Systems and Control 2 (SC2)
Albert-Ludwigs-Universität Freiburg – Wintersemester 2015/2016

Exercises 2: Review of control theory - Loopshaping, BIBO-stability, robust stability

(Thursday 05.11.2015 at 15:00 in Room SR 00 014)

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3. The DC motor drives different loads, that not only operate as 'inputs' to the system, but that also influence the inertial moment J of the motor shaft. The change of J changes the dynamic behavior of the system and hence leads to different transfer function models of the closed loop system. We want to analyze if the designed controller robustly stabilizes the DC motor system, under consideration of all expected load conditions. The load has a variation span of $J \in [0.7, 1.2]\,\mathrm{kg}\cdot\mathrm{m}^2$. Inspect whether the controller $K(s)$ designed for the nominal inertial moment ($\hat{J} = 1\mathrm{kg}\cdot\mathrm{m}^2$) robustly stabilizes the closed loop system.

   (a) Find the upper bound deviation $\bar{\Delta}_{\mathrm{A}}(j\omega)$ and the corresponding inertial moment $\bar{J}$.

   (b) Plot the upper bound $\bar{\Delta}_{\mathrm{A}}(j\omega)$ together with the robust stability limit. Is the closed loop robustly BIBO-stable?

   SOLUTION:

   (a) The actual transfer function of the DC motor takes the form

$$G(s) = \hat{G}(s) + \Delta_{\mathrm{A}}(s),$$

where $\hat{G}(s)$ is the nominal model of the motor, and $\Delta_{\mathrm{A}}(s)$ is the deviation with respect to the actual transfer function. This deviation has an upper bound, for which holds that

$$|\Delta_{\mathrm{A}}(s)| \leq \bar{\Delta}_{\mathrm{A}}(s)$$

for every possible value of J. Since we have an explicit expression for both $G(s)$ and $\hat{G}(s)$, we can write the deviation as

$$\begin{aligned}\Delta_{\mathrm{A}}(s) &= G(s) - \hat{G}(s) \\ &= \frac{1}{s(Js+1)} - \frac{1}{s(\hat{J}s+1)} \\ &= \frac{(\hat{J}-J)}{(Js+1)(\hat{J}s+1)} \quad .\end{aligned}$$

Of special interest for robust stability, is the frequency response of the upper bound $\bar{\Delta}_{\mathrm{A}}(j\omega)$. This frequency response can be found by examining the magnitude bode plot of the deviation $\Delta_{\mathrm{A}}(j\omega)$ for the whole range of J, as shown in Fig. 1. The

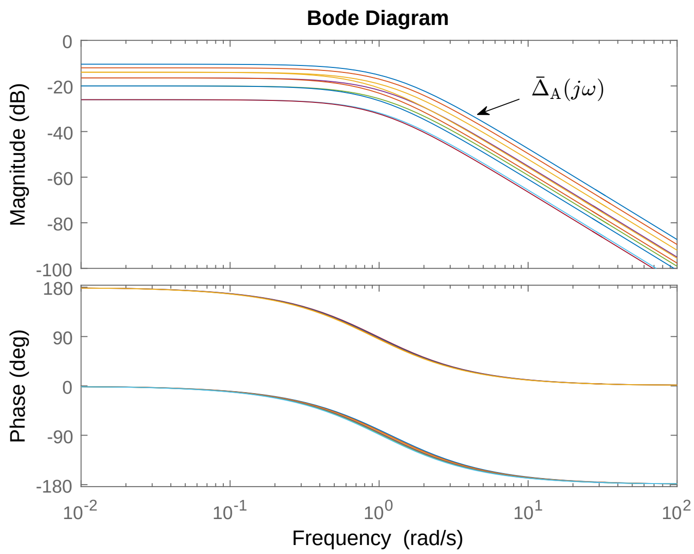

Figure 1: Deviation $\Delta_{\mathrm{A}}(j\omega)$ for $J \in [0.7, 1.2]\,\mathrm{kg}\cdot\mathrm{m}^2$

upper bound $\bar{\Delta}_{\mathrm{A}}(j\omega)$ corresponds, for all frequencies, to the lowest value of the inertia moment ($\bar{J} = 0.7$).
This can also be seen by analysing the transfer function of the deviation $\Delta_{\mathrm{A}}(s)$. The DC-gain $\Delta_{\mathrm{A}}(0) = |\hat{J} - J|$ is in this case maximal for $J = 0.7$. Furthermore, for the same value of J, the bandwidth of the deviation ($\omega = \frac{1}{J}$) is maximal, thereby ensuring that the deviation for $\bar{J} = 0.7$ is the upper bound for all frequencies.

(b) The condition for robust stability of the closed loop, given a certain upper bound deviation, implies that

$$\bar{\Delta}_{\mathrm{A}}(j\omega) < \left| \frac{1 + \hat{G}(j\omega)K(j\omega)}{K(j\omega)} \right| \tag{1}$$

for all frequencies. Fig. 2 shows the magnitude bode plot of the right hand side of the stability condition (1), together with the magnitude bode plot of the upper bound $\bar{\Delta}_{\mathrm{A}}(j\omega)$.

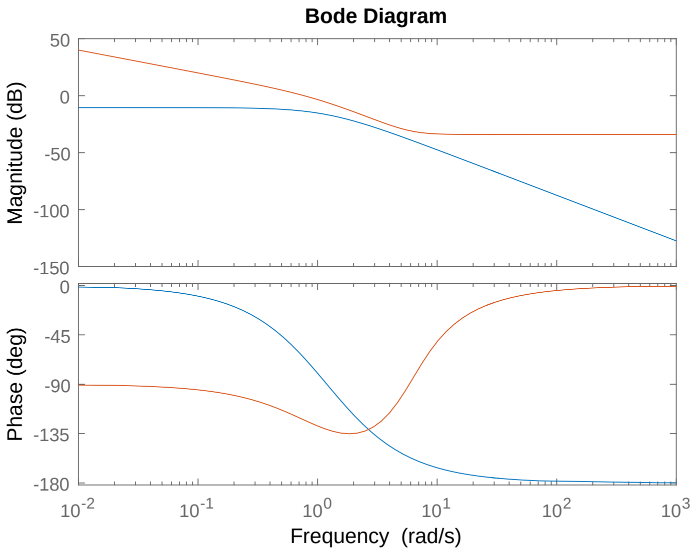

Figure 2: Bode diagram of the upper bound $\bar{\Delta}_{\mathrm{A}}$ and the stability limit.

The upper bound meets (1), therefore the closed-loop system is stable in a robust way for the given variation of J and the designed controller $K(s)$.