Exercises 1: Review of linear systems theory

(Thursday 29.10.2015 at 15:00 in Room SR 00 014)

Dr. Jörg Fischer and Prof. Dr. Moritz Diehl

1. A LTI-System is described by the ODE $2\ddot{y} + 6\dot{y} + 8y = 27\ddot{u}$.
   What is the characteristic polynomial $p_A(\lambda)$?

(a) ☐ $2\lambda^2 - 6\lambda - 8$	(b) ☐ $2\lambda^3+6\lambda^2+8\lambda-27$	(c) ☐ $\lambda^2 + 3\lambda + 4$	(d) ☐ $\lambda^2 - 6\lambda + 2$

2. Determine the poles of the system that is described by the following ODE:
   $3\ddot{y} - 15\dot{y} + 18y = 4\dot{u} - 5u$.

(a) ☐ $(2, 3)$	(b) ☐ $(6, 9)$	(c) ☐ $(-6, -9)$	(d) ☐ $(-2, -3)$

3. What is the solution $x(t)$ of the ODE $\dot{x}(t) = u(t) - x(t)$, with an initial state $x(0) = 1$?

(a) ☐ $e^{-t} + e^{t} \int_0^t u(\tau)d\tau$	(b) ☐ $e^{-t} + e^{-t} \int_0^t e^{\tau} u(\tau)d\tau$
(c) ☐ $u(t) + \int_0^t x(\tau)d\tau$	(d) ☐ $1 + e^{u(t)}$

4. The Flückiger See has a temperature T. During the day, solar radiation heat Q warms the lake. At the same time, the cool lake soil (constant temperature T_0) extracts heat from the lake. The temperature of the lake during the day is approximately described by the ODE $\dot{T} = k_1 \cdot Q - k_2 \cdot (T - T_0)$. What is the steady state temperature T_{ss}, that sets in due to a constant heat input Q_{ss}?

(a) ☐ $\frac{Q_{\text{ss}} + k_1 \cdot T_0}{k_2}$	(b) ☐ $k_1 \cdot Q_{\text{ss}} + T_0 \cdot k_2$	(c) ☐ $\frac{k_2 \cdot Q_{\text{ss}} + T_0}{k_1}$	(d) ☐ $\frac{k_1 \cdot Q_{\text{ss}}}{k_2} + T_0$

5. An electrical oscillator is described by the ODEs $\frac{\mathrm{d}v_C}{\mathrm{d}t} = \frac{i}{C}$ and $\frac{\mathrm{d}i}{\mathrm{d}t} = \frac{1}{L}(v_E - iR - v_C)$. Take $x = \begin{bmatrix} i \\ v_C \end{bmatrix}$ as a state and $u = v_E$ as an input. Transform the system into the form $\dot{x} = Ax + Bu$. Specify A and B.

(a) ☐ $A = \begin{bmatrix} 0 & 1/C \\ -1/L & -R/L \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1/L \end{bmatrix}$	(b) ☐ $A = \begin{bmatrix} -1/L & 0 \\ -R/L & 1/C \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1/L \end{bmatrix}$
(c) ☐ $A = \begin{bmatrix} -R/L & -1/L \\ 1/C & 0 \end{bmatrix}, B = \begin{bmatrix} 1/L \\ 0 \end{bmatrix}$	(d) ☐ $A = \begin{bmatrix} 1/C & -R/L \\ 0 & -1/L \end{bmatrix}, B = \begin{bmatrix} 1/L \\ 0 \end{bmatrix}$

6. A farm tractor is described by the state equations $\dot{x}_1 = V\cos(x_2)$ and $\dot{x}_2 = \frac{V}{L}\tan(u)$. In these equations, x_1 is the X-coordinate and x_2 is the angel of orientation of the tractor. The Y-coordinate is not of interest in this example. Linearise the system in the equilibrium point $u_{\text{ss}} = 0$ and $x_{\text{ss}} = \begin{bmatrix} 0 & \frac{\pi}{2} \end{bmatrix}^{\mathrm{T}}$. Transform the linearised system in the form $\dot{x} = Ax + Bu$, and specify A and B.

(a) ☐ $A = \begin{bmatrix} 0 & -V \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ V/L \end{bmatrix}$	(b) ☐ $A = \begin{bmatrix} V/L & 1 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ V\cos(1) \end{bmatrix}$
(c) ☐ $A = \begin{bmatrix} V/L & 0 \\ o & V \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$	(d) ☐ $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 \\ -V/L \end{bmatrix}$

7. A LTI-system is described by the ODE $6\ddot{y} + 3\dot{y} + y = 4\ddot{u} + 2u$. What is the corresponding transfer function $G(s)$?

