Contactless current sensing with position detection using model-based nonlinear optimization

Lukas Klar,
Dr. Stefan Albrecht,
Prof. Dr. Moritz Diehl

TDK-Micronas GmbH
A TDK Group Company

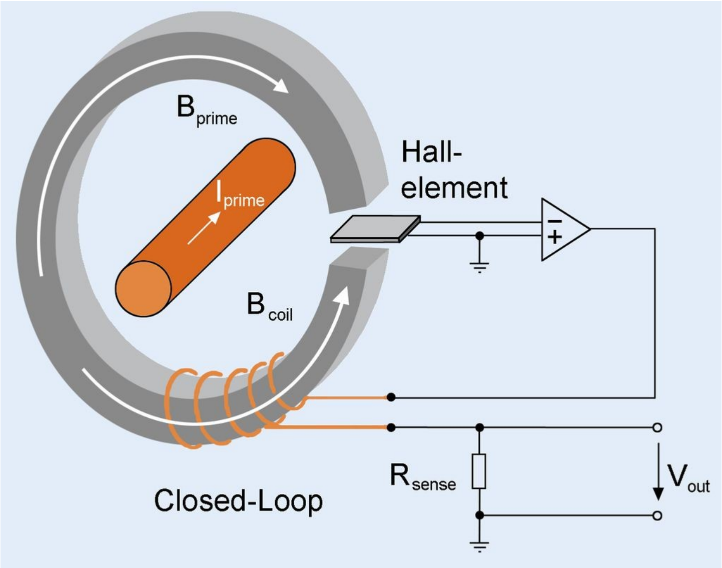
Magnetic Sensors Business Group • Application Engineering
Freiburg, Germany
January 30, 2018

State of the art – Contactless current sensing

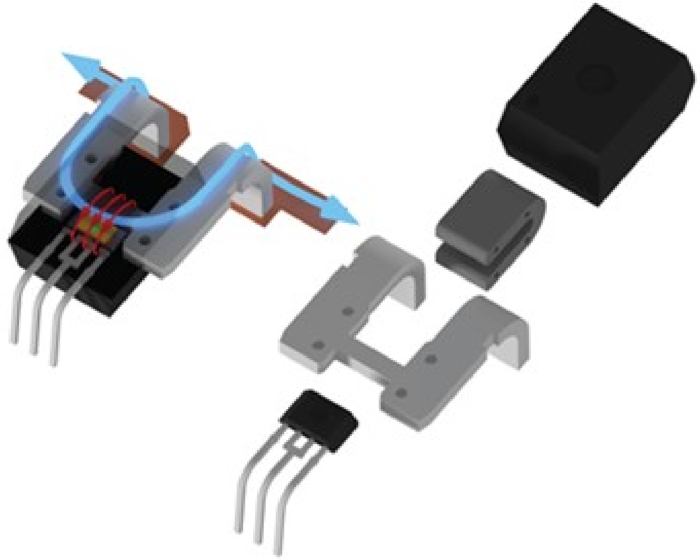
Enclosure of conductor by a core
- Big size
- High cost of core
- Bandwidth limitation
- Remanence
- Complicated mounting

Integrated conductor on leadframe
- Dual molding
- High cost of conductor
- Complex assembly process

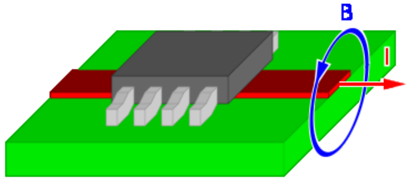
Coreless placement
- Low stray field immunity
- Precise placement or end of line calibration



www.elektronikpraxis.vogel.de/sensorik/articles/299736/index2.html



www.allegromicro.com/en/Products/Current-Sensor-ICs/Fifty-To-Two-Hundred-Amp-Integrated-Conductor-Sensor-ICs.aspx



