdot),u(\cdot),\\ t_\mathrm{f}}} \quad & P_\mathrm{gen}(x,u,t_\mathrm{f}) \\
\text{s.t.} \quad & \dot{x} = f(x(t),u(t)), \quad \forall t \in [0,t_\mathrm{f}], \\
& 0 \leq h(x(t),u(t)), \quad \forall t \in [0,t_\mathrm{f}]
\end{aligned}$$

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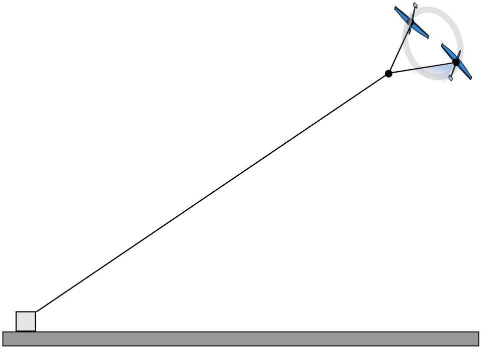

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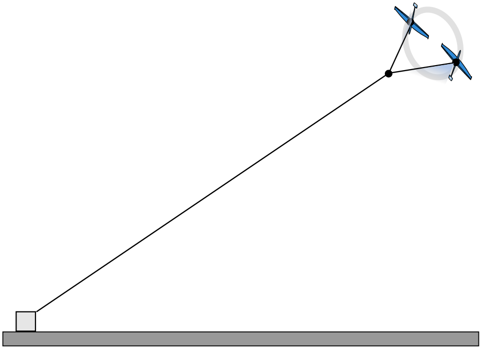

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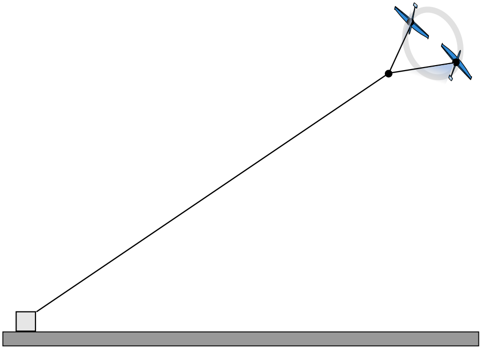

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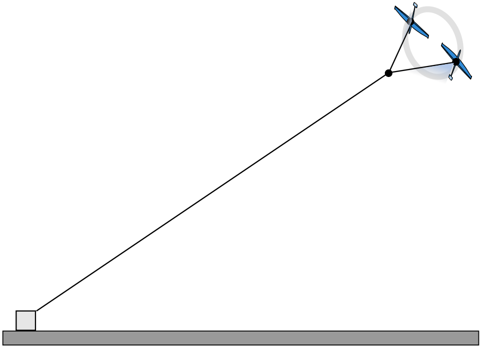
- Software Packages such as the AWEBox[2]

Model



- Dual Kite Setup, two AP2 planes

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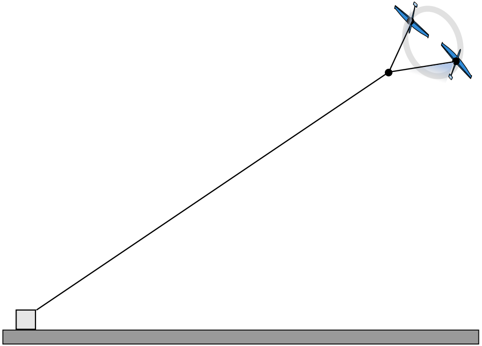

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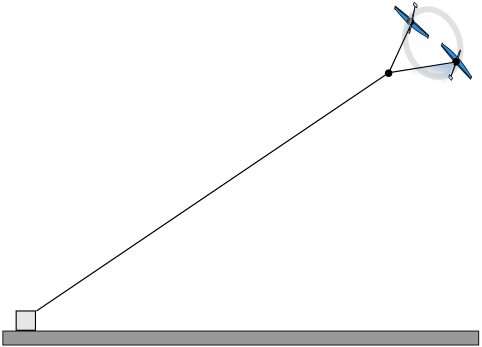

- Dual Kite Setup, two AP2 planes
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- System Dynamics (Index-1 DEA)

$$f(x, u, z, t) = 0$$

with $x \in \mathbb{R}^{51}$, $u \in \mathbb{R}^{19}$, $z \in \mathbb{R}^{3}$, based on Lagrangian dynamics

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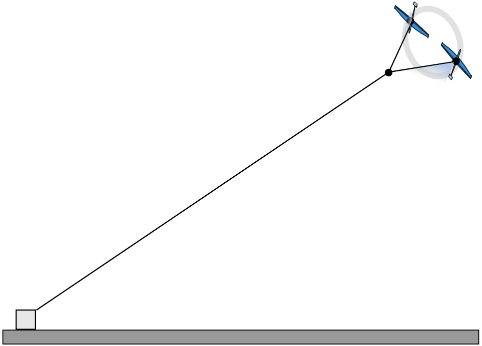

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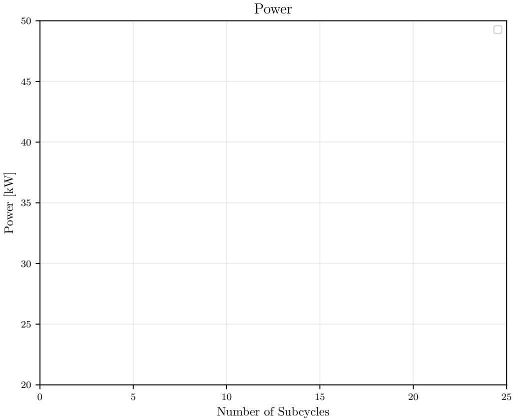
- x: 21 states per kite + 6 states connection point + 3 auxillary

Experiments



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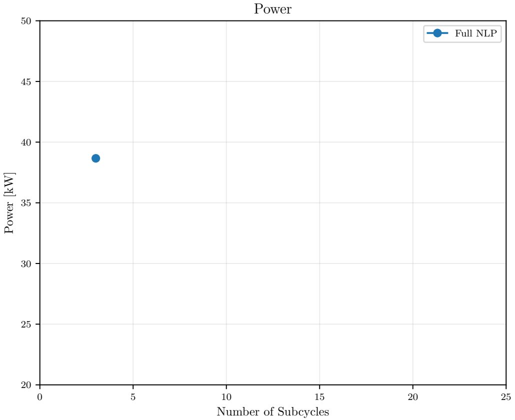

