Dynamic Soaring in Wind Turbine Wakes

ECC 2023 Presentation

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Systems Control and Optimization Laboratory

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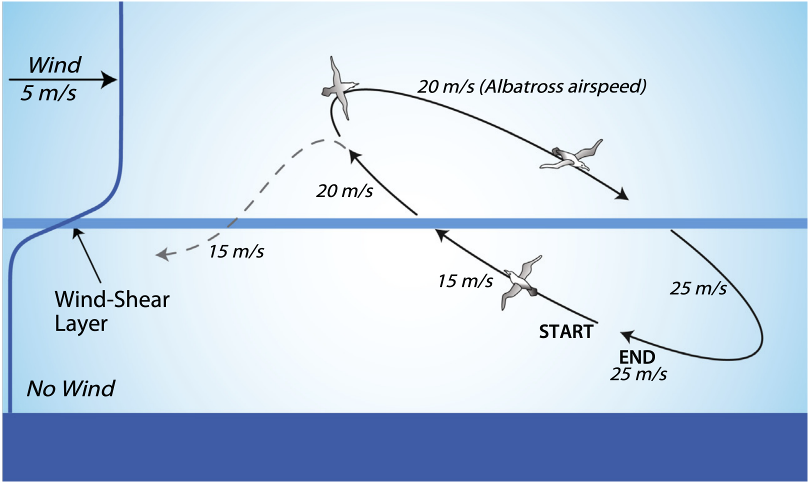
Motivation

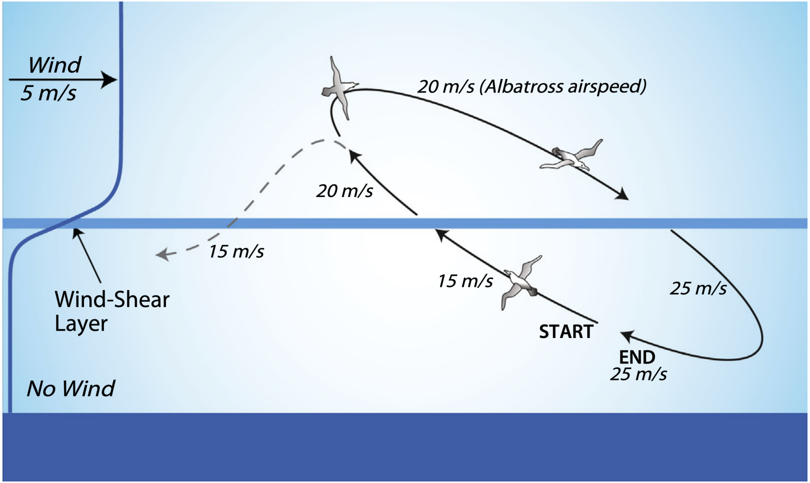
https://www.rechargenews.com/wind/blockage-effect-insight-shows-science-of-wind-still-evolving/2-1-713787

Dynamic Soaring



https://doi.org/10.1016/j.pocean.2014.11.002

Dynamic Soaring



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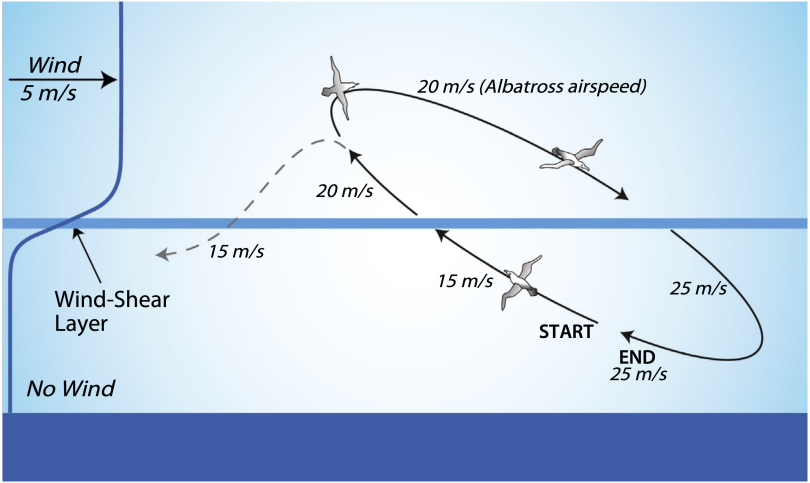
1. Can we find a periodic soaring orbit for a glider in the wake of a windturbine?

