Efficient Collision Modelling for Numerical Optimal Control

Final Presentation of Master’s Thesis Project

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April 5, 2022

Background

- University: Technical University of Munich (TUM)
- Study course: Master Mathematics in Science and Engineering
- Supervisor: Prof. Dr. Michael Ulbrich (TUM)
- Advisors: Dr. Sebastian Albrecht (Siemens),
  Armin Nurkanović (University of Freiburg)

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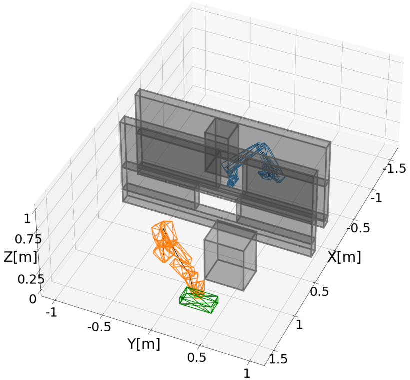
Motivational example

Kuka LBR iiwa 7 & Ur5 execute an item passing task



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Contribution of the master's thesis

Master's thesis is based on:

X. Zhang, A. Liniger, and F. Borrelli. **Optimization-based collision avoidance.** *IEEE Transactions on Control Systems Technology*, 29:972-983, 2021.

Contributions:

- Derivation of different forms of collision constraints
- Performance evaluation of the constraints through numerical experiments
- Application to manipulator models

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Outline

1. Optimal Control Problem
2. Derivation of Collision Constraints
3. Numerical Benchmarks
4. Trajectory Planning for Manipulators: Methodology and Challenges

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1. Optimal Control Problem
2. Derivation of Collision Constraints
3. Numerical Benchmarks
4. Trajectory Planning for Manipulators: Methodology and Challenges

Starting point is a dynamic model of a robot

Dynamic equations of motion:

$$\tau = M(q)\ddot{q} + C(q, \dot{q}) + G(q)$$

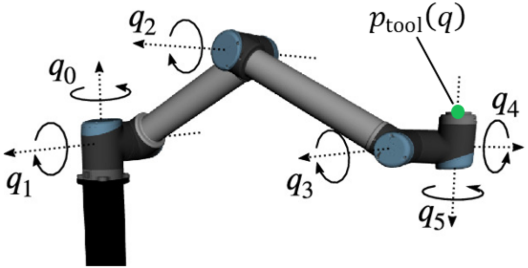
where

q_i - joint angles

$\dot{q}_i$ - joint velocities

$\ddot{q}_i$ - joint accelerations

τ_i - torques



From: https://github.com/giusenso/robot-manipulator-dynamics

Starting point is a dynamic model of a robot

Rewrite as first-order ODE

$$\frac{d}{dt}\begin{pmatrix} q \\ \dot{q} \end{pmatrix} = \begin{pmatrix} \dot{q} \\ M(q)^{-1}(\tau - C(q,\dot{q}) - G(q)) \end{pmatrix}$$

set

$$z = (q, \dot{q}), \quad u = \tau$$

Compact form:

$$\dot{z} = f(z, u)$$

Starting point is a dynamic model of a robot

Rewrite as first-order ODE

$$\frac{d}{dt}\begin{pmatrix} q \\ \dot{q} \end{pmatrix} = \begin{pmatrix} \dot{q} \\ M(q)^{-1}(\tau - C(q,\dot{q}) - G(q)) \end{pmatrix}$$

set

$$z = (q, \dot{q}), \quad u = \tau$$

Compact form:

$$\dot{z} = f(z, u)$$

Alternatively:

Double integrator dynamic

$$\frac{d}{dt}\begin{pmatrix} q \\ \dot{q} \end{pmatrix} = \begin{pmatrix} \dot{q} \\ \ddot{q} \end{pmatrix}$$

set

$$z = (q, \dot{q}), \quad u = \ddot{q}$$

Compact form:

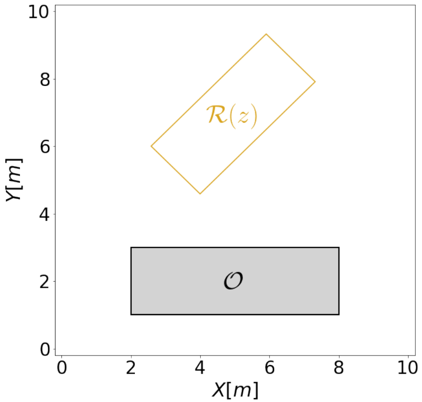
$$\dot{z} = f(z, u)$$

A discrete optimal control problem (OCP)

$$
\begin{aligned}
\min_{z,u,\lambda,\mu} \quad & \sum_{k=0}^{N-1} L(z_k, u_k) + M(z_N) & & \text{(objective term)} \\
\text{s.t.} \quad & z_0 = \bar{z}_0, & & \text{(initial condition)} \\
& p_{\text{tool}}(z_N) = \bar{p}_N, & & \text{(terminal constraint)} \\
& z_{k+1} = F(z_k, u_k), \quad k = 0, \dots, N-1, & & \text{(dynamical conditions)} \\
& g_{\text{Path}}(z_k, u_k) \leq 0, \quad k = 0, \dots, N-1, & & \text{(path constraints)} \\
& g_{\text{Coll}}(z_k, \lambda_k, \mu_k) \leq 0, \quad k = 0, \dots, N-1. & & \text{(collision constraints)}
\end{aligned}
$$