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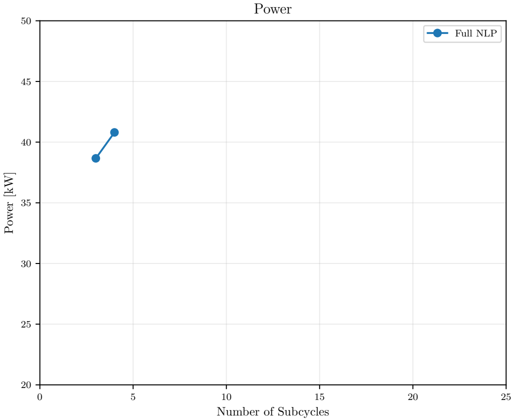

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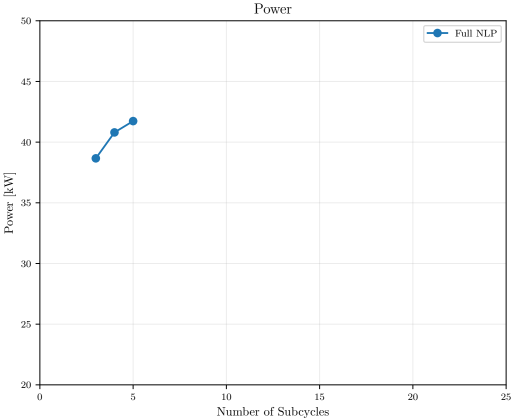

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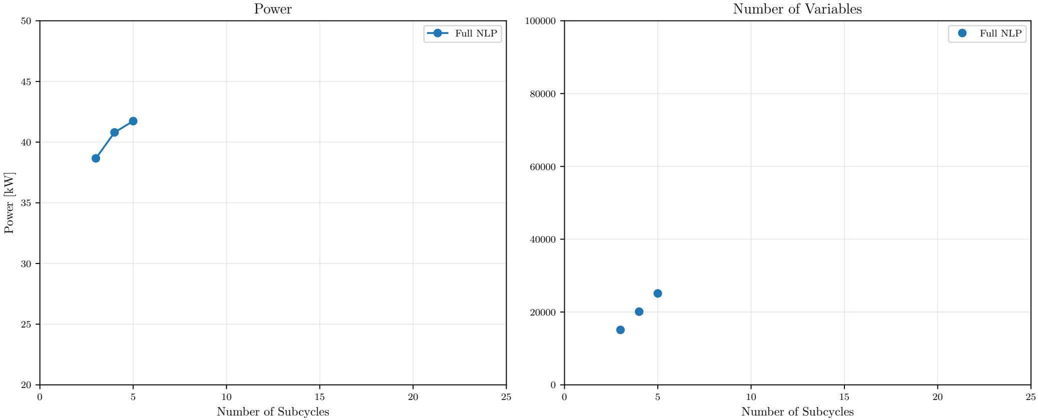

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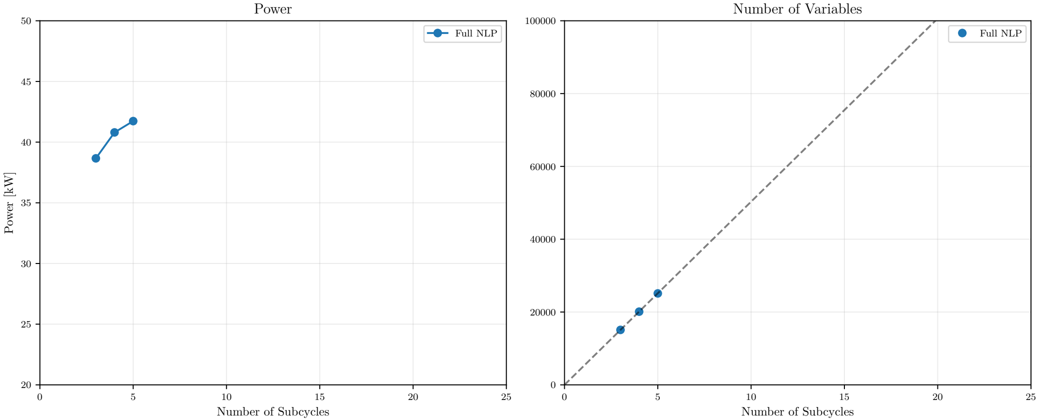

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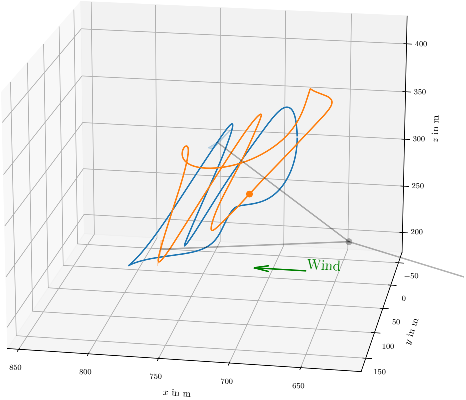

$N = 2$

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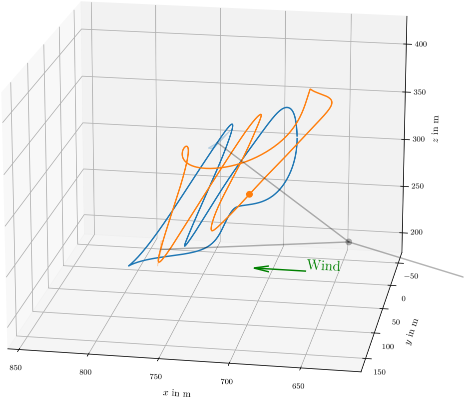

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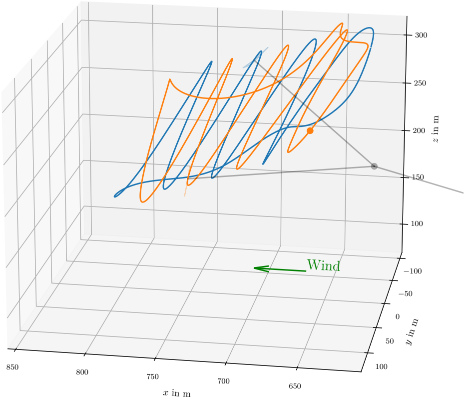