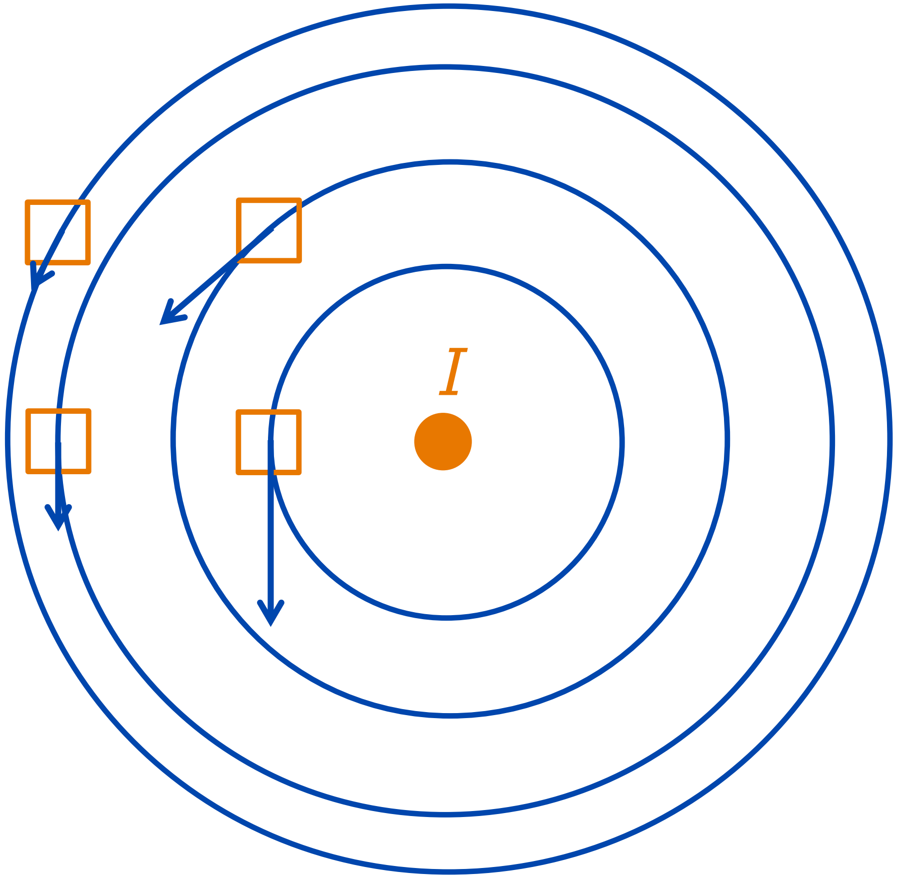
de.wikipedia.org/wiki/Stromsensor

An innovative idea

Requirements
- Non-contact sensor ➔ free of potential
- No exact mounting ➔ position tolerant
- No current carrying leadframe ➔ low cost
- No enclosure ➔ easy to mount

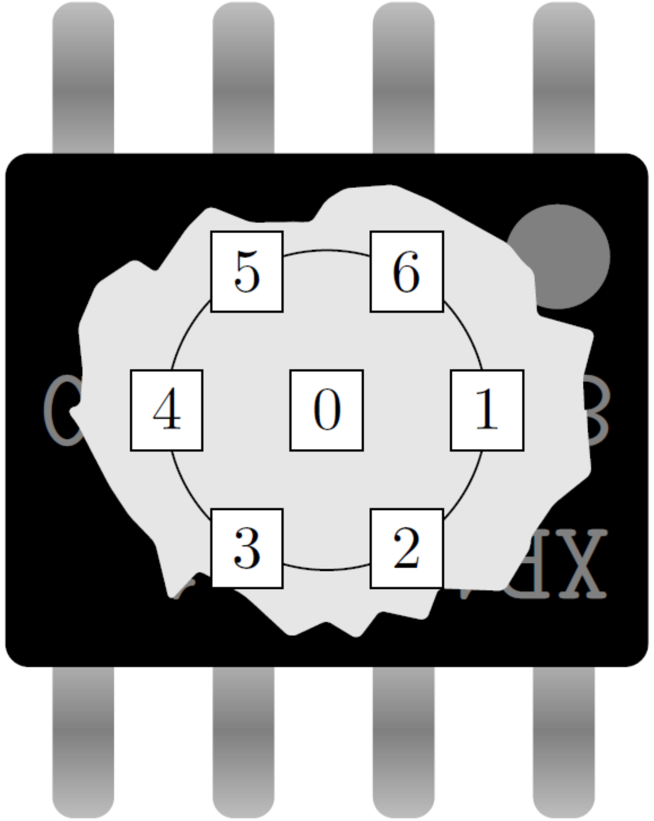
Implementation
- Measurement of the magnetic field at multiple locations in all three spatial dimensions
- Modelling of the expected magnetic field based on used setup e.g. straight, round conductor, distance between sensors,…
- Estimation of the current by fitting the model to the measurement data using non-linear optimization