Dynamic Soaring



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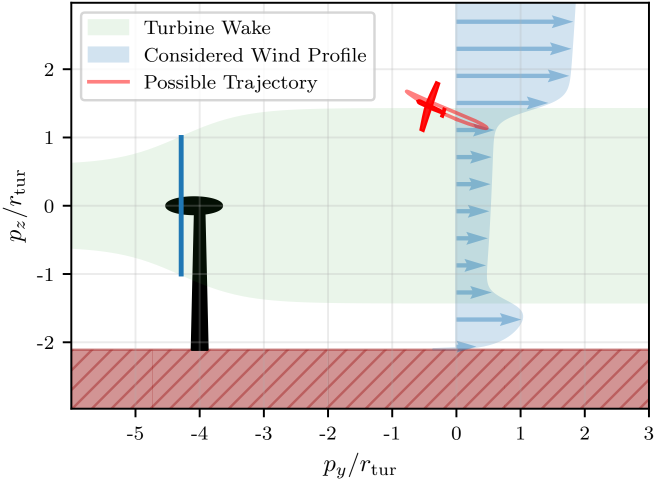
1. Can we find a periodic soaring orbit for a glider in the wake of a windturbine?
2. How much can we 'revitalize' the flow of the turbine?

Setup



Symbol	Value	Unit
r_{tur}	57.85	m
h_{tur}	122	m
r_{wake}	82	m
$v_{\mathrm{w,ref}}$	10	$\mathrm{m\,s^{-1}}$

Table: Setup Parameter

Glider Model

States:

$$x = \begin{bmatrix} p \\ v \\ F_{\mathrm{L}} \\ \dot{F}_{\mathrm{L}} \end{bmatrix} \in \mathbb{R}^{12}$$

Controls:

$$u = \ddot{F}_{\mathrm{L}} \in \mathbb{R}^3$$

Parameter	Symbol	Value	Unit
mass	m	36.8	kg
max. lift coefficient	$C_{\mathrm{L,max}}$	1.5	-
parasitic drag	$C_{\mathrm{D,0}}$	0.01	-
aspect ratio	AR	10	-
Oswald efficiency factor	O_e	0.8	-
wing area	A_{W}	3	m^2
wing span	b	5.5	m

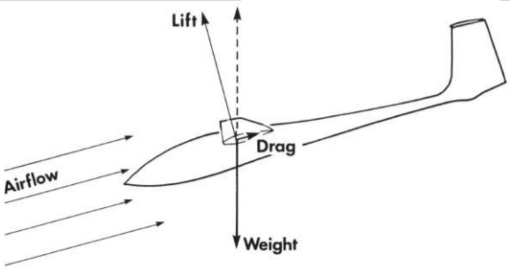
Table: AP2 Glider Parameter

Glider Model

$$f(x,u) = \frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} p \\ v \\ F_\mathrm{L} \\ \dot{F}_\mathrm{L} \end{bmatrix} = \begin{bmatrix} v \\ \frac{1}{m}(F_\mathrm{L} + F_\mathrm{D} + F_\mathrm{g}) \\ \dot{F}_\mathrm{L} \\ u \end{bmatrix}$$

$$0 = F_\mathrm{L}^T v_\mathrm{a}$$

$$\|F_\mathrm{L}\|_2 \leq \frac{1}{2}\rho A_\mathrm{W} C_{\mathrm{L,max}} \|v_\mathrm{a}\|_2^2$$



where

$$v_\mathrm{a} = v - v_w(p) \qquad C_\mathrm{L} = \frac{\|F_L\|_2}{\frac{1}{2}\rho A_\mathrm{W} \|v_\mathrm{a}\|_2^2} \qquad F_\mathrm{D} = \frac{v_\mathrm{a}}{\|v_\mathrm{a}\|_2}\frac{1}{2}\rho A_\mathrm{W}(C_{\mathrm{D},0} + \beta C_\mathrm{L}^2)\|v_\mathrm{a}\|_2^2$$