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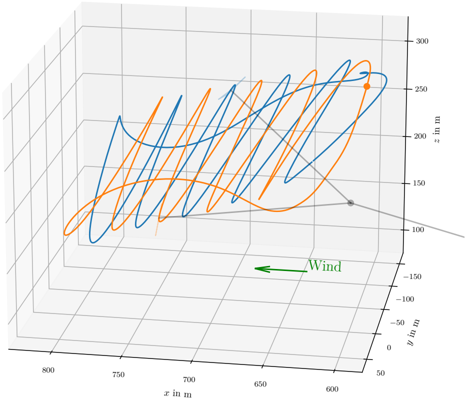

$$N = 5$$

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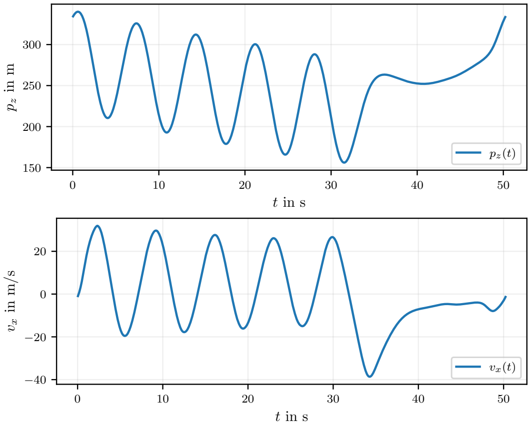

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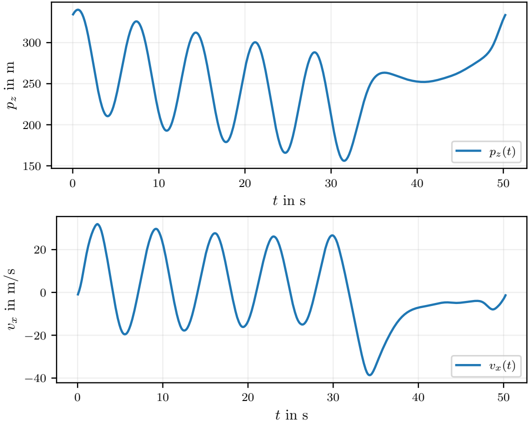

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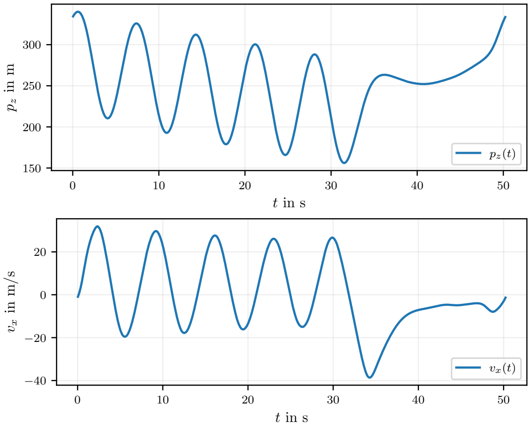

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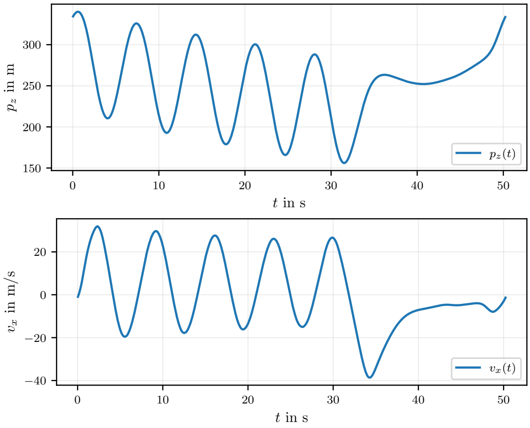

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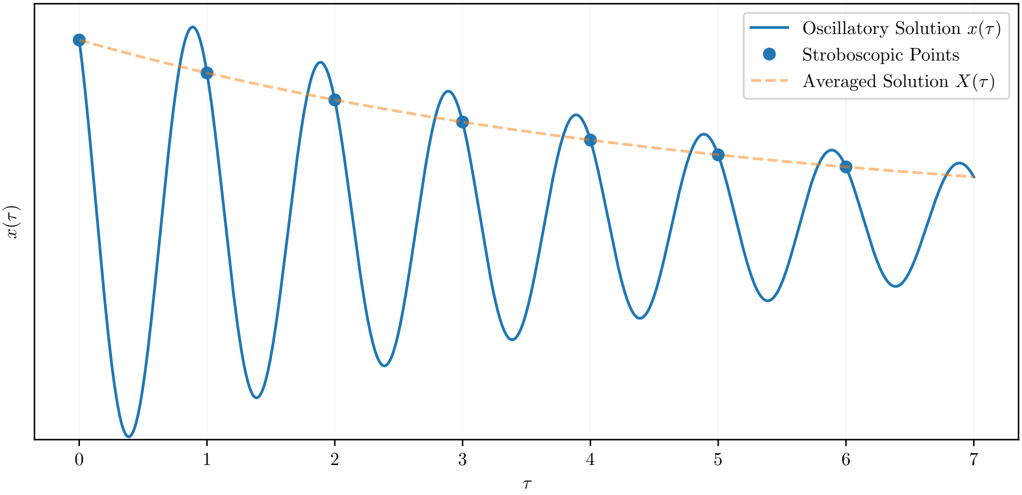
Strong Assumption

In the reel-out phase, the power optimal trajectory $x^*(t)$ and the corresponding control $u^*(t)$ consist of many similar, slowly changing cycles.

Stroboscopic Averaging Method



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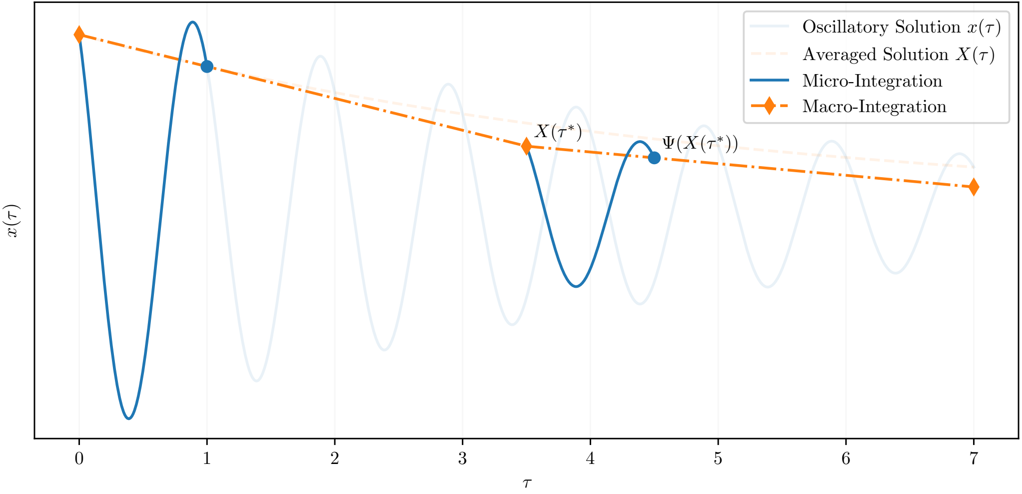
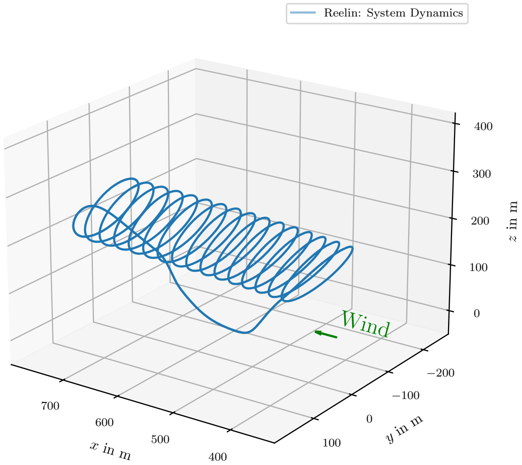