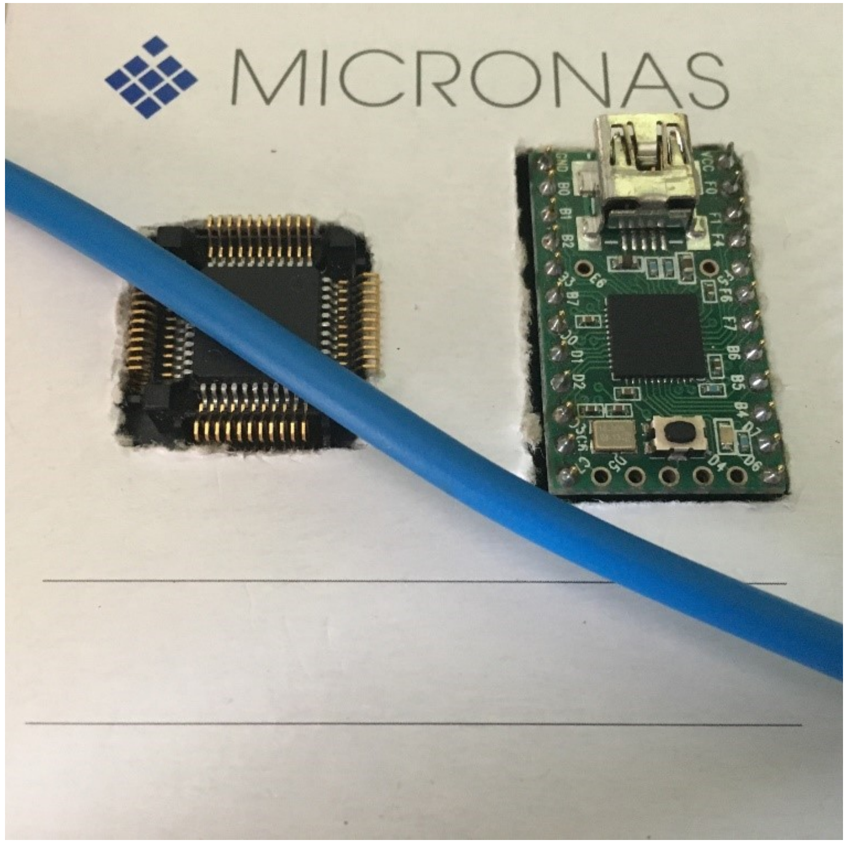
$$\underset{\theta}{\text{minimize}} \quad f(\theta) = \frac{1}{2}||m(\theta) - y||_2^2$$



The Hardware

Sensor chip with seven 3D Hall pixel cells ➔ 21 magnetic field values measured at once