Objective

- Divide periodic trajectory phases outside and inside of the wake of durations T_o and T_i such that $T_\text{o} + T_\text{i} = t_\text{f}$

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$$F_\mathrm{Rev} = \frac{1}{t_\mathrm{f}} \int_0^{T_\mathrm{o}} e_y^\top F(\tau)\ \mathrm{d}\tau$$

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$$J_\mathrm{reg} = \epsilon_u \int_0^{t_\mathrm{f}} \|u(\tau)\|^2\ \mathrm{d}\tau$$

Optimal Control Problem

$$
\begin{aligned}
\underset{\substack{x(\cdot),u(\cdot),\\ T_\mathrm{o},T_\mathrm{i}}}{\text{minimize}} \quad & -J_\mathrm{eco} + J_\mathrm{reg} && && \text{(1a)}\\
\text{subject to} \quad & 0 = p_y(0), && && \text{(1b)}\\
& 0 = \dot{x}(t) - f(x(t), u(t)) && \forall t \in [0, t_\mathrm{f}], && \text{(1c)}\\
& 0 = v_\mathrm{a}(t)^\top F_\mathrm{L}(t) && \forall t \in [0, t_\mathrm{f}], && \text{(1d)}\\
& 0 \geq \|F_\mathrm{L}(t)\|_2 - F_\mathrm{L,max}(t) && \forall t \in [0, t_\mathrm{f}], && \text{(1e)}\\
& 0 \geq \alpha_\mathrm{roll}(t) - \alpha_\mathrm{roll,max} && \forall t \in [0, t_\mathrm{f}], && \text{(1f)}\\
& 0 \leq \alpha_\mathrm{roll}(t) + \alpha_\mathrm{roll,max} && \forall t \in [0, t_\mathrm{f}], && \text{(1g)}\\
& 0 \leq r(p(t)) - r_\mathrm{wake} && \forall t \in [0, T_\mathrm{o}], && \text{(1h)}\\
& 0 \geq r(p(t)) - r_\mathrm{wake} && \forall t \in [T_\mathrm{o}, t_\mathrm{f}], && \text{(1i)}\\
& 0 = x(t_\mathrm{f}) - x(0) && && \text{(1j)}
\end{aligned}
$$

Discretization

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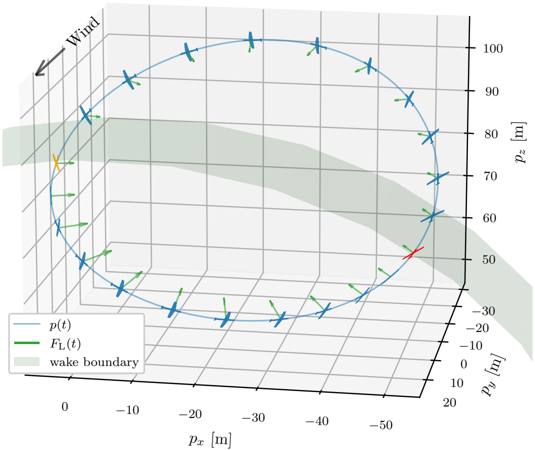
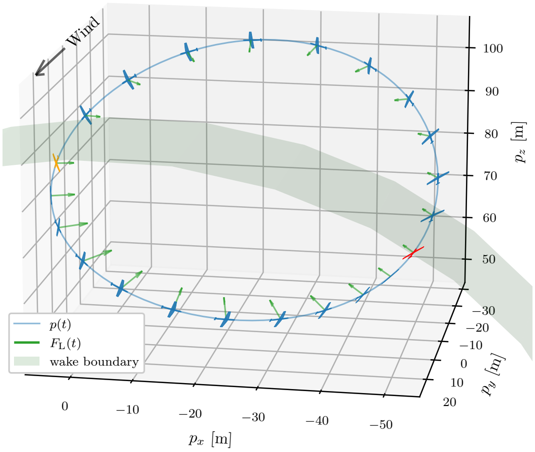
- One Control at each collocation point $\rightarrow$ polynomial instead of piecewise constant controls
- Solve using a homotopy that moves from a tracking objective to the economic objective
- IPOPT converges to a solution with ~ 10k variables ~ 70s.