... Applied to Dual Kites



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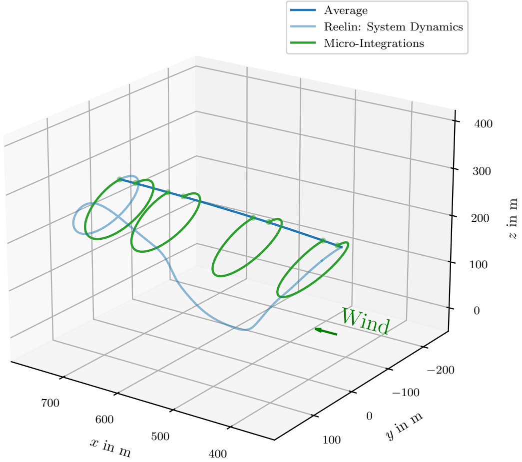

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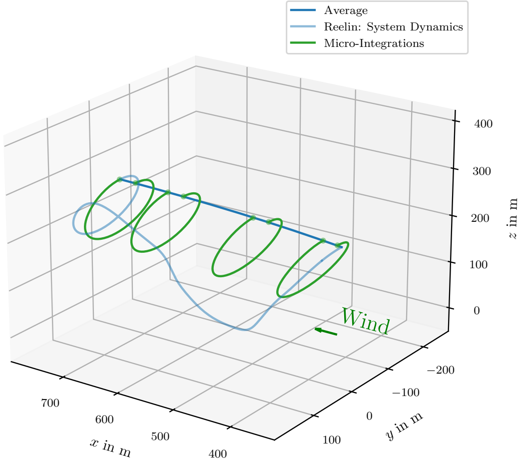

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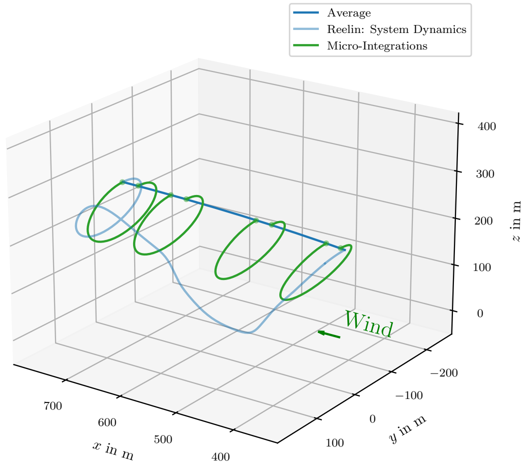

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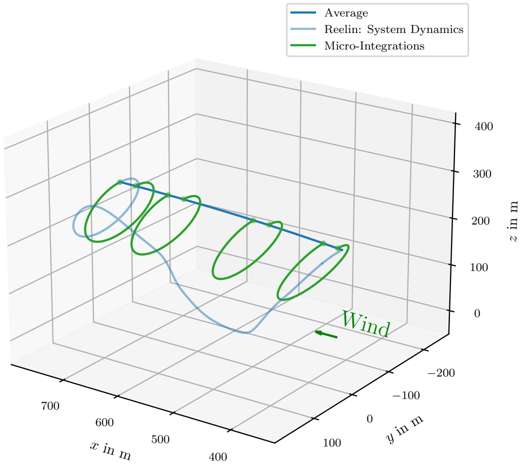

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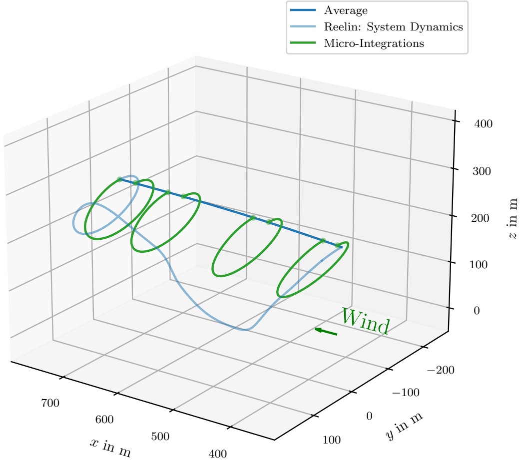

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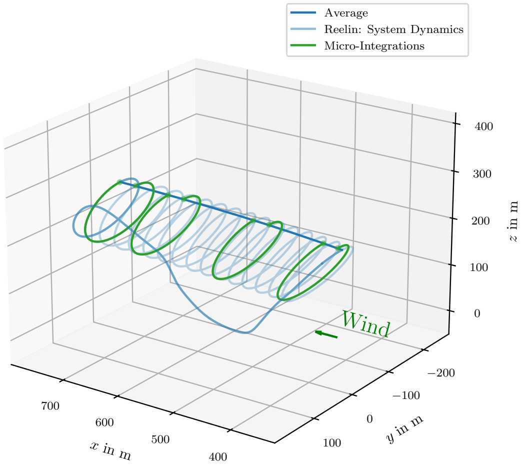

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Picture from freePik.com

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Researcher

- reward similarity in micro-integrations

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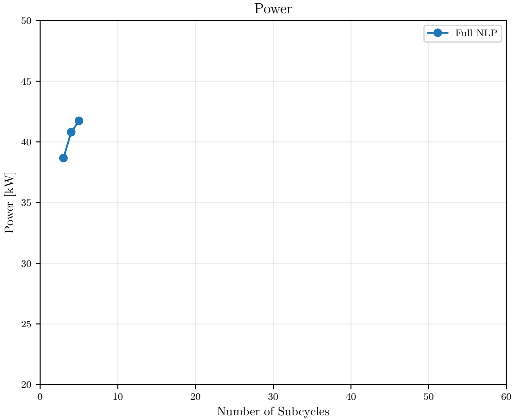