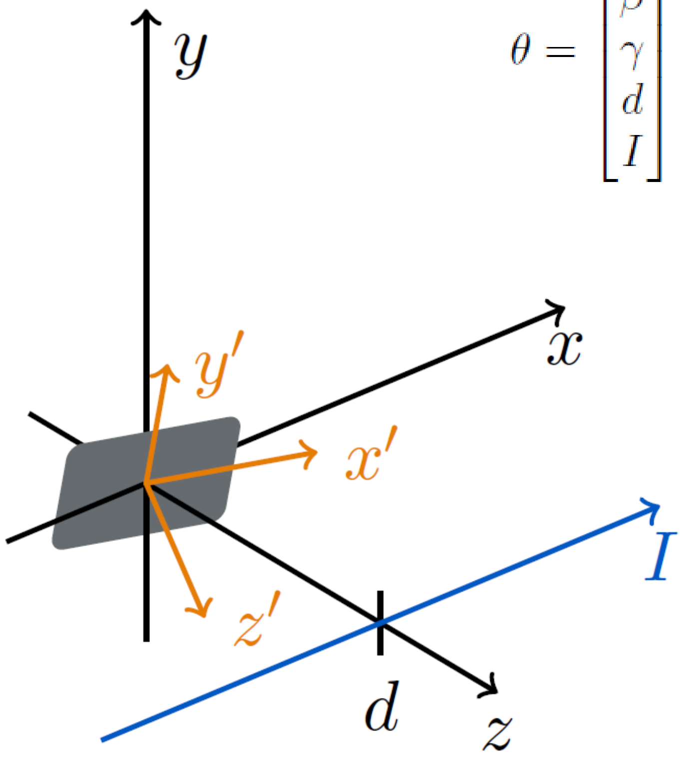




The Model

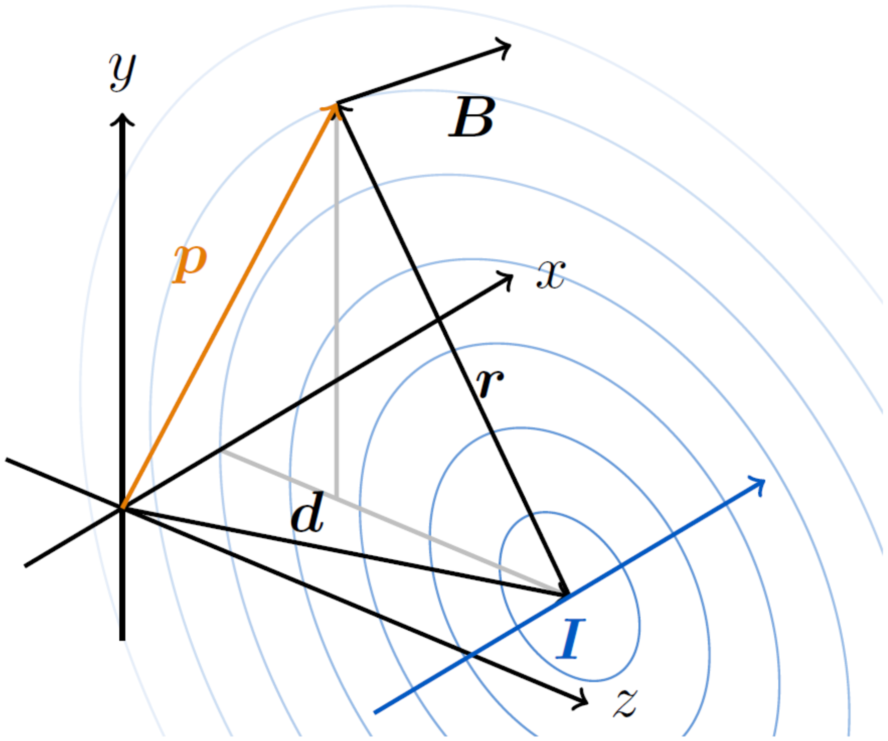
Parameter space consists of 5 variables
- 4 Spatial degrees of freedom
  - Distance between sensor and conductor – *d*
  - Rotation of sensor (3 axis) – *α, β and γ*
- Current flow in the conductor

$$\theta = \begin{bmatrix} \alpha \\ \beta \\ \gamma \\ d \\ I \end{bmatrix}$$



Magnetic field model according to Biot-Savart:
- Infinitely long
- Straight conductors
- Round cross-section

$$\mathbf{B} = \frac{\mu_0\mu_r}{2\pi r^2}\mathbf{I} \times \mathbf{r}$$



Model – Equation

Position of *i*-th sensing elements after rotation:

$$
\begin{aligned}
p_{y,i} &= \cos(\beta)\sin(\gamma)p_{0,x,i} + \cos(\alpha)\cos(\gamma)p_{0,y,i} \\
&\quad + (\sin(\beta)\sin(\gamma) - \sin(\alpha)\cos(\gamma))p_{0,z,i}, \\
p_{z,i} &= -\sin(\beta)p_{0,x,i} + \sin(\alpha)\cos(\beta)p_{0,y,i} \\
&\quad + \cos(\alpha)\cos(\beta)p_{0,z,i},
\end{aligned}
$$