Outline

Squared distance function: Definition



Squared distance between robot $\mathcal{R}(z)$ and obstacle $\mathcal{O}$:

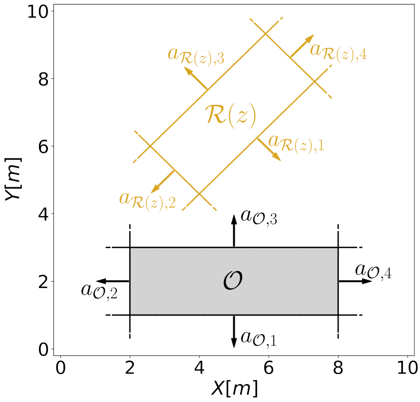
$$\begin{aligned} \Delta^2(\mathcal{R}(z), \mathcal{O}) := \min_{x,y} \quad & \|x - y\|_2^2 \\ \text{s.t.} \quad & x \in \mathcal{R}(z), \\ & y \in \mathcal{O}. \end{aligned}$$

Theoretically: Enforce

$$\Delta^2(\mathcal{R}(z_k), \mathcal{O}) \geq \Delta^2_{\min}, \quad k = 1, \ldots, N,$$

in the discrete OCP.

Squared distance function: Half-space representation

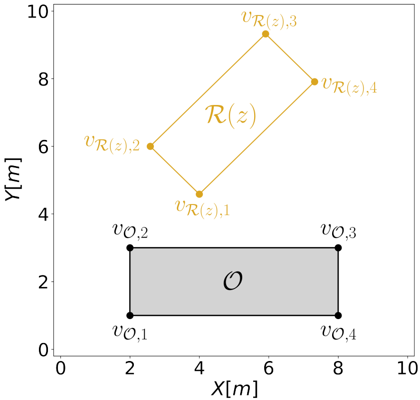


$$A_{\mathcal{R}(z)} = \begin{bmatrix} a^T_{\mathcal{R}(z),1} \\ a^T_{\mathcal{R}(z),2} \\ a^T_{\mathcal{R}(z),3} \\ a^T_{\mathcal{R}(z),4} \end{bmatrix} \in \mathbb{R}^{4\times 2}, \ b_{\mathcal{R}(z)} = \begin{bmatrix} b_{\mathcal{R}(z),1} \\ b_{\mathcal{R}(z),2} \\ b_{\mathcal{R}(z),3} \\ b_{\mathcal{R}(z),4} \end{bmatrix} \in \mathbb{R}^4,$$

$A_{\mathcal{O}} \in \mathbb{R}^{4\times 2}$ and $b_{\mathcal{O}} \in \mathbb{R}^4$ similarly structured.

$$\begin{aligned} \Delta^2(\mathcal{R}(z), \mathcal{O}) = \min_{x,y} \quad & \|x - y\|_2^2 \\ \text{s.t.} \quad & A_{\mathcal{R}(z)} x \leq b_{\mathcal{R}(z)}, \\ & A_{\mathcal{O}} y \leq b_{\mathcal{O}}. \end{aligned}$$

Squared distance function: Vertex representation



$$V_{\mathcal{R}(z)} = \begin{bmatrix} v_{\mathcal{R}(z),1} & v_{\mathcal{R}(z),2} & v_{\mathcal{R}(z),3} & v_{\mathcal{R}(z),4} \end{bmatrix} \in \mathbb{R}^{2\times 4},$$

$V_{\mathcal{O}} \in \mathbb{R}^{2\times 4}$ similarly structured.

$$\begin{aligned} \Delta^2(\mathcal{R}(z), \mathcal{O}) = \min_{\theta,\eta} \quad & \|V_{\mathcal{R}(z)}\theta - V_{\mathcal{O}}\eta\|_2^2 \\ \text{s.t.} \quad & 1^T\theta = 1,\ 1^T\eta = 1, \\ & \theta \geq 0,\ \eta \geq 0. \end{aligned}$$