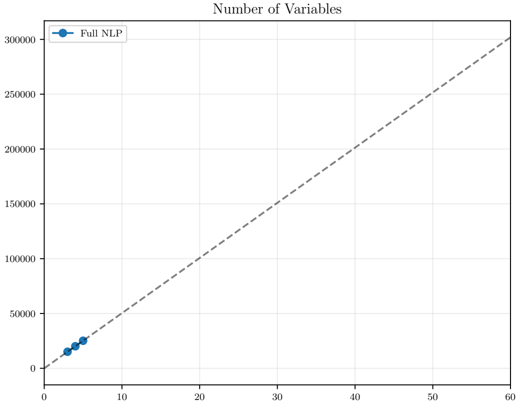

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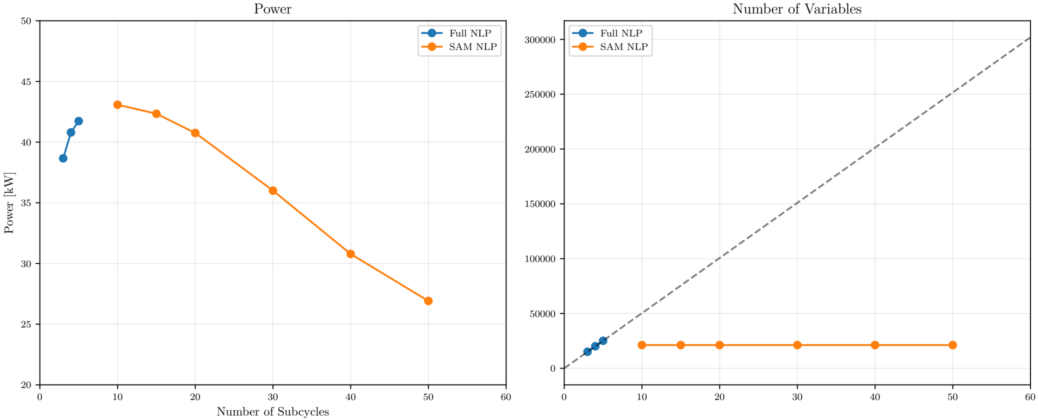
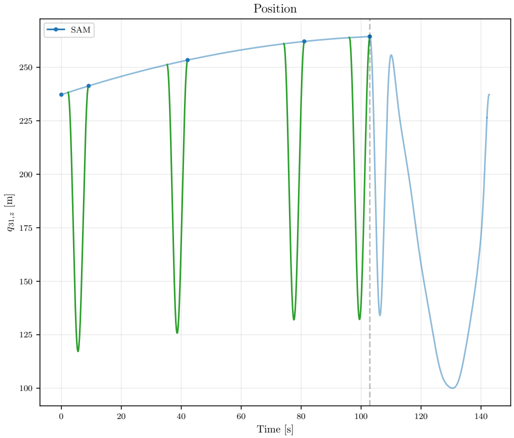


What about the full trajectory?



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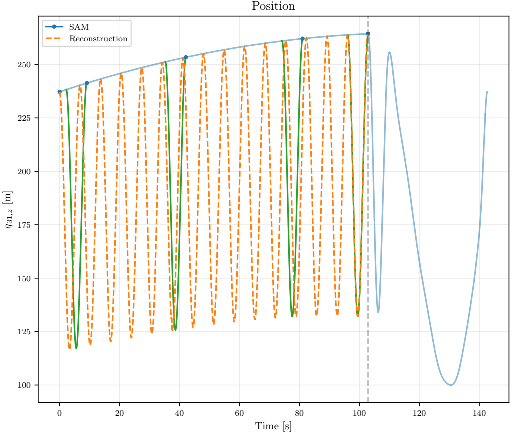

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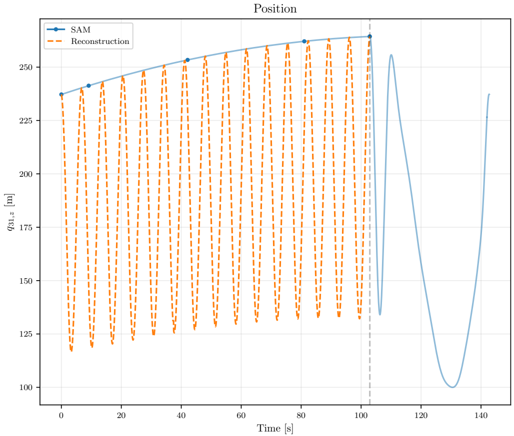

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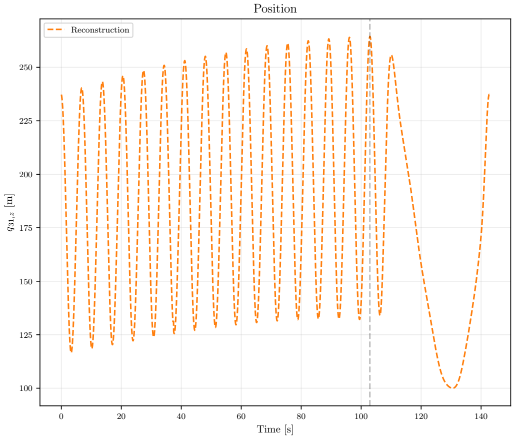

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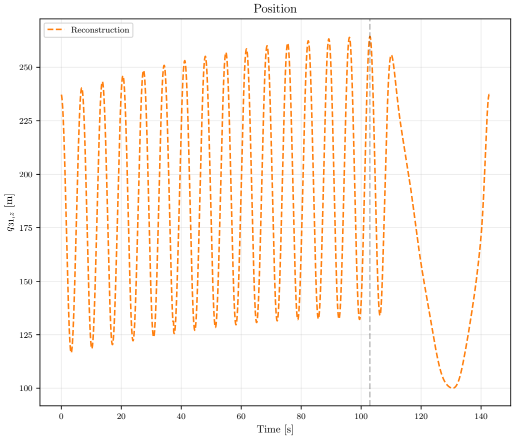

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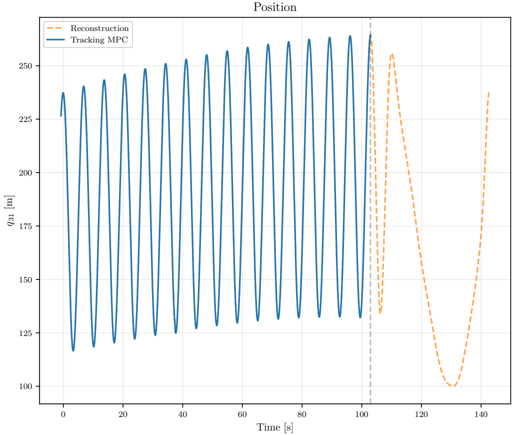
- 'Validate' the trajectory using a tracking MPC (very expensive)

