Systems and Control 2 (SC2)
Albert-Ludwigs-Universität Freiburg – Wintersemester 2015/2016

Exercises 4: Root locus method for controller design
(Thursday 19.11.2015 at 15:00 in Room SR 00 014)

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1. Sketch the root locus of the following systems. Verify your sketches by using MATLAB.

   (a) $G(s) = \dfrac{s+1}{s^2}$

   (b) $G(s) = \dfrac{s+1}{s^2(s+4)}$

   (c) $G(s) = \dfrac{1}{s(s+4-4j)(s+4+4j)}$

2. Design a controller for a DC-motor, so that the closed-loop system has an overshoot of no more than 20% and a rise time $T_\mathrm{r} < 0.3$ sec. Use the root locus method to evaluate different controller types and to tune the parameters of the appropriate controller. The DC-motor can be approximately described by

$$G(s) = \frac{1}{s(s+1)} \quad .$$

   (a) Translate the design objectives into specifications for the dominant poles of the closed-loop system, using the dynamic behaviour heuristics in Table 1.

   (b) Consider a proportional controller $K(s) = K_\mathrm{p}$. Sketch the root locus of the closed-loop system with respect to K_p. Which value of K_p allows us to meet the design objectives?

   (c) Consider a PD-controller $D(s) = K_\mathrm{p}(s + z_1)$. Choose the parameter z_1 so that the design objectives can be met.
   Hint: Use the MATLAB-function rltool.

   (d) In order to attain a realizable system, as well as to achieve acceptable noise suppression, we must convert the PD-controller into a lead compensator $D_{\mathrm{c1}}(s) = K_\mathrm{p}\frac{s+z_1}{s+p_1}$. Choose a suitable value for the parameter p_1 and iteratively adapt the value of z_1 so that the design objectives can be met by an appropriate choice of K_p.

   (e) An additional control objective is to achieve a steady state error of 0.01 for a ramp input. Does the closed-loop system with the lead compensator $D_{\mathrm{c1}}(s)$ designed in (d), meet this objective?

   (f) We expand the controller with a lag compensator $D_{\mathrm{c2}}(s) = \frac{s+z_2}{s+p_2}$, resulting in an overall controller $K(s) = D_{\mathrm{c2}}(s)D_{\mathrm{c1}}(s)$. Design the lag compensator in order to attain the desired steady state error, without influencing the dynamics of the lead compensator $D_{\mathrm{c1}}(s)$.

Peak time T_m	$\frac{\pi}{\omega_0\sqrt{1-\zeta^2}}$
Rise time T_r	$\frac{1.8}{\omega_0}$
Settling time $T_{5\%}$	$\frac{3}{\zeta\omega_0}$
Settling time $T_{2\%}$	$\frac{4.5}{\zeta\omega_0}$

ϕ_ζ	ζ	Δh
$66°$	0.4	25%
$54°$	0.58	10%
$45°$	0.7	5%
$37°$	0.8	2%

Table 1: Dynamic behaviour heuristics of a second order system with complex conjugate poles $\zeta\omega_0 \pm j\omega_0\sqrt{1-\zeta^2}$.