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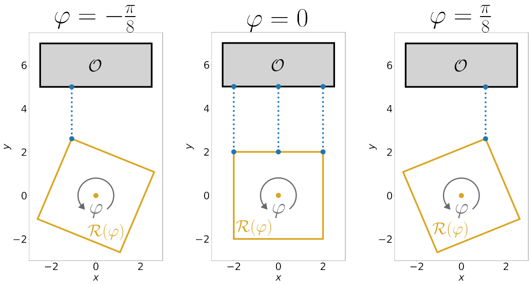
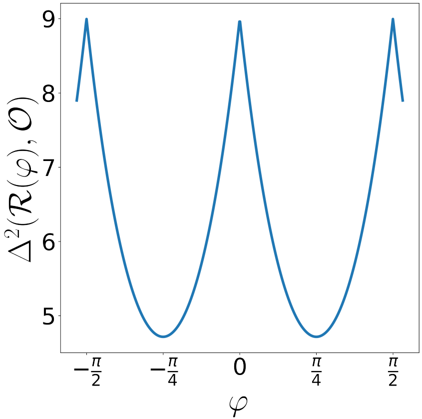
The squared distance function is not differentiable

- $\Delta^2(\mathcal{R}(\varphi), \mathcal{O})$: squared distance between square and rectangle
- square depends on orientation φ





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Resolving non-differentiability[1]: Deriving the dual problem

Primal distance problem:

$$
\begin{aligned}
\min_{x,y,w} \quad & \|w\|_2^2 \\
\text{s.t.} \quad & w = x - y, \\
& A_{\mathcal{R}(z)} x \leq b_{\mathcal{R}(z)}, \\
& A_{\mathcal{O}} y \leq b_{\mathcal{O}}.
\end{aligned}
$$

Dual function:

$$
\begin{aligned}
d_z(\nu, \lambda, \mu) &= \inf_{x,y,w} L_z(x, y, w, \nu, \lambda, \mu) \\
&= \inf_{x,y,w} \Big(\|w\|_2^2 + \nu^T (x - y - w) \\
&\quad + \lambda^T (A_{\mathcal{R}(z)} x - b_{\mathcal{R}(z)}) + \mu^T (A_{\mathcal{O}} y - b_{\mathcal{O}}) \Big)
\end{aligned}
$$

[1] X. Zhang, A. Liniger, and F. Borrelli. Optimization-based collision avoidance. *IEEE Transactions on Control Systems Technology*, 29:972- 983, 2021.

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Resolving non-differentiability[1]: Deriving the dual problem

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&= \inf_{x,y,w} \big(\|w\|_2^2 + \nu^T(x - y - w) \\
&\quad + \lambda^T(A_{\mathcal{R}(z)} x - b_{\mathcal{R}(z)}) + \mu^T(A_{\mathcal{O}} y - b_{\mathcal{O}})\big) \\
&= \inf_{w} \big(\|w\|_2^2 - \nu^T w\big) + \inf_{x} \big((\lambda^T A_{\mathcal{R}(z)} + \nu^T)x\big) \\
&\quad + \inf_{y} \big((\mu^T A_{\mathcal{O}} - \nu^T)y\big) - \lambda^T b_{\mathcal{R}(z)} - \mu^T b_{\mathcal{O}}
\end{aligned}
$$

[1] X. Zhang, A. Liniger, and F. Borrelli. Optimization-based collision avoidance. *IEEE Transactions on Control Systems Technology*, 29:972- 983, 2021.

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Resolving non-differentiability[1]: Deriving the dual problem

Dual function:

$$d_z(\nu, \lambda, \mu) = \inf_w \left(\|w\|_2^2 - \nu^T w\right) + \inf_x \left((\lambda^T A_{\mathcal{R}(z)} + \nu^T)x\right) \\ + \inf_y \left((\mu^T A_{\mathcal{O}} - \nu^T)y\right) - \lambda^T b_{\mathcal{R}(z)} - \mu^T b_{\mathcal{O}}$$

Infima values:

$$\inf_w \left(\|w\|_2^2 - \nu^T w\right) = -\frac{1}{4}\|\nu\|_2^2,$$

$$\inf_x \left((\lambda^T A_{\mathcal{R}(z)} + \nu^T)x\right)$$

$$= \begin{cases} 0, & \text{if } A_{\mathcal{R}(z)}^T \lambda + \nu = 0, \\ -\infty, & \text{if } A_{\mathcal{R}(z)}^T \lambda + \nu \neq 0. \end{cases}$$

$\inf_y \left((\mu^T A_{\mathcal{O}} - \nu^T)y\right)$ similarly.

[1] X. Zhang, A. Liniger, and F. Borrelli. Optimization-based collision avoidance. *IEEE Transactions on Control Systems Technology*, 29:972- 983, 2021.

Resolving non-differentiability[1]: Deriving the dual problem

Dual problem:

$$\begin{aligned} \max_{\nu,\lambda,\mu} \quad & -\frac{1}{4}\|\nu\|_2^2 - \lambda^T b_{\mathcal{R}(z)} - \mu^T b_{\mathcal{O}} \\ \text{s.t.} \quad & A_{\mathcal{R}(z)}^T \lambda + \nu = 0, \\ & A_{\mathcal{O}}^T \mu - \nu = 0, \\ & \lambda \geq 0, \ \mu \geq 0. \end{aligned}$$

Reduced dual problem:

$$\begin{aligned} \max_{\lambda,\mu} \quad & -\frac{1}{4}\|A_{\mathcal{O}}^T \mu\|_2^2 - \lambda^T b_{\mathcal{R}(z)} - \mu^T b_{\mathcal{O}} \\ \text{s.t.} \quad & A_{\mathcal{R}(z)}^T \lambda + A_{\mathcal{O}}^T \mu = 0, \\ & \lambda \geq 0, \ \mu \geq 0. \end{aligned}$$

[1] X. Zhang, A. Liniger, and F. Borrelli. Optimization-based collision avoidance. *IEEE Transactions on Control Systems Technology*, 29:972- 983, 2021.