Validation with Tracking MPC



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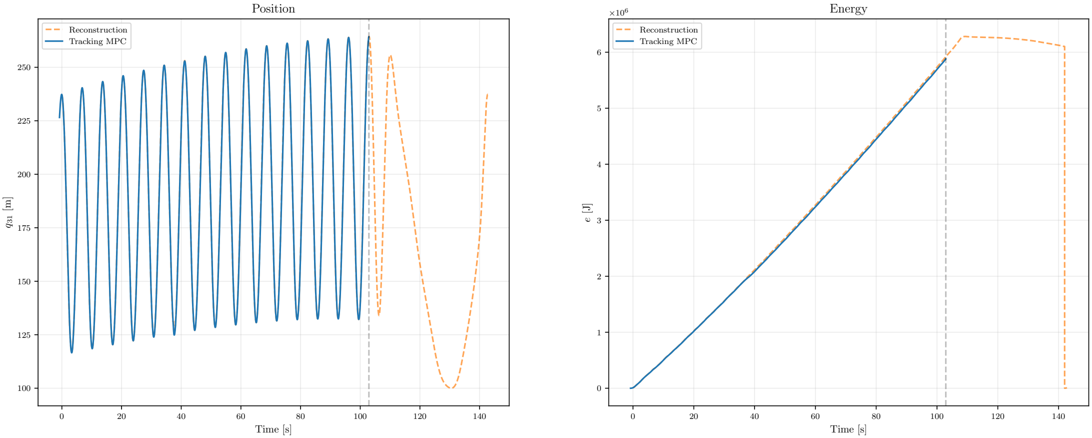

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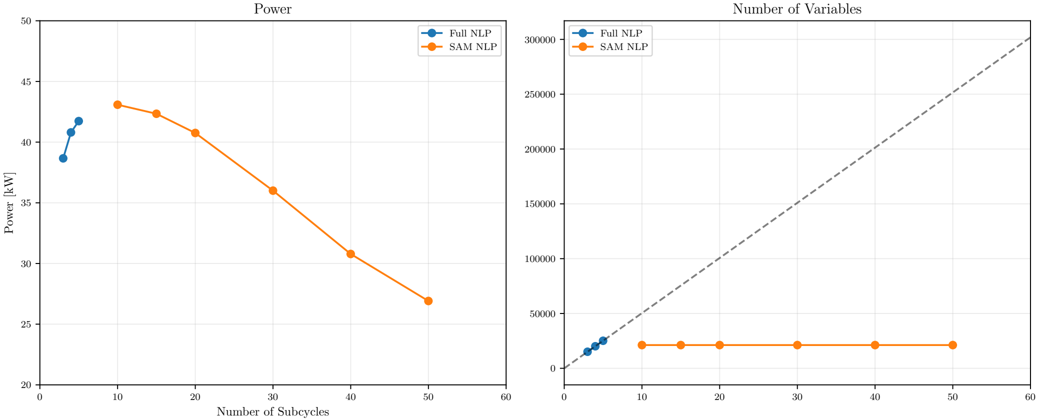

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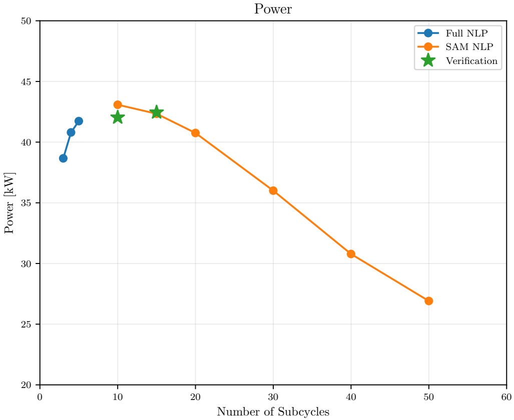

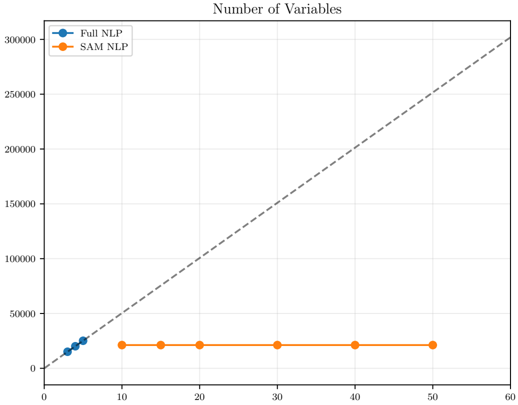

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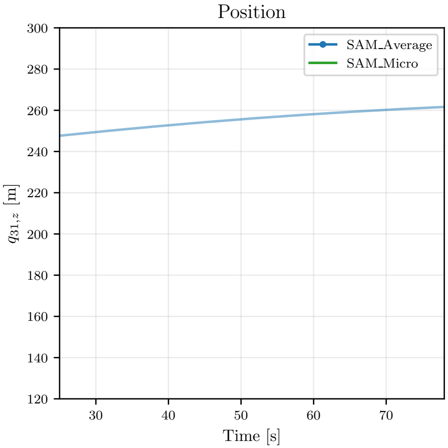
Thank you for your attention!

Useful Sources

Mari Paz Calvo, Philippe Chartier, Ander Murua, and Jesús María Sanz-Serna.
A stroboscopic numerical method for highly oscillatory problems.
In Björn Engquist, Olof Runborg, and Yen-Hsi R. Tsai, editors, Numerical Analysis of Multiscale Computations, pages 71–85, Berlin, Heidelberg, 2012. Springer Berlin Heidelberg.

Jochem De Schutter, Rachel Leuthold, Thilo Bronnenmeyer, Elena Malz, Sebastien Gros, and Moritz Diehl.
Awebox: An optimal control framework for single- and multi-aircraft airborne wind energy systems.
Energies, 16(4), 2023.

