Systems and Control II (SC2)
Albert-Ludwigs-Universität Freiburg – Wintersemester 2015/2016

Exercises 13: LQR, state space regulation and output feedback control

(Thursday 04.02.2016 at 15:00 in Room SR 00 014)

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A magnetic ball with mass M is suspended in midair by an electromagnet, that exerts a magnetic force on the ball in order to compensate for the gravitational force working on it, as shown in Fig. 1. The magnetic force is induced by the current $i(t)$ going through the electromagnet.

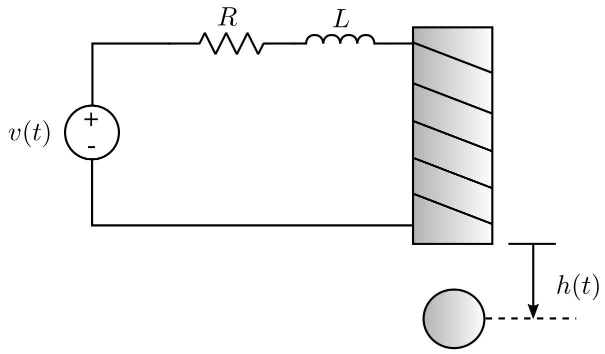

Figure 1: Set-up of the magnetic suspension system.

The state equations of the system are given by

$$M\frac{\mathrm{d}^2h(t)}{\mathrm{d}t^2} = Mg - \frac{Ki(t)^2}{h(t)}$$
$$v(t) = L\frac{\mathrm{d}i(t)}{\mathrm{d}t} + i(t)R\,,$$

where $h(t)$ is the distance of the ball with respect to the electromagnet, K a constant that determines the magnetic force, $v(t)$ the applied voltage and where L and R are respectively an inductance and a resistance. The parameter values are $M = 0.05\mathrm{kg}$, $K = 0.0001$, $L = 0.01\mathrm{H}$ and $R = 1\Omega$. The system is linearised around the point $h_{\mathrm{nom}} = 0.01\mathrm{m}$, with a nominal current $i_{\mathrm{nom}} = 7\mathrm{A}$. The states of the system are chosen as $\mathbf{x}(t) = \begin{bmatrix}\Delta h(t) & \Delta\dot{h}(t) & \Delta i(t)\end{bmatrix}^T$, with $\Delta h(t) = h(t) - h_{\mathrm{nom}}$, and $\Delta i(t) = i(t) - i_{\mathrm{nom}}$. The input $u(t)$ of the linearised system is the applied voltage deviation $\Delta v(t) = v(t) - v_{\mathrm{nom}}$, and the output $y(t)$ is the height deviation of the ball $\Delta h(t)$. The dynamics of this system are described by the following state space model:

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} 0 & 1 & 0 \\ 980 & 0 & -2.8 \\ 0 & 0 & -100 \end{bmatrix}\mathbf{x}(t) + \begin{bmatrix} 0 \\ 0 \\ 100 \end{bmatrix}u(t)$$
$$y(t) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}\mathbf{x}(t)\,.$$

The different variables are subject to following limitations:

$$|\Delta v(t)| \leq 13\mathrm{V}$$
$$|\Delta h(t)| \leq 0.005\mathrm{m}$$
$$|\Delta i(t)| \leq 7\mathrm{A}\,.$$

There is no explicit limitation on $\Delta\dot{h}(t)$. The goal of this exercise is to design an optimal controller that is able to regulate and stabilize the output of this system around a certain constant reference height deviation $r \neq 0$, by using state space controller design methods.

1. **Design of an LQR-controller**
   (a) Is the system in this set-up BIBO- and/or Lyapunov stable?
   (b) Is the system controllable?
   (c) Assume that the system states are all measurable. Formulate the LQ control design task for an optimal full-state feedback controller $\mathbf{K}$ and choose appropriate weighting matrices $\mathbf{Q}_{\mathrm{x}}$ and $\mathbf{Q}_{\mathrm{u}}$. Consider the evolution of the state $\Delta\dot{h}(t)$ as unimportant when choosing $\mathbf{Q}_{\mathrm{x}}$.
   (d) (MATLAB) Compute the LQR-controller that minimizes the cost function $J(\mathbf{x}_0, u(t))$ formulated in (1.c), by using the function `lqr`.

(e) (MATLAB) Write down the closed-loop state equation for this system and simulate the natural response.

2. **Design of the feedforward controller for regulation**

   (a) Design a feedforward controller for a constant reference input $r \neq 0$. Evaluate the feedforward matrix in MATLAB.

   (b) Write down the closed-loop state equation for the feedback system that combines the LQR state feedback controller with the feedforward controller designed in (2.a).

3. **Design of the Luenberger observer**

   (a) Assume now that the states of the system cannot be directly measured. Only the output $y(t) = \Delta h(t)$ is measured. Is the system observable?

   (b) Determine the appropriate pole locations for a Luenberger observer that feeds back an estimate $\hat{\mathbf{x}}(t)$ of the actual system state $\mathbf{x}(t)$ to the state feedback controller computed in (1.d).

   (c) (MATLAB) Compute the Luenberger gain $\mathbf{L}$ that places the observer poles in their desired locations. Do we need to iterate on the state feedback controller design?

   (d) Write down the state equation for the closed-loop system with feedback and feedforward controller as well as with the state estimator. Is this closed-loop system stable?

   (e) (MATLAB) Simulate with the function `lsim` the step response of the regulating closed-loop system with and without the observer and compare. The reference input $r = 0.002\text{m}$. In the simulation, set the initial state of the system $\mathbf{x}_0 = \begin{bmatrix} -0.0025\text{m} & 0 & 1\text{A} \end{bmatrix}^T$, and set the observer initial state $\hat{\mathbf{x}}_0 = 0$. What would happen if we would set these initial states equal?