Solution



Wind
100
90
80
70
60
50
p_z [m]
−30
−20
−10
0
10
20
p_y [m]
0
−10
−20
−30
−40
−50
p_x [m]
$p(t)$
$F_{\mathrm{L}}(t)$
wake boundary

Solution



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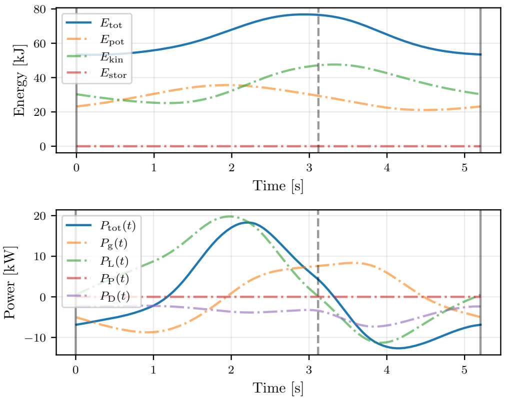
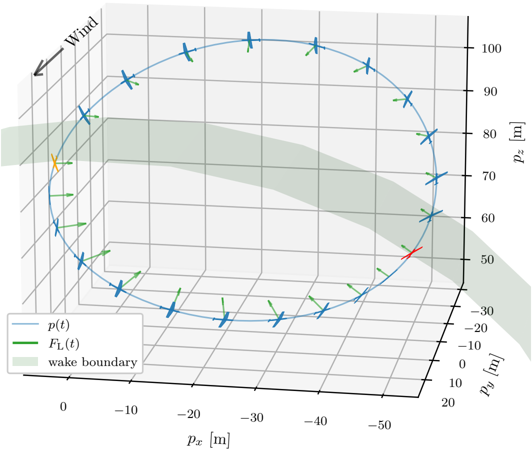


E_{tot}
E_{pot}
E_{kin}
E_{stor}
Energy [kJ]
Time [s]
$P_{tot}(t)$
$P_g(t)$
$P_L(t)$
$P_P(t)$
$P_D(t)$
Power [kW]
Time [s]

Solution



- Revitalization force:

$$F_{\text{rev}} = 0.585\,\text{kN}$$

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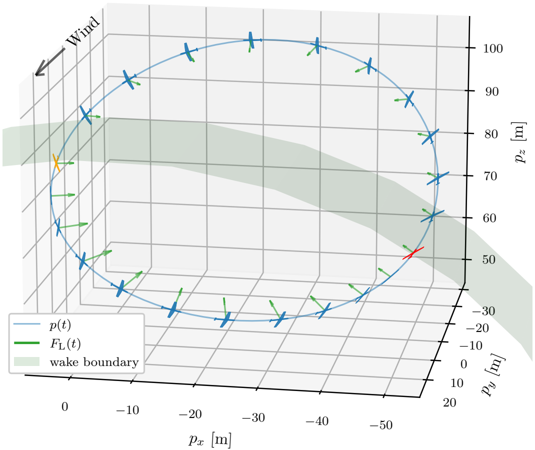

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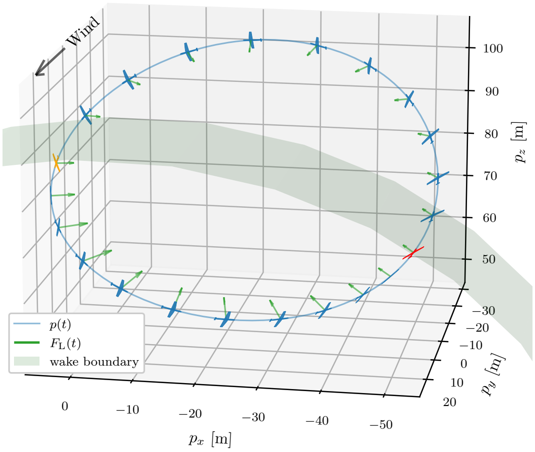
- Compare with Turbine thrust force:

$$F_{\text{tur}} = 4a(1-a)\frac{1}{2}\rho A_{\text{tur}} u_{\text{ref}}^2 = 286\,\text{kN}$$