Resolving non-differentiability[1]: Deducing differentiable constraints

Dual distance constraints:

$$\Delta^2(\mathcal{R}(z_k), \mathcal{O}) \geq \Delta^2_{\min}$$

$$\Delta^2(\mathcal{R}(z_k), \mathcal{O}) :=$$

$$\begin{aligned} \max_{\lambda,\mu} \quad & -\frac{1}{4}\|A^T_{\mathcal{O}}\mu\|^2_2 - \lambda^T b_{\mathcal{R}(z_k)} - \mu^T b_{\mathcal{O}} \\ \text{s.t.} \quad & A^T_{\mathcal{R}(z)}\lambda + A^T_{\mathcal{O}}\mu = 0, \\ & \lambda \geq 0, \ \mu \geq 0. \end{aligned}$$

Differentiable distance constraints:

$$-\frac{1}{4}\|A^T_{\mathcal{O}}\mu\|^2_2 - \lambda^T b_{\mathcal{R}(z_k)} - \mu^T b_{\mathcal{O}} \geq \Delta^2_{\min},$$

$$A^T_{\mathcal{R}(z)}\lambda + A^T_{\mathcal{O}}\mu = 0,$$

$$\lambda \geq 0, \ \mu \geq 0.$$

[1] X. Zhang, A. Liniger, and F. Borrelli. Optimization-based collision avoidance. *IEEE Transactions on Control Systems Technology*, 29:972- 983, 2021.

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Different possible primal formulations; and resulting dual formulations

Polytopic Robot	$\mathcal{R}(z) = \{V_{\mathcal{R}(z)}\theta \mid 1^T\theta = 1, \theta \geq 0\}$	$\mathcal{R}(z) = \{x \mid A_{\mathcal{R}(z)}x \leq b_{\mathcal{R}(z)}\}$
Polytopic Obstacles	$\mathcal{O} = \{V_{\mathcal{O}}\eta \mid 1^T\eta = 1, \eta \geq 0\}$ *(V-Pol)*[1]	$\mathcal{O} = \{y \mid A_{\mathcal{O}}y \leq b_{\mathcal{O}}\}$ *(H-Pol)*[2]

[1] New contributions

[2] X. Zhang, A. Liniger, and F. Borrelli. Optimization-based collision avoidance. *IEEE Transactions on Control Systems Technology*, 29:972- 983, 2021.

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Different possible primal formulations; and resulting dual formulations

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Ellipsoidal Obstacles	$\mathcal{O} = \{y \mid \hat{A}_{\mathcal{O}}y \preceq_{\mathcal{K}_{\text{so}}} \hat{b}_{\mathcal{O}}\}$ *(V-GI)*[1]	$\mathcal{O} = \{y \mid \hat{A}_{\mathcal{O}}y \preceq_{\mathcal{K}_{\text{so}}} \hat{b}_{\mathcal{O}}\}$ *(H-GI)*[2]

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[2] X. Zhang, A. Liniger, and F. Borrelli. Optimization-based collision avoidance. *IEEE Transactions on Control Systems Technology*, 29:972- 983, 2021.

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Polytopic Obstacles	$\mathcal{O} = \{V_{\mathcal{O}}\eta \mid 1^T\eta = 1, \eta \geq 0\}$ *(V-Pol)*[1]	$\mathcal{O} = \{y \mid A_{\mathcal{O}}y \leq b_{\mathcal{O}}\}$ *(H-Pol)*[2]
Ellipsoidal Obstacles	$\mathcal{O} = \{y \mid \hat{A}_{\mathcal{O}}y \preceq_{\mathcal{K}_{\text{so}}} \hat{b}_{\mathcal{O}}\}$ *(V-GI)*[1]	$\mathcal{O} = \{y \mid \hat{A}_{\mathcal{O}}y \preceq_{\mathcal{K}_{\text{so}}} \hat{b}_{\mathcal{O}}\}$ *(H-GI)*[2]
Ellipsoidal Obstacles	$\mathcal{O} = \{y \mid (y - c_{\mathcal{O}})^T P_{\mathcal{O}}(y - c_{\mathcal{O}}) \leq 1\}$ *(V-Sqrt)*[1], *(V-MD)*[1]	$\mathcal{O} = \{y \mid (y - c_{\mathcal{O}})^T P_{\mathcal{O}}(y - c_{\mathcal{O}}) \leq 1\}$ *(H-Sqrt)*[1], *(H-Sqrt+L)*[1], *(H-MD)*[1]

[1] New contributions

[2] X. Zhang, A. Liniger, and F. Borrelli. Optimization-based collision avoidance. *IEEE Transactions on Control Systems Technology*, 29:972- 983, 2021.