Micro-Integration



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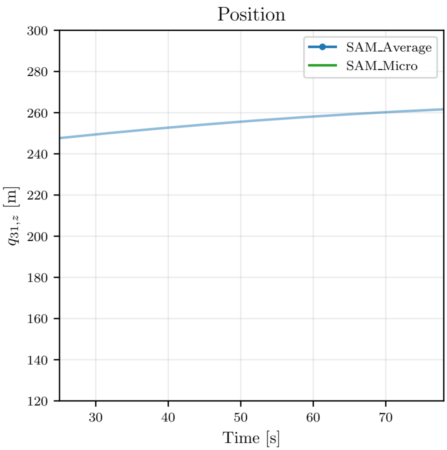

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$$\dot{X} = F(X, Z) \tag{1a}$$

$$0 = G(X, Z, T) \tag{1b}$$

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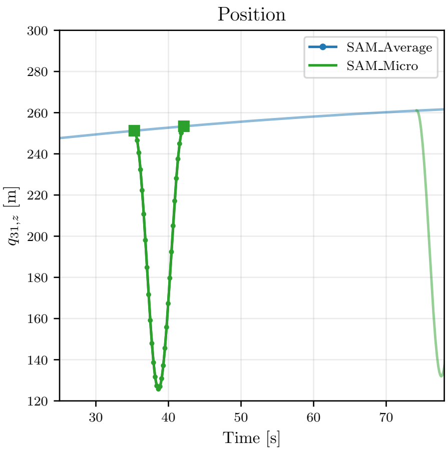

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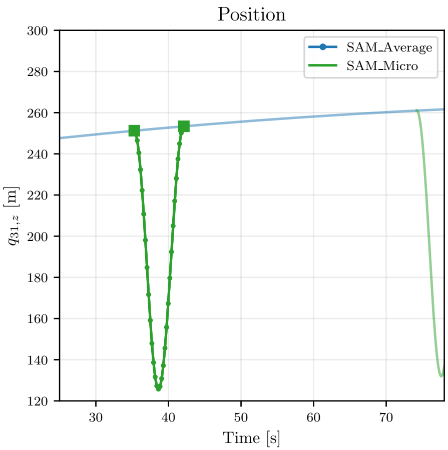

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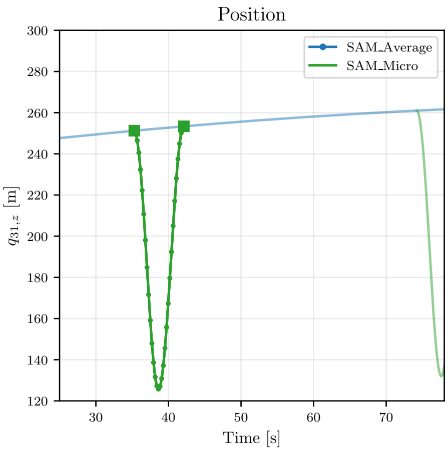

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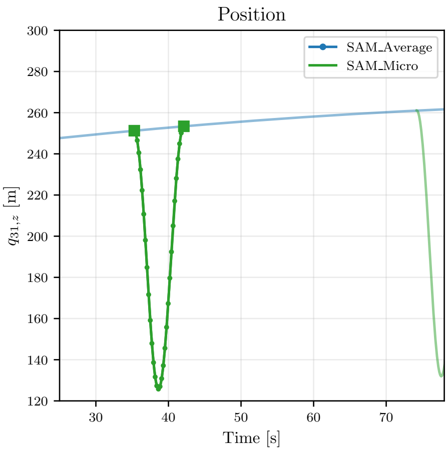
- Algebraic cycle variables Z :
  - Startpoint x^-, Endpoint x^+

Micro-Integration



- Approximate the 'average dynamics' with DAE:

$$\dot{X} = F(X, Z) \tag{1a}$$

$$0 = G(X, Z, T) \tag{1b}$$

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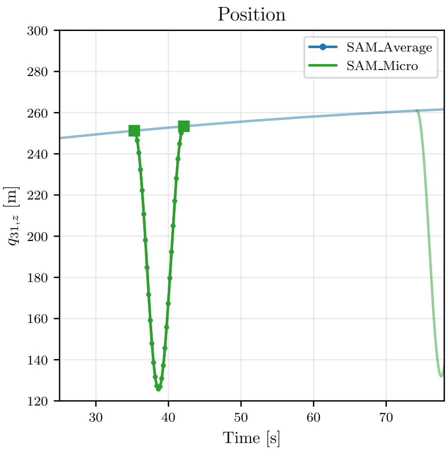
Micro-Integration



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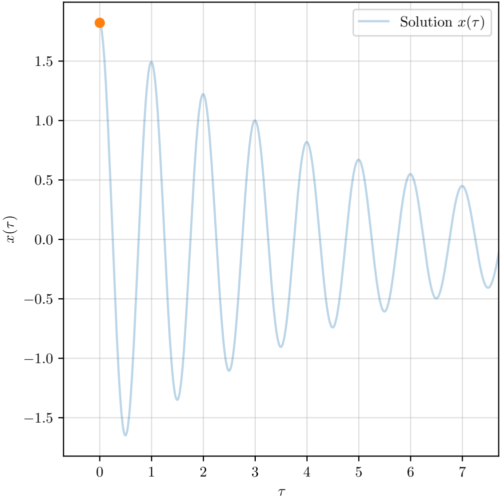
- Regularization to keep the start and endpoint 'close' to each other

$$J_{\text{reg}} = \cdots + \|x^+ - x^-\|_W^2 + \ldots \qquad (2)$$

Numerical Method for Efficient Simulation [1]



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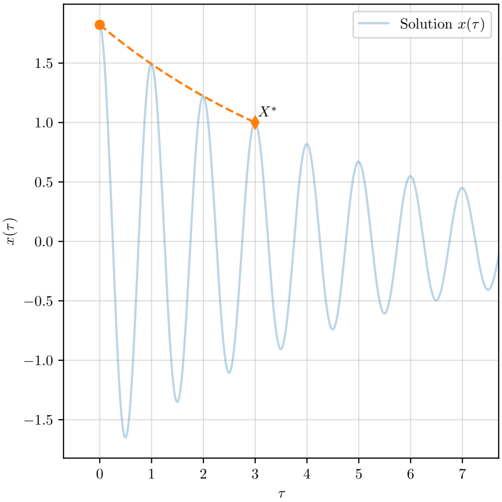
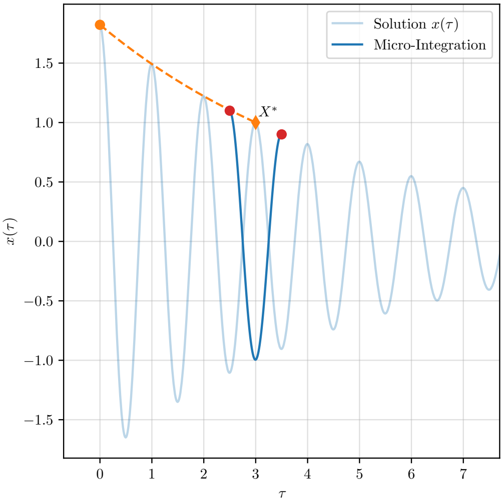


- At some point (τ^*, X^*):

Numerical Method for Efficient Simulation [1]