Solution



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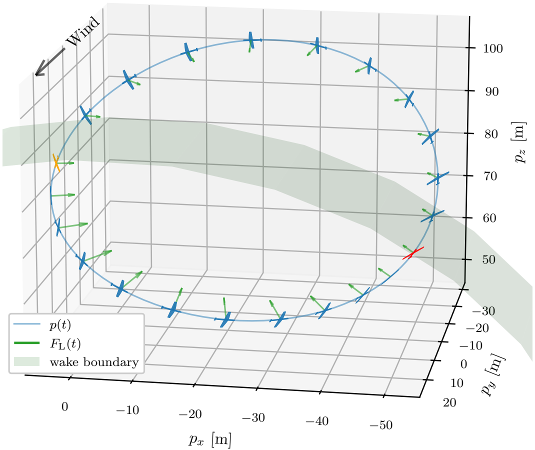

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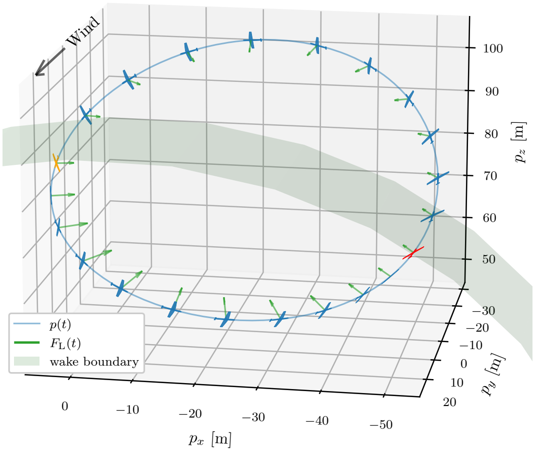

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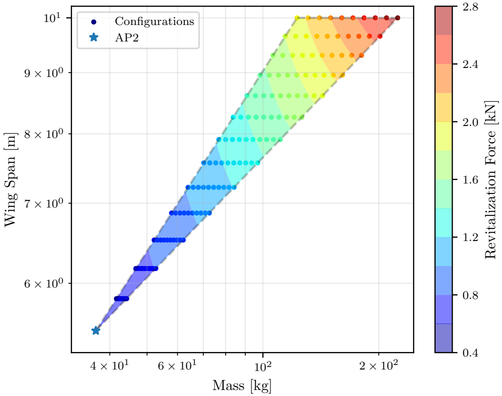
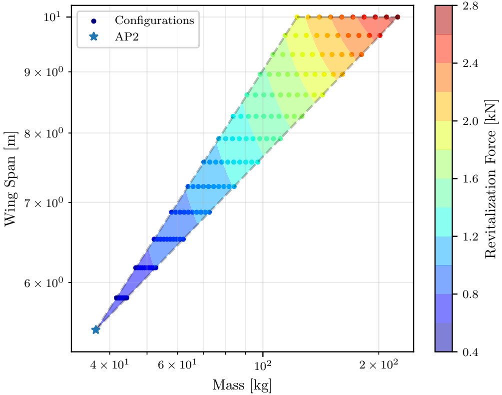
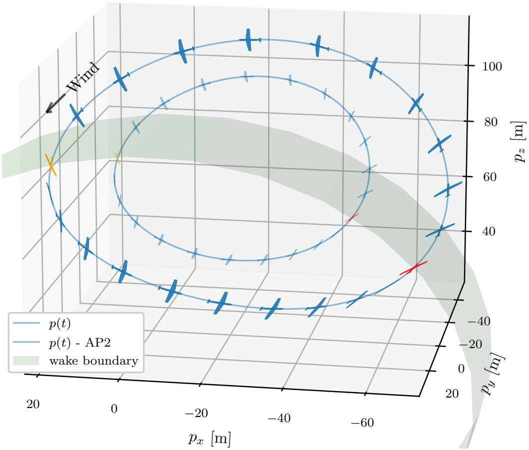
- Option 2: Propeller + Energy Storage