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Different possible primal formulations; and resulting dual formulations

Additional variables/constraints - Scaling w.r.t. number (#) of vertices / half-spaces:

Formulation Type	# Optimization Variables	# Inequality Constraints	# Equality Constraints
"V-"	Constant	Linear	None
"H-"	Linear	Linear	Constant / None

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1. Optimal Control Problem
2. Derivation of Collision Constraints
3. Numerical Benchmarks
4. Trajectory Planning for Manipulators: Methodology and Challenges

Computational framework

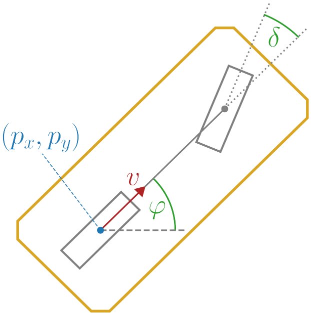
- Code is implemented in **Python**
- OCP problems are formulated via **CasADi**[1]
- Numerical solver used is **IPOPT**[2]

[1] J. A. E. Andersson, J. Gillis, G. Horn, J. B. Rawlings, and M. Diehl. CasADi - A software framework for nonlinear optimization and optimal control. *Mathematical Programming Computation*, 11:1-36, 2019.

[2] A. Wächter and L. T. Biegler. On the implementation of an interior-point filter line-search algorithm for large-scale nonlinear programming. *Mathematical Programming*, 106:25-57, 2006.

Considered robotic systems for benchmark problems

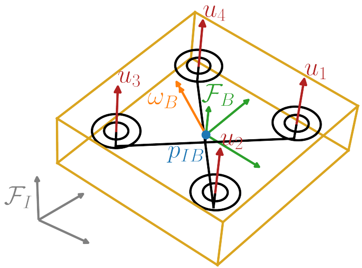
Bicycle-based Car Model:



State variables: $z = (p_x, p_y, \varphi, v, \dot{v}, \delta)$

Control variables: $u = (\ddot{v}, \dot{\delta})$

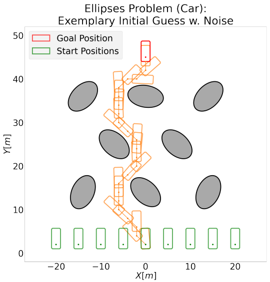
Drone Model:

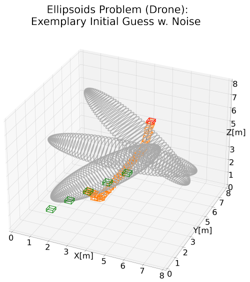


State variables: $z = (p_{IB}, \dot{p}_{IB}, q_{IB}, \omega_B)$

Control variables: $u = (u_1, u_2, u_3, u_4)$

Initial guesses generated by an A* algorithm





- A* samples through nodes in discrete space
- Next node to explore is chosen based on lowest cost
- cost $\hat{=}$ path length so far + remaining path length estimation (lower bound)

$\Rightarrow$ A* provides initial guess for positions

- Initializing orientation with noise provides a **family of initial guesses**

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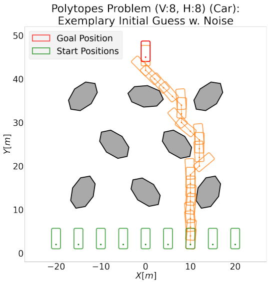
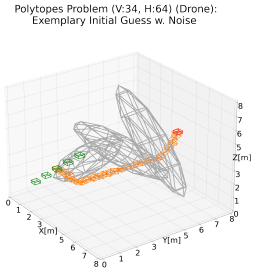
Initial guesses generated by an A* algorithm





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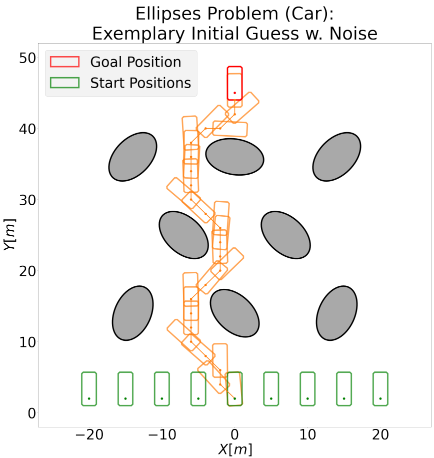
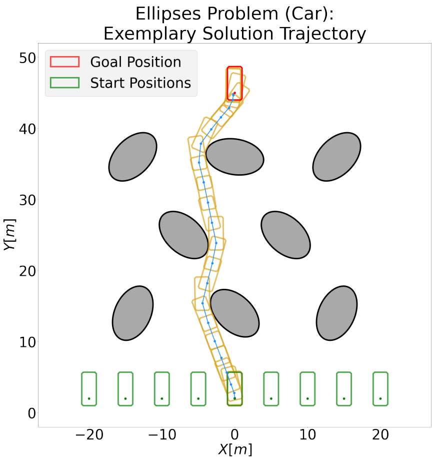
Example solutions of the benchmark problems