- At some point (τ^*, X^*):
  (a) perform one or more micro-integrations to evaluate the one-cycle map

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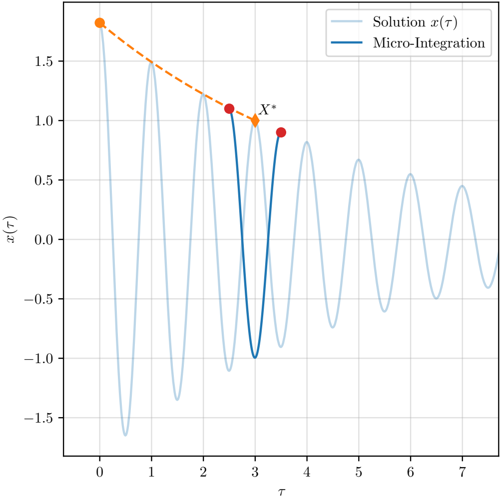

- At some point (τ^*, X^*):
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  (b) approximate the averaged dynamics

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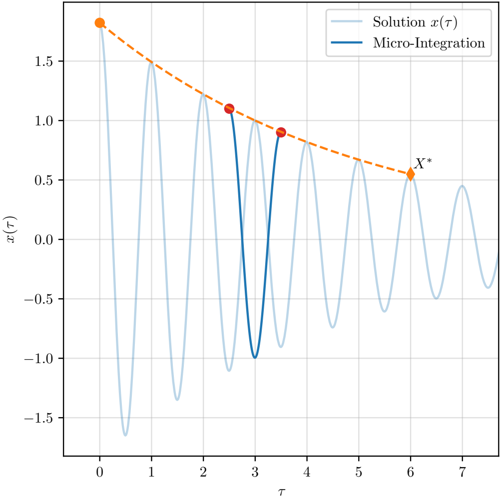
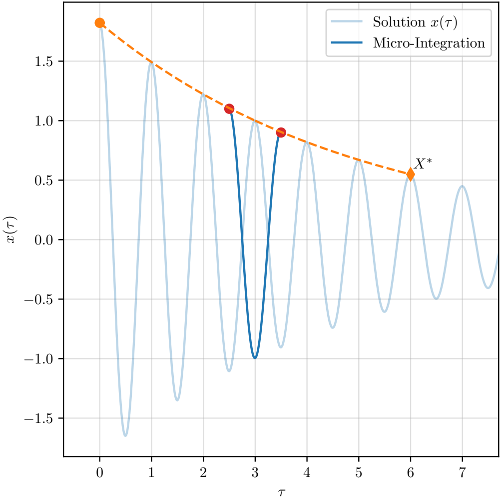


- At some point (τ^*, X^*):
  - (a) perform one or more micro-integrations to evaluate the one-cycle map
  - (b) approximate the averaged dynamics
- Macro-integrate the averaged dynamics

Numerical Method for Efficient Simulation [1]



- At some point (τ^*, X^*):
  - (a) perform one or more micro-integrations to evaluate the one-cycle map
  - (b) approximate the averaged dynamics
- Macro-integrate the averaged dynamics
- Integration horizon of integer size N cycles since

$$x(N) = X(N)$$

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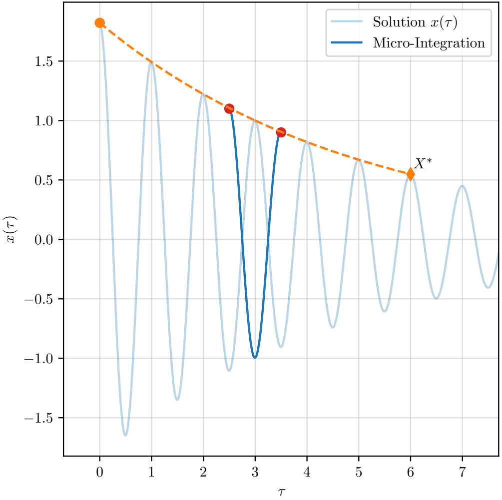

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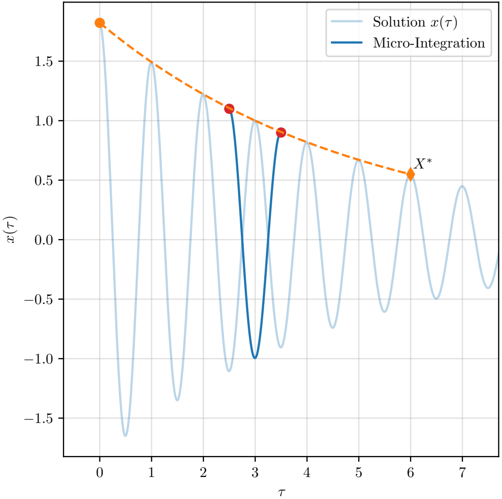

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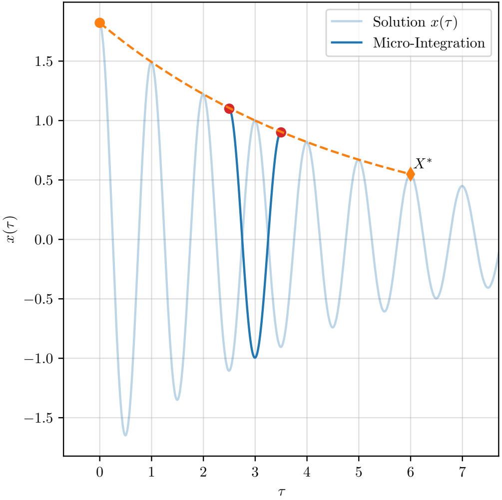

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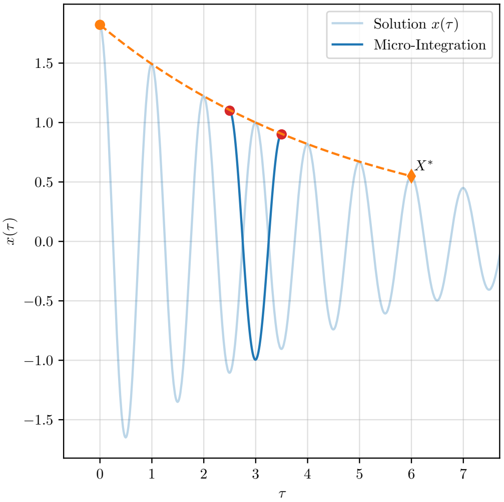

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- Macro-integrate the averaged dynamics
- Integration horizon of integer size N cycles since

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- Three sources of error:
  - (a) errors in the micro-integration
  - (b) errors in the approximation of the dynamics
  - (c) errors in the macro-integration