Bigger Glider

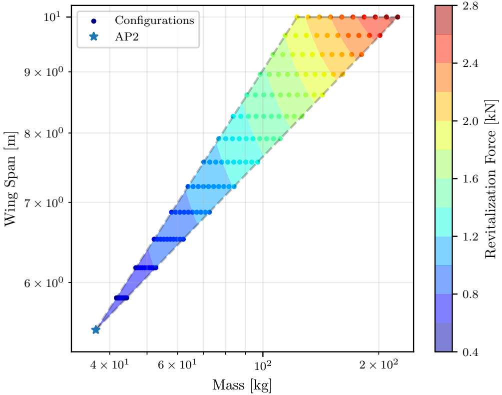


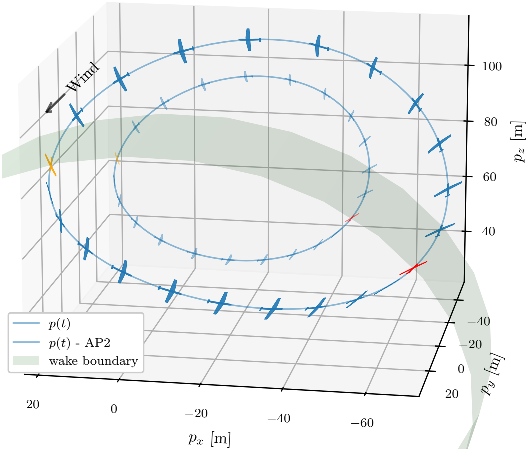
Bigger Glider





Bigger Glider





- $F_{\text{rev}}/F_{\text{tur}} \approx 1\,\%$

Propeller + Energy Storage

- Add a Propeller and a capacitor to the glider to store energy

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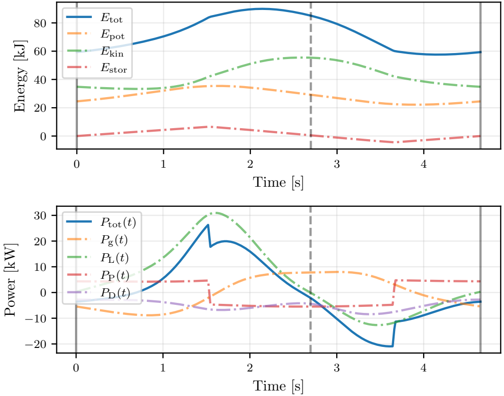
- Add periodic state for the stored energy

Propeller + Energy Storage



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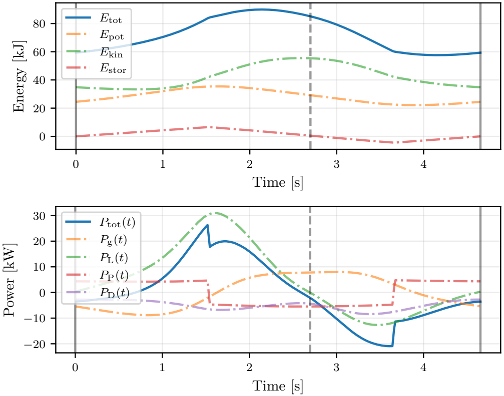