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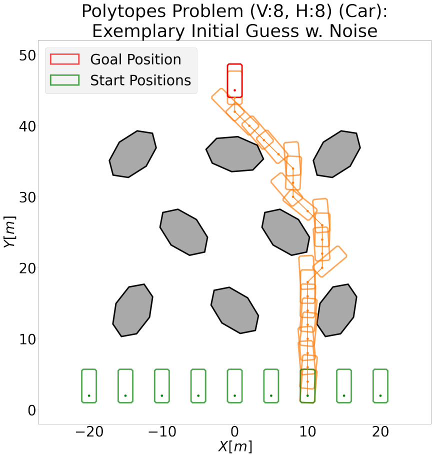
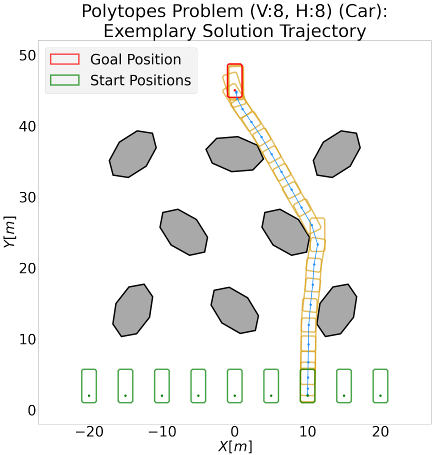
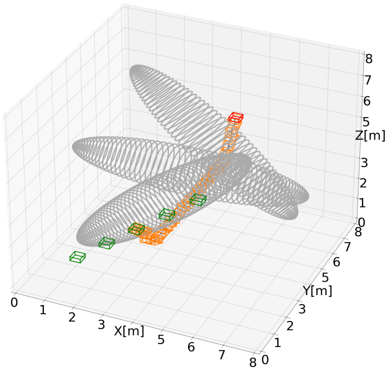


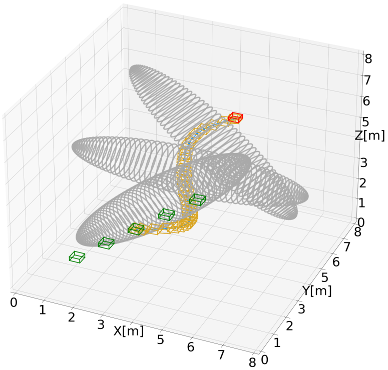


Example solutions of the benchmark problems

Ellipsoids Problem (Drone):
Exemplary Initial Guess w. Noise

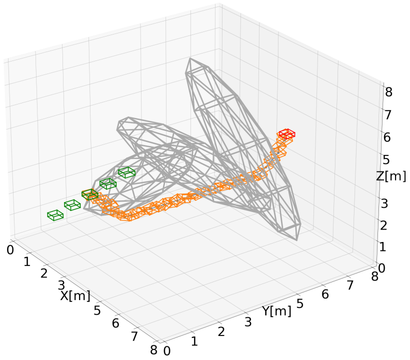


Ellipsoids Problem (Drone):
Exemplary Solution Trajectory

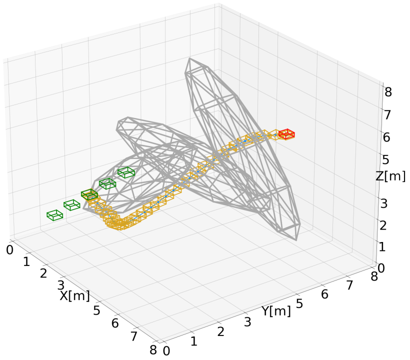


Example solutions of the benchmark problems

Polytopes Problem (V:34, H:64) (Drone):
Exemplary Initial Guess w. Noise



Polytopes Problem (V:34, H:64) (Drone):
Exemplary Solution Trajectory



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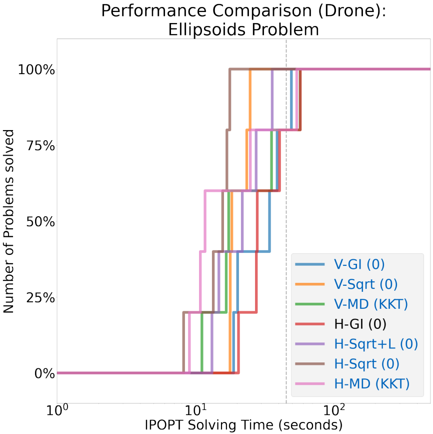
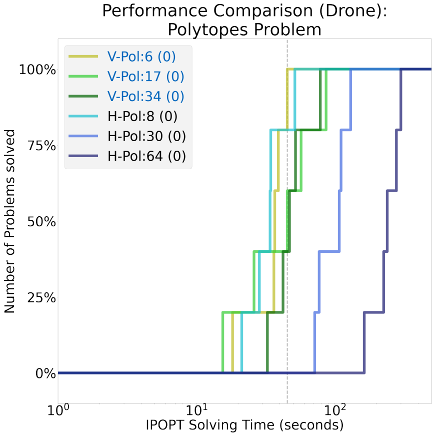
Computational performance comparison





