Systems and Control II (SC2)
Albert-Ludwigs-Universität Freiburg – Wintersemester 2015/2016

Exercises 14: Course revision and exam preparation

(Thursday 11.02.2016 at 15:00 in Room SR 00 014)

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1. Consider a process plant that is described by the nominal transfer function

$$\hat{G}(s) = \frac{(3-s)}{(s+1)(s+5)} \ .$$

with a multiplicative model uncertainty that has as an upper bound

$$\bar{\Delta}_{\mathrm{M}}(s) = 0.1 \cdot \frac{s+1}{\frac{s}{100}+1} \ .$$

We want to design a robust and stable discrete-time controller $K_{\mathrm{z}}(z)$ with internal model control (IMC). The goal is to achieve the maximal possible bandwidth for the closed-loop system, while still guaranteeing robust stability. The controller design is done in the continuous-time domain. Afterwards the controller is digitized.

(a) Is the nominal plant model $\hat{G}(s)$ minimum-phase?

(b) Determine the ideal IMC-controller $K^*_{\mathrm{IMC}}(s)$ that minimizes the integral square error (ISE) for step reference inputs. Factorize $\hat{G}(s)$ if necessary.

(c) We now add a filter $V_{\mathrm{f}}(s) = \frac{1}{(Ts+1)^n}$ in order to make the controller $K_{\mathrm{IMC}}(s)$ proper. Choose a filter of the lowest possible order to achieve this. Determine a lower limit for the filter time constant T, by taking into consideration the condition for robust stability.
Hint: Draw the Bode-diagram of the robustness limit, computed with the upper bound of the multiplicative uncertainty $\bar{\Delta}_{\mathrm{M}}(s)$.

(d) Determine the transfer function of the overall controller $K(s)$.

(e) Determine the discrete equivalent $K_{\mathrm{z}}(z)$ of the overall controller $K(s)$ designed above. Use the zero-pole matching method. Use the sampling period $T_{\mathrm{p}} = 25\mathrm{ms}$

2. Consider a process plant described by the transfer function

$$G(s) = \frac{e^{-T_{\mathrm{d}}s}}{(5s+1)(2s+1)} \ .$$

Design a Smith Predictor that realizes a feedback loop with a settling time $T_{\mathrm{s}} \leq T_{\mathrm{d}} + 3$ sec, and with an overshoot $M_{\mathrm{p}} \leq 10\%$. The dead time $T_{\mathrm{d}} = 42$ sec.

(a) Design a lead compensator $K_{\mathrm{R}}(s) = k_{\mathrm{D}} \frac{T_{\mathrm{D}}s+1}{T'_{\mathrm{D}}s+1}$ for the rational part $G_{\mathrm{R}}(s)$ of the process plant, using the root locus method.

i. What are the desired pole locations of the closed-loop system poles?

ii. Determine the lead zero $\frac{1}{T_{\mathrm{D}}}$ in order to cancel the slowest process pole.

iii. Determine the lead pole $\frac{1}{T'_{\mathrm{D}}}$ in order to fix the real part of the closed-loop poles.

iv. Determine the gain k_{D} in order to place the closed-loop poles in their desired locations.

(b) Compute the resulting closed-loop transfer function $G_{\mathrm{r}}(s)$. Assume that the model of the plant is perfect.

(c) What is the steady state error of the closed-loop system for a step input? And for a ramp input?

(d) Design a prefilter feedforward controller $K_{\mathrm{ff,p}}(s)$ for a servo application, that minimizes the ISE for step reference inputs. The prefilter should reduce the steady state error for a ramp input to $e_{\mathrm{ss}} \leq 0.1$. Check if the dynamic requirements are still met.

3. A process plant $G(s)$ can be divided into two partial processes $G_1(s)$ and $G_2(s)$ in series. The transfer functions of these partial processes are given by

$$G_1(s) = \frac{3}{(2s+1)(4s+1)} \quad \text{and} \quad G_2(s) = \frac{4}{15s+1} \ .$$

Design a cascaded controller that realizes a closed-loop system with a bandwidth $\omega_{\mathrm{BW,cl}} \approx 0.04 \frac{\mathrm{rad}}{\mathrm{s}}$.

(a) Design a causal controller $K_1(s)$ with internal model control (IMC) for the inner loop process $G_1(s)$. Minimize the Integral Square Error (ISE). Make sure that the inner loop bandwidth is approximately $\omega_{\mathrm{BW},1} \approx 0.25 \frac{\mathrm{rad}}{\mathrm{s}}$.

(b) Approximate the obtained inner closed-loop system by a first order system $G_{\mathrm{a}}(s) = \frac{k_{\mathrm{a}}}{T_{\mathrm{a}}s+1}$.

(c) Design a realizable controller $K_2(s)$ with internal model control (and with minimal ISE) for the overall process $G_{\mathrm{a}}(s)G_2(s)$. Make sure that the bandwidth requirement is met.

(d) Evaluate the validity of the approximation of the inner closed-loop system, computed in 3b.

4. Consider the following system, of which the state space model is decomposed into Kalman form:

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} 2 & -3 & 0 \\ 0 & -1 & 0 \\ 2.8 & 0 & -2 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} \sqrt{2} \\ 0 \\ -2 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{bmatrix} \mathbf{x}(t) \; .$$

(a) Evaluate the controllability and observability of the different states.

(b) Is the system controllable? If not, is it stabilizable? Is the system observable? (If not, is it detectable?)

(c) Compute the transfer function of this system. Also, determine that state space model that is a minimal realization of this transfer function.

(d) Given the state space model obtained in 4c, determine the desired pole locations $\lambda_{\mathrm{cl,i}}$, so that the closed-loop system has a settling time $T_{\mathrm{s}} \leq 0.3$ sec.

(e) If the system is *at least* stabilizable and detectable, determine by 'comparison of coefficients' the feedback controller matrix $\mathbf{K}$ that places the closed-loop poles at the desired locations $\lambda_{\mathrm{cl,i}}$.

Peak time T_{m}	$\frac{\pi}{\omega_0\sqrt{1-\zeta^2}}$
Rise time T_{r}	$\frac{1.8}{\omega_0}$
Settling time $T_{5\%}$	$\frac{3}{\zeta\omega_0}$
Settling time $T_{2\%}$	$\frac{4.5}{\zeta\omega_0}$

ϕ_ζ	ζ	Δh
66°	0.4	25%
54°	0.58	10%
45°	0.7	5%
37°	0.8	2%

Table 1: Dynamic behaviour heuristics of a second order system with complex conjugate poles $\zeta\omega_0 \pm j\omega_0\sqrt{1-\zeta^2}$.

Rise time T_{r}	$\frac{2.2}{\lvert s_1 \rvert}$
Settling time $T_{5\%}$	$\frac{3}{\lvert s_1 \rvert}$

Table 2: Dynamic behaviour heuristics of a first order system.

Rise time T_{r}	$\frac{3.36}{\lvert s_{1/2} \rvert}$
Settling time $T_{5\%}$	$\frac{4.8}{\lvert s_{1/2} \rvert}$

Table 3: Dynamic behaviour heuristics of a second order system with a double negative real pole.