Sensing direction of *i*-th elements after rotation:

$$
\begin{aligned}
e_{y,i} &= \cos(\beta)\sin(\gamma)e_{0,x,i} + \cos(\alpha)\cos(\gamma)e_{0,y,i} \\
&\quad + (\sin(\beta)\sin(\gamma) - \sin(\alpha)\cos(\gamma))e_{0,z,i}, \\
e_{z,i} &= -\sin(\beta)e_{0,x,i} + \sin(\alpha)\cos(\beta)e_{0,y,i} \\
&\quad + \cos(\alpha)\cos(\beta)e_{0,z,i}.
\end{aligned}
$$

Magnetic flux measured at the *i*-th sensing element:

$$
y_i = m_i(\theta) + \epsilon_i = \frac{\mu_0 I(p_{y,i}e_{z,i} + (d - p_{z,i})e_{y,i})}{2\pi(p_{y,i}^2 + (d - p_{z,i})^2)} + \epsilon_i
$$

Complete system model:

$$
m(\theta) = \begin{bmatrix} m_1(\theta) \\ \vdots \\ m_{21}(\theta) \end{bmatrix}
$$

Model – Measurement noise

Sensor noise is dependent on the implemented sensing direction:

Direction	σ-noise
X and Y	3.0 μT
Z	1.2 μT

The residual is the mismatch between model and measurement scaled by the covariance matrix

$$R(\theta) = \Sigma_{\epsilon}^{-\frac{1}{2}} (m(\theta) - y)$$

Resulting optimization problem:

$$\underset{\theta}{\text{minimize}} \quad f(\theta) = \frac{1}{2} ||R(\theta)||_2^2$$

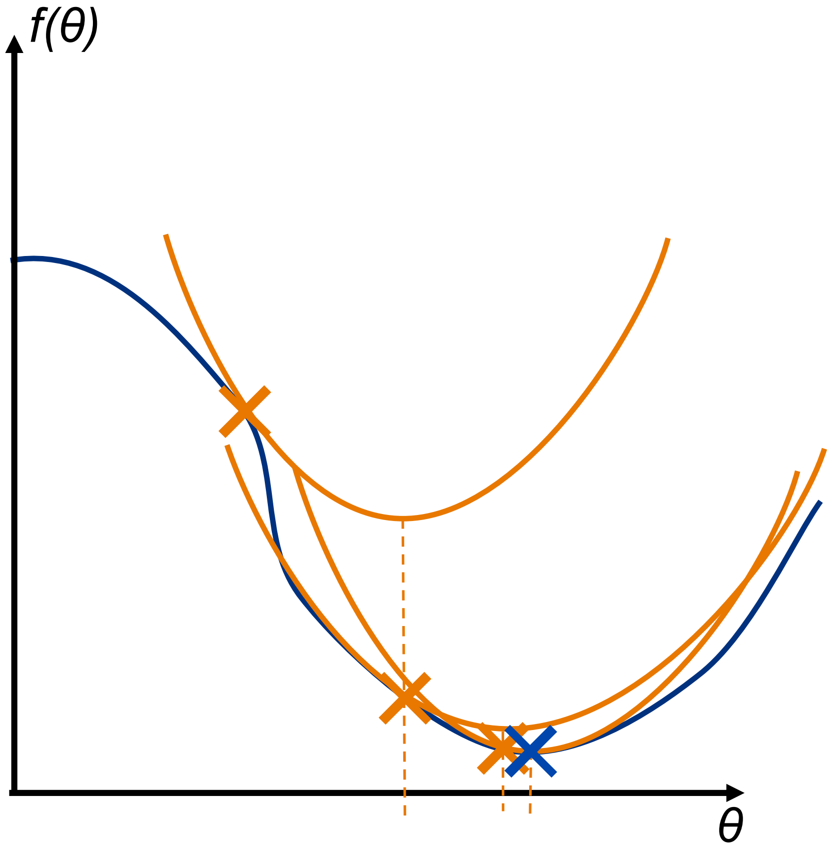
Nonlinear parameter estimation – Overview