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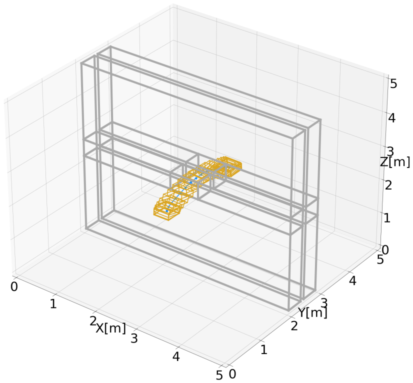
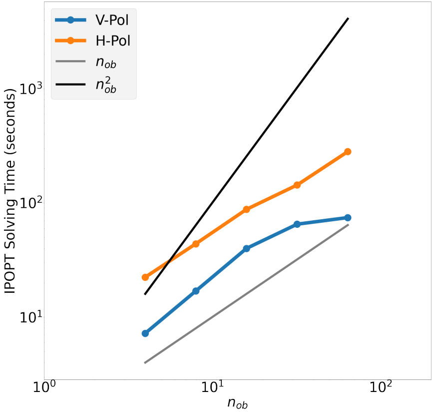
Scaling w.r.t. number of obstacles

Exemplary Problem



Computation Time according to Number of Obstacles



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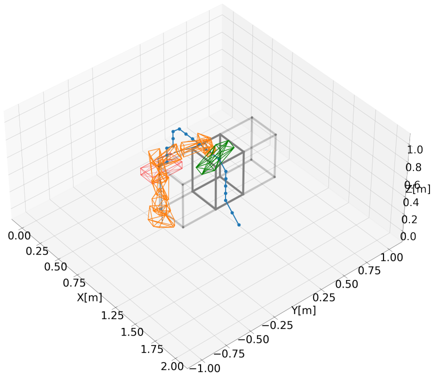
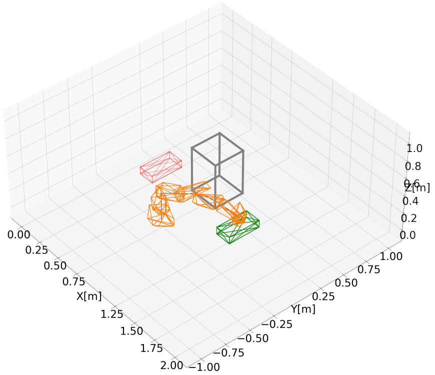
Generating initial guesses

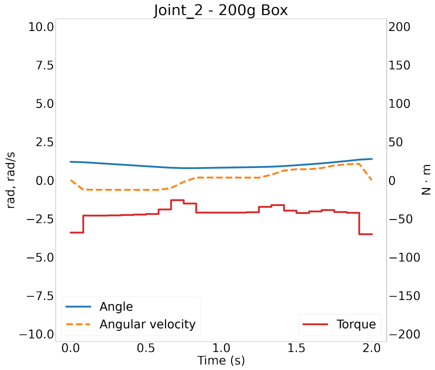
Solving Inverse Kinematics for A-star points:
Solve $p_{\text{tool}}(q) = p_{\text{Astar},\,i}$

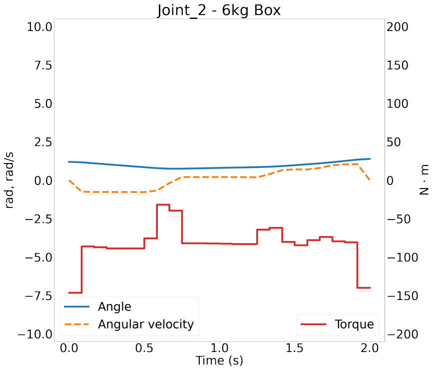


Solving with Full Dynamic:
Box weighs 200g



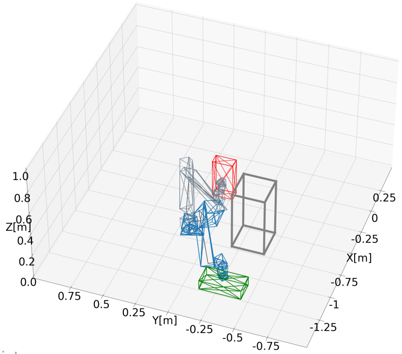
Using the "full" dynamic yields realistic controls