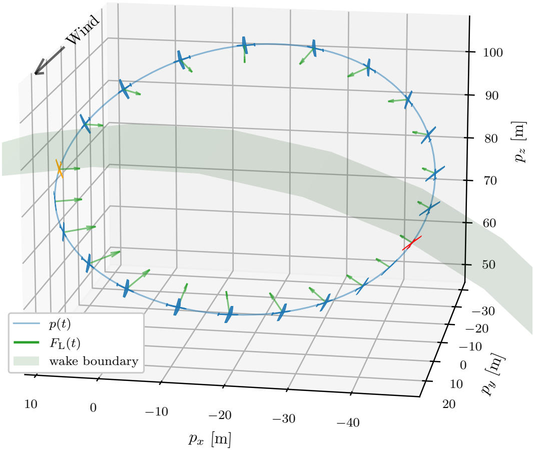

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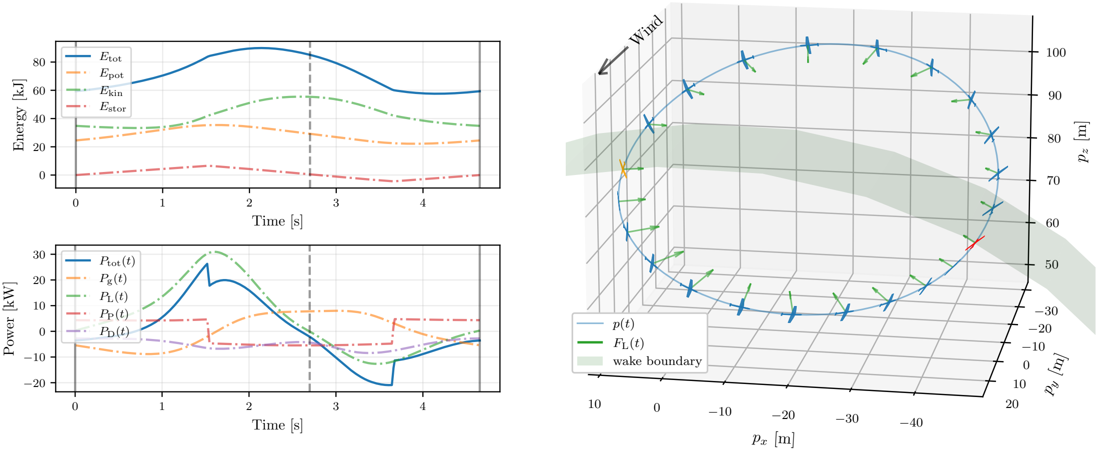

- $F_{\mathrm{rev}}/F_{\mathrm{tur}} \approx 0.5\,\%$

Conclusion

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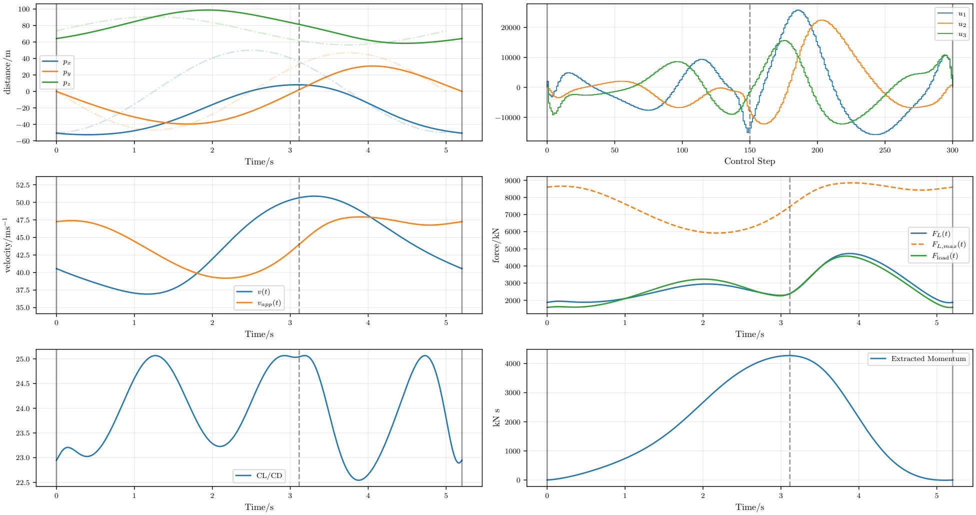
- Other Applications: monitoring, wind measurements etc., since you 'fly for free' in the wake of a turbine

Conclusion

Thank you for your attention

Appendix


distance/m
Time/s
p_x
p_y
p_z
Control Step
u_1
u_2
u_3
velocity/ms^{-1}
$v(t)$
$v_{app}(t)$
force/kN
$F_L(t)$
$F_{L,max}(t)$
$F_{load}(t)$
CL/CD
kN s
Extracted Momentum