(a) ☐ $\frac{4s^2+2}{6s^2+3s+1}$	(b) ☐ $6s^2 + 3s + 1$	(c) ☐ $\frac{2s}{3s+1}$	(d) ☐ $\frac{6s^2+3s+1}{4s+2}$

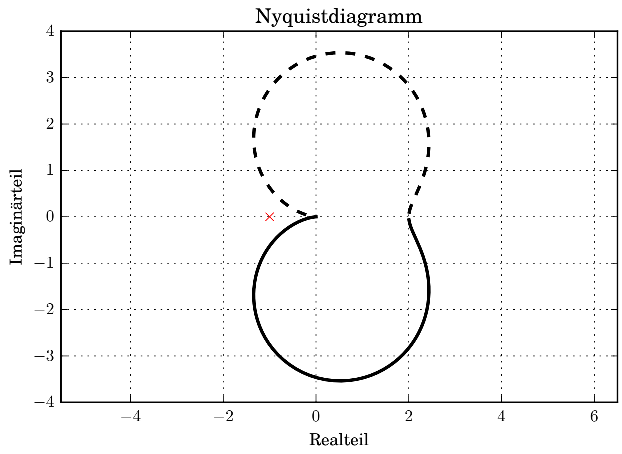
8. A LTI-System is described by the state space model $\dot{x} = Ax+Bu$, $y = Cx+Du$, where $A = \begin{bmatrix} 0 & 2 \\ -1 & -2 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 1 \end{bmatrix}$, $D = \begin{bmatrix} 0 \end{bmatrix}$. What is the corresponding transfer function $G(s)$ of the system?

(a) ☐ $\frac{2s+1}{s+1}$	(b) ☐ $\frac{s+4}{s^2+2s+2}$	(c) ☐ $\frac{s+2}{s+1}$	(d) ☐ $\frac{2s+1}{(s+1)^2}$

9. Which LTI-system is described by the transfer function $G(s) = \frac{s^2+5s-1}{2s^2+3}$?

(a) ☐ $2\ddot{y} + 3y = \ddot{u} + 5\dot{u} - u$	(b) ☐ $\ddot{y} + 5\dot{y} - y = 2\ddot{u} + 3u$
(c) ☐ $2\dot{y} + 3 = \dot{u} + 5u - 1$	(d) ☐ $\dot{y} + 5y - 1 = 2\dot{u} + 3$

10. Consider the following Nyquist diagram.

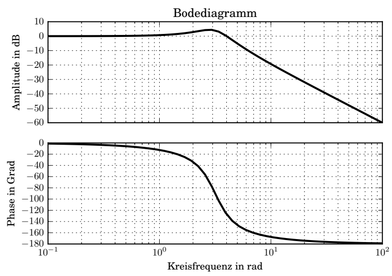


What is the corresponding transfer function $G(s)$?

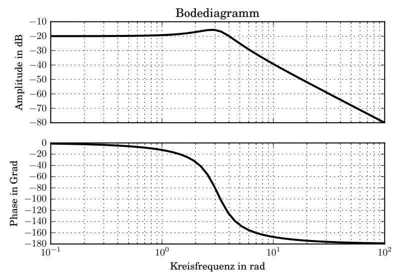
(a) ☐ $\frac{1}{s+2}$	(b) ☐ $\frac{6}{s^2+s+3}$	(c) ☐ $\frac{s}{s^2+s+2}$	(d) ☐ $\frac{1}{s^2+s+1}$

11. Consider the transfer function $G(s) = \frac{1}{s^2+2s+10}$, what is the corresponding Bode diagram?

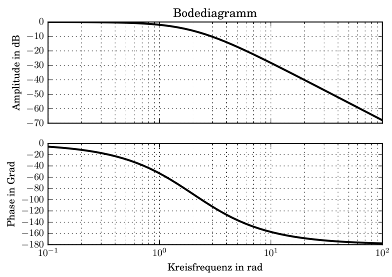
(a) ☐



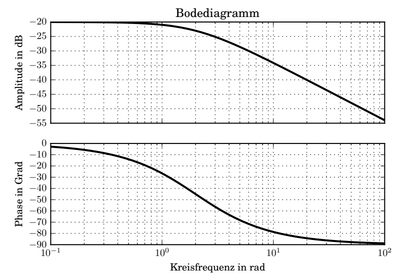
(b) ☐



(c) ☐



(d) ☐



12. A system has a step response $h(t) = 1 + e^{-t}$ (for $t \geq 0$, and $h(t) = 0$ otherwise). What is the impulse response $g(t)$ (for $t \geq 0$)?

(a) ☐ $2\delta(t) - e^{-t}$	(b) ☐ $-e^{-t}$	(c) ☐ $\delta(t) - e^{-t}$	(d) ☐ $-te^{-t}$

13. Which of the following systems, described by their respective step responses, is not BIBO stable?

(a) ☐ $\cos(t)$	(b) ☐ $(1+t)^{-2}$	(c) ☐ $\sin(t)e^{-t}$	(d) ☐ $1 - e^{-t}$

14. Which of the following LTI-SISO systems is not BIBO-stable ?

(a) ☐ $\ddot{y} + \dot{y} + y = \ddot{u} + u$	(b) ☐ $\dot{y} = \dot{u} + u$	(c) ☐ $\ddot{y} + y = \dot{u} - u$	(d) ☐ $\dot{y} + y = \dot{u}$