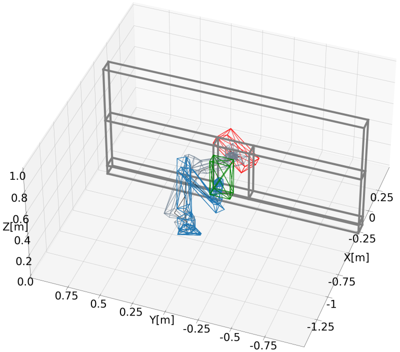


Motivational example revisited: Each manipulator has four phases

Ur5 - Phase 1
Checking Collision for Parts: 3-8
Checking 10 Self-Collisions



Ur5 - Phase 2
Checking Collision for Parts: 4-8
Checking 0 Self-Collisions



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Summary

- Using problem specific formulations increases computational performance
  - $\rightarrow$ Vertex representation for polytopes
  - $\rightarrow$ Quadratic inequality representation for ellipsoids
- Representing polytopes through vertices increases computational performance especially for intricate polytopes
- Considered constraints are applicable together with different robotic systems:
  - $\rightarrow$ Car model
  - $\rightarrow$ Drone model
  - $\rightarrow$ Manipulator models

References

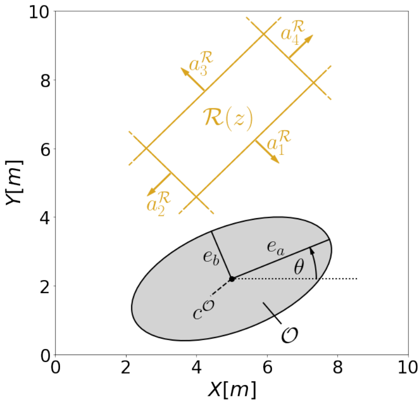
[1] J. A. E. Andersson, J. Gillis, G. Horn, J. B. Rawlings, and M. Diehl. **CasADi – A software framework for nonlinear optimization and optimal control.** *Mathematical Programming Computation*, 11:1–36, 2019.

[2] L. M. G. Johannessen, M. H. Arbo, and J. T. Gravdahl. **Robot dynamics with URDF & CasADi.** In *2019 7th International Conference on Control, Mechatronics and Automation*. IEEE, 2019.

[3] A. Wächter and L. T. Biegler. **On the implementation of an interior-point filter line-search algorithm for large-scale nonlinear programming.** *Mathematical Programming*, 106:25–57, 2006.

[4] X. Zhang, A. Liniger, and F. Borrelli. **Optimization-based collision avoidance.** *IEEE Transactions on Control Systems Technology*, 29:972–983, 2021.

Representation for ellipsoidal obstacles



Let

$$P_{\mathcal{O}} = R(\theta) \begin{pmatrix} \frac{1}{e_a^2} & 0 \\ 0 & \frac{1}{e_b^2} \end{pmatrix} R(\theta)^T$$

where

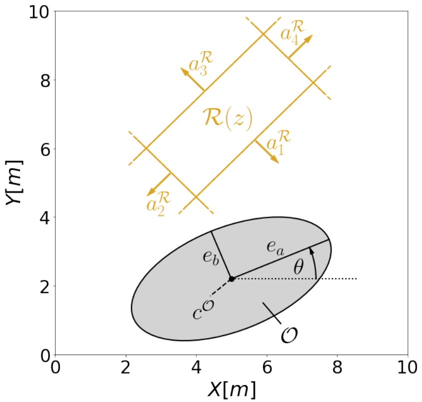
$$R(\theta) = \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix}$$

Then

$$\mathcal{O} = \{y \mid (y - c_{\mathcal{O}})^T P_{\mathcal{O}} (y - c_{\mathcal{O}}) \leq 1\}$$

and e.g. $\mathcal{R}(z) = \{x \mid A_{\mathcal{R}(z)} x \leq b_{\mathcal{R}(z)}\}$

Representation for ellipsoidal obstacles



Possible distance function:

$$\Delta^2(\mathcal{R}(z), \mathcal{O}) = \min_{x,y} \quad \|x - y\|_2^2$$

$$\text{s.t.} \quad A_{\mathcal{R}(z)} x \leq b_{\mathcal{R}(z)},$$

$$(y - c_{\mathcal{O}})^T P_{\mathcal{O}} (y - c_{\mathcal{O}}) \leq 1.$$

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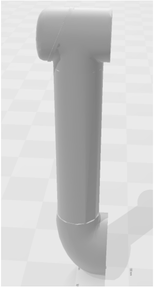
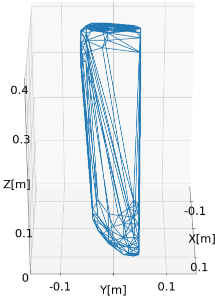
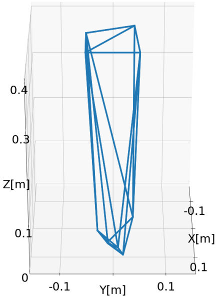
Representing manipulator components as convex sets







Urdf files come with .stl meshes.

By taking the convex hull of the .stl mesh we get a convex set.

Randomly sampling subsets of vertices and taking the best volume match.

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IPOPT warm start behaviour (V-Pol)