Optimization problem:
Which parameters fit best to the measured values?
➔ Where is the minimum of the sum of squares of the residual?

$$\underset{\theta}{\text{minimize}} \quad f(\theta) = \frac{1}{2}||R(\theta)||_2^2$$

Gauss-Newton Algorithm:

1. Start at initial value $\theta_{[0]}$
2. Linearize the residual at the given point
3. Solve the linear optimization problem with the least squares method
4. Repeat from step 2 with solution as new linearization point



Nonlinear parameter estimation – Linearization

First order Taylor series at $\theta_{[k]}$:

$$R_{[k]}(\theta) = R(\theta_{[k]}) + \frac{\partial R}{\partial \theta}(\theta_{[k]})(\theta - \theta_{[k]})$$

Jacobian matrix:

$$J_{[k]} = \frac{\partial R}{\partial \theta}(\theta_{[k]}) \qquad J_{[k]} = \left[\frac{\partial R}{\partial \theta_1}(\theta_{[k]}) \quad \cdots \quad \frac{\partial R}{\partial \theta_5}(\theta_{[k]})\right]$$

Imaginary step derivation:

$$\frac{\partial R}{\partial \theta_n}(\theta_{[k]}) = \frac{\mathrm{Im}(R(\theta_{[k]} + i \cdot \varepsilon \cdot e_n))}{\varepsilon}$$

Nonlinear parameter estimation – LLS

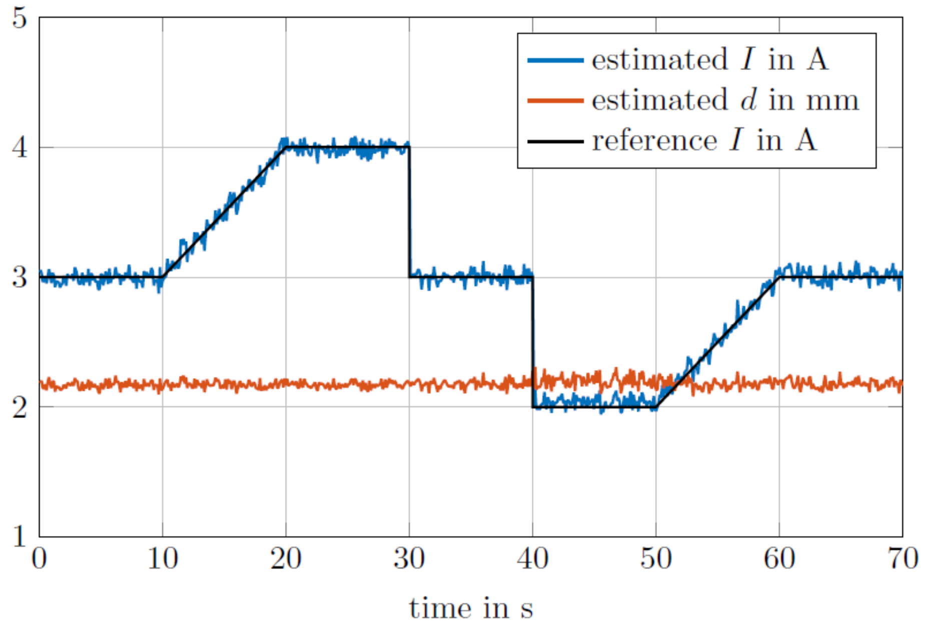
Linear least squares problem:

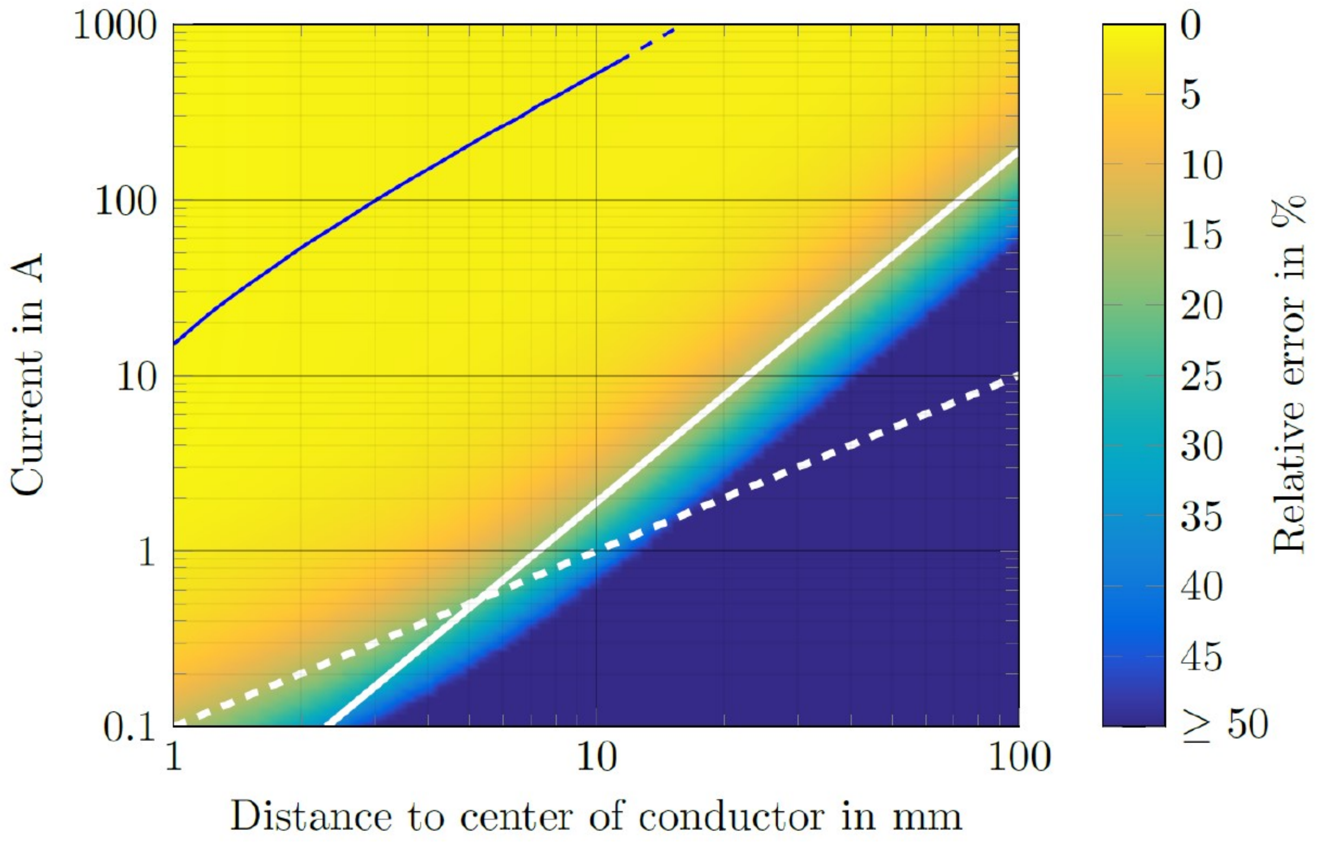
$$\underset{\theta}{\text{minimize}} \quad f_{[k]}(\theta) = \frac{1}{2}||R_{[k]}(\theta)||_2^2$$

Analytical solution with the general Moore-Penrose pseudo-inverse:

$$\theta_{[k+1]} = \theta_{[k]} - J_{[k]}^{+} \cdot R(\theta_{[k]})$$

Experimental results





TDK

